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A new Monte Carlo study of evolution equation with coherence

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ABSTRACT

We solve CCFM evolution equation numerically using the CohRad program based on Monte Carlo methods. We discuss the effects of removing soft emissions and non-Sudakov form factor by comparing the obtained distributions as functions of accumulated transverse momenta or fractions of proton's longitudinal momenta. We also compare the solution of the CCFM with the DGLAP equation in the gluonic channel. Finally, we analyze the infrared behaviour of solutions using the so-called diffusion plots.

1. Introduction

The Large Hadron Collider opened up the possibility to scan parton densities over a wide domain of partons kinematics. This allows for detailed studies of various theoretically interesting and phenomenologically relevant dynamical effects taking place during partons evolution, such as: coherence [1], saturation [2] or both [3–5]. In the present work we are in particular interested in the coherence effects in the initial state gluon cascade which is modelled by CCFM [1,6] evolution equation. These effects are taken into account by summing up dominant contribution in angular ordered regions of phase space. Therefore, in comparison with DGLAP [7] equation, CCFM includes some of the interference effects that are subleading from the point of view of approximation to leading-order in the ordering in the hard scales $\alpha_S \ln q_T^2/\mu^2$ where q_T is the transverse momentum of a t-channel gluon. Because of this type of ordering, the CCFM equation is applicable also in the domain of low x and can be viewed as a bridge between low x and large x physics. The CCFM equation, due to the coherence effects included, provides parton distribution function (PDF) not only at a given fraction of proton longitudinal momentum x and transverse momentum accumulated in the gluonic ladder, but accounts also for additional argument $\bar{p}$, related to the maximal angle of gluon emissions. Hence, it allows for matching a PDF with a hard process matrix element given the scale of the last emission. The last feature makes it particularly interesting for extextension of the BFKL [8] approach. The classical CCFM equation is linear and predicts unlimited growth of parton densities at small x. It can, however, be extended in order to account for saturation by extending it by a nonlinear term [3,9] or impose absorptive boundary conditions [10,11]. The CCFM equation has been already studied theoretically in some limiting cases i.e. in the low x limit [10,12–14] and within Monte Carlo formulation [15–18]. It has been also used in phenomenological applications [19–22]. However, the open issues concerning the CCFM are numerous to mention here just the proper form of the initial conditions, the details of violation of unitarity, the role of the soft emissions in the low x limit. Also the efficient algorithm for solving it together with evolution of quark densities is still an open problem. In the present study we reinvestigate systematically aspects of the CCFM equation using the Monte Carlo Markov chain approach focusing on the role played by the non-Sudakov form factor and the soft parts of the splitting function. The understanding of the effects coming from both parts of the splitting functions is important for investigations of unitarity violation effects in evolution of partons [5] since the nonlinearities affect the soft emissions. Such investigation is possible since the Monte Carlo numerical integration we use [23] allows for easy handling of singular integrals and therefore for taking into account effects from the full splitting function i.e. soft and hard emissions and corresponding form factors. The second reason linked to the use of Markov chain Monte Carlo is to provide a new scheme for performing numerically efficient parton shower based on a forward evolution and extend it in the future to account for a nonlinear term allowing for saturation of gluons as well as for quarks.

The Letter is organized as follows. In Section 2 we introduce an iterative formulation of the CCFM equation suitable for the Markov Chain implementation. In Section 3 we present the Monte Carlo algorithm we apply to solve the CCFM equation using the CohRad program. In the Section 4 we analyze properties of the CCFM equation i.e. the effect of neglecting the Sudakov form factor and soft real emissions, and we compare the CCFM to angular ordered DGLAP cascade. Finally we perform the analysis of population of different regions of phase space focusing on the diffusion aspects of the considered equations.

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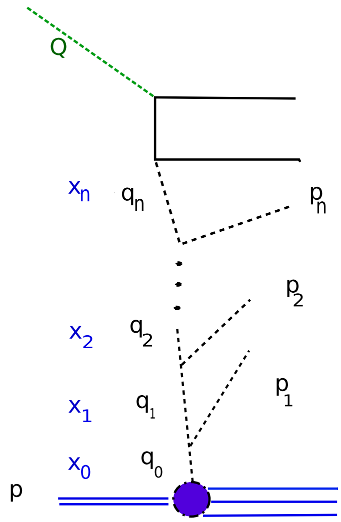

Fig. 1. Notation used in the text.

2. CCFM evolution equation

We use notation as in Fig. 1: p_i and q_i denote four-momenta of real and virtual gluons, respectively, and $x_{i+1} = z_{i+1}x_i$ are fractions of longitudinal momenta of the gluon initiating the cascades. If $z \simeq 0$, a large momentum fraction has been carried out by a real emitted gluon (hard emission), while $z \simeq 1$ corresponds to a soft emission. 2-vectors of transverse momenta of the emitted gluons are denoted by $\mathbf{p}_{iT}$ and transverse momenta accumulated on the emitting line by $q_{iT} = |\mathbf{q}_{0T} - \sum_{j=1}^{i} \mathbf{p}_{jT}|$. It is also convenient to use rescaled transverse momenta $\tilde{\mathbf{p}}_i = \frac{\mathbf{p}_{iT}}{1-z_i}$, with their modulus being related to angles of emissions: $\tilde{p}_i = |\tilde{\mathbf{p}}_i| = \frac{p_{iT}}{1-z_i} \simeq E_i\theta_i$.

The CCFM equation imposes angular ordering in the real emissions, that can be expressed either using angles: $\tilde{p}_{i+1} \geqslant z_i\tilde{p}_i$ or rapidities of the emitted gluons: $\ln \frac{p_i^+}{p_{iT}} \equiv \eta_i < \eta_{i+1}$.

In the following we present solution of the CCFM equation for the unintegrated gluon density in the iterative form:

$$
\begin{aligned}
&\mathcal{A}(x, q_T, \bar{p}) \\
&\quad = \mathcal{A}(x_0, q_{0_T}, p_0)\Delta_S(\bar{p}, p_0)\delta(x - x_0)\delta(q_T - q_{0_T}) \\
&\qquad + \sum_{n=1}^{\infty} \int dx_0 \int d^2q_{0T}\, \mathcal{A}(x_0, q_{0_T}, p_0) \\
&\qquad \times \int\limits_{\tilde{p}_1<\bar{p}} \frac{d^2\tilde{p}_1}{\pi\tilde{p}_1^2} \int dz_1\, \Theta(\tilde{p}_1 - \tilde{p}_0)\Delta_S(\tilde{p}_1, p_0)P_{gg}(z_1, \tilde{p}_1, q_0) \\
&\qquad \times \int\limits_{\tilde{p}_2<\bar{p}} \frac{d^2\tilde{p}_2}{\pi\tilde{p}_2^2} \int dz_2\, \Theta(\tilde{p}_2 - z_1\tilde{p}_1)\Delta_S(\tilde{p}_2, z_1\tilde{p}_1) \\
&\qquad \times P_{gg}(z_2, \tilde{p}_2, q_{1T}) \\
&\qquad \ldots \\
&\qquad \times \int\limits_{\tilde{p}_n<\bar{p}} \frac{d^2\tilde{p}_n}{\pi\tilde{p}_n^2} \int dz_n\, \Theta(\tilde{p}_n - z_{n-1}\tilde{p}_{n-1}) \\
&\qquad \times \Delta_S(\tilde{p}_n, z_{n-1}\tilde{p}_{n-1})P_{gg}(z_i, \tilde{p}_n, q_{n-1,T}) \\
&\qquad \times \Delta_S(\bar{p}, z_n\tilde{p}_n)\delta(x - x_0z_1\cdots z_n)\delta\left(q_T - \left|\mathbf{q}_{0T} - \sum_{i=1}^{n} \mathbf{p}_{iT}\right|\right)
\end{aligned}
\tag{1}
$$

convenient for a Markov chain Monte Carlo implementation. The $\Theta(\tilde{p}_i - z_{i-1}\tilde{p}_{i-1})$ functions impose angular ordering of emissions. The scale $\bar{p}$ related to rapidity position of the hard process will be defined more precisely in the following. The variable $p_0 = 1$ GeV plays the role of the minimal scale and the infrared cutoff on transverse momenta, $p_{iT} > p_0$. The initial transverse momentum of a gluon coming from a proton is denoted by q_{0T}. In the above

$$
P_{gg}(z, \tilde{p}, q_T) = \frac{\alpha_S N_C}{\pi}\left(\frac{1}{1-z} + \frac{\Delta_{NS}(\tilde{p}, z, q_T)}{z}\right) \tag{2}
$$

is the CCFM splitting function. Its form is similar to the DGLAP [7] splitting:

$$
P_{gg}^{DGLAP}(z) = \frac{\alpha_S N_C}{\pi}\left(\frac{1}{1-z} + \frac{1}{z} + z(1-z) - 2\right) \tag{3}
$$

yet does not include terms non-singular in $z \to 0$. The non-Sudakov form factor $\Delta_{NS}(p, z, q_T)$ reads:

$$
\begin{aligned}
\Delta_{NS}(\tilde{p}, z, q_T) &= \exp\left\{-\frac{\alpha_S N_C}{\pi}\int\limits_z^1 \frac{dz'}{z'}\int\limits_{z'^2\tilde{p}^2}^{q_T^2} \frac{dp_T^2}{p_T^2}\right\} \\
&= \exp\left\{-\frac{\alpha_S N_C}{\pi}\ln\frac{1}{z}\ln\frac{q_T^2}{z\tilde{p}^2}\right\}.
\end{aligned}
\tag{4}
$$

The particular form of the non-Sudakov form factor we study here and which allows the form factor to be larger than unity was motivated by the investigations in [11]. This formulation which we wanted to reproduce in Monte Carlo neglects kinematical constraint effects and leads to fast growth of gluon density towards small values of q_T. Studies taking into account kinematical constraint effects we postpone for future investigations. In both Δ_{ns} and Δ_s as well as in the splitting function we kept α_s constant, $\alpha_s = 0.2$, for simplicity at this stage. In the future we plan to use coupling constant as suggested by NLO BFKL results [24]. This form factor enters the CCFM equation from resummation of virtual emissions that are harder than either of the emitting lines. It regulates $z \to 0$ singularity of the splitting function. A detailed discussion on the physical interpretation of the region of integration can be found for instance in Ref. [10].

The Sudakov form factor is given by

$$
\Delta_S(q, q') = \exp\left(-\frac{\alpha_S N_C}{\pi}\int\limits_{(q')^2}^{q^2} \frac{d\tilde{p}^2}{\tilde{p}^2}\int\limits_0^{1-\varepsilon(\tilde{p}_i)} \frac{dz}{1-z}\right). \tag{5}
$$

It can be interpreted as resummed soft virtual emissions on the emitter line. With $\tilde{p}_0$ being the minimal allowed transverse momentum[1] the soft (IR) cutoff in CCFM $\varepsilon(\tilde{p}_i) = \tilde{p}_0/(\tilde{p}_i x_0)$ is evolution scale dependent. At the beginning of the evolution ($\tilde{p}_i \simeq \tilde{p}_0$) only hard, $z_i \to 0$, emissions are allowed. In DGLAP this cutoff is constant, $\varepsilon(\tilde{p}_i) = \varepsilon$.

Finally let us also mention that the distribution $\mathcal{A}$ in Eq. (1) is related to the gluon density $g(x)$ through the relation

$$
xg(x, Q^2) = \int \frac{d^2q_T}{\pi q_T^2} x\mathcal{A}(x, q_T, Q^2)\Theta(Q^2 - q_T^2).
$$

3. Monte Carlo algorithm

We implemented CCFM evolution equation (1) in CohRad as a Markov chain in (η_i, x_i). Due to the similarities between the

[1] We set $\tilde{p}_0 = 1$ GeV, see below for more discussion.