Physics Letters B 721 (2013) 136–140

Contents lists available at SciVerse ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Neutron matter instabilities induced by strong magnetic fields

R. Aguirre, E. Bauer *

Departamento de Física, Facultad de Ciencias Exactas, Universidad Nacional de La Plata and IFLP-CCT-La Plata, CONICET, Argentina

ARTICLE INFO

Article history:
Received 22 January 2013
Accepted 25 February 2013
Available online 1 March 2013
Editor: W. Haxton

Keywords:
Neutron matter
Magnetic instability

ABSTRACT

We study some properties of spin-polarized neutron matter in the presence of a strong magnetic field at finite temperature. Using the Skyrme model together with the Hartree–Fock approximation we obtain an energy density functional that is employed to extract the spin polarization, the effective mass and the magnetic free energy of the system. In order to find the equilibrium state, we have analyzed different global spin configurations over a wide range of matter density ($0 < n/n_0 \leqslant 3$), magnetic field intensity (10^{14} G $\leqslant B < 10^{19}$ G) and temperature ($T \leqslant 80$ MeV). The outcome is that the system can be either completely spin-down polarized or partially polarized. A change in any of the (n, T, B)-variables can induce a transition from one polarization state to the other. The transition takes place in a surface in the (n, T, B)-phase space, which represents an instability of the system. We have also found a discontinuity in the internal energy associated with this change in the state of magnetization.

1. Introduction

The effects of magnetic fields on dense matter have been a subject of interest from long time ago [1], particularly in relation to astrophysical issues. The equation of state for magnetized matter is important for the neutron star structure [2] and for the cooling of magnetized stars [3–5]. Moreover, since neutrinos are a fundamental piece in cooling processes, its emission and transport properties in the presence of magnetic fields were studied in detail [4,5].

A wide range of observational data of periodic or irregular radiation from localized sources has been related to the presence of very intense magnetic fields in compact stellar objects. These manifestations have been classified as pulsars, soft gamma ray repeaters and anomalous X-ray pulsars, according to the energy released and the periodicity of the episodes. They have been associated with different stages of the neutron star evolution. The intensity of these magnetic fields could reach 10^{14} G in the star surface and could grow up several orders of magnitude in its dense interior. The origin of such unusually large fields is still uncertain. The source of the quasi-periodic frequency observed for the radiation phenomena has been attributed to different causes, such as the breakdown of the star crust or to sudden rearrangements of the magnetization [6]. In any case, a detailed knowledge of the magnetic susceptibility is necessary to clarify this question. As the properties of the radiation measured are closely related to the equation of state of nuclear matter, this subject offers a valuable opportunity to constrain theoretical models.

The study of the high density nuclear equation of state in the presence of very strong magnetic fields and the structure of neutron stars was developed within the covariant field theory (see for example [7,8]). Non-relativistic effective models have also been used in this context, as they are specially suited to deal with dense nuclear systems. In particular, for homogeneous matter they give rise to energy density functionals of low computational complexity. Furthermore, they have been applied to describe non-homogeneous low density environments, such as the neutron star crust.

In this Letter we study infinite homogeneous neutron matter in the presence of an external magnetic field at finite temperature, using the Skyrme model. There are previous works which used the Skyrme model to study neutron matter under similar conditions [9–11]. We consider densities up to three times the normal nuclear density, temperatures up to 80 MeV, and field intensities $10^{14} \leqslant B < 10^{19}$ G, which are appropriate for some astrophysical studies.

The equilibrium state for a system in the presence of an external magnetic field is characterized by a minimum of the appropriate thermodynamical potential [12]. Therefore we examine the possible global configurations of the spin polarization of neutron matter in terms of the field intensity, the density and the temperature.

This Letter is organized as follows. In Section 2 we outline the formalism used to study neutron matter under a strong magnetic field, the outcomes are discussed in Section 3, and the conclusions are drawn in Section 4.

---

* Corresponding author.
E-mail address: bauer@fisica.unlp.edu.ar (E. Bauer).

0370-2693/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.physletb.2013.02.046

2. The Skyrme model for neutron matter in strong magnetic field

The Skyrme model is an effective formulation of the nuclear interaction [13]. It consists of a two-body contact potential plus some terms having an explicit density dependence. Using this interaction and the Hartree–Fock approximation, one builds up an energy density functional. The associated single-particle spectrum can be expressed in such a way that the interaction contributes partly to the definition of an effective mass and partly to a remaining potential energy. This approximation provides a simple and useful scheme to study neutron matter properties like the magnetization, effective mass, etc. There are several parameterizations for the Skyrme model, according to the different applications, which cover issues from exotic nuclei to stellar matter.

We are particularly interested in the contributions coming from terms containing time-reversal-odd densities and currents, since they are active when spin states are not symmetrically occupied. A derivation of these terms can be found in [14].

In working with an external magnetic field B, the energy density functional is the sum of the Skyrme density functional $\mathcal{E}_{Skm}$, plus the interacting term between matter and the magnetic induction B,

$$\mathcal{E} = \mathcal{E}_{Skm} - \mu_N \chi W B, \tag{1}$$

where μ_N is the Bohr magneton, the Lande factor $\chi = -1.913$ takes account of the anomalous magnetic moment and

$$\mathcal{E}_{Skm} = \sum_s \frac{K_s}{2m_s^*} + \frac{1}{16} a\big(n^2 - W^2\big) \tag{2}$$

is the standard Skyrme density functional. Here we have introduced the effective nucleon mass m_s^* for a neutron with spin up $(s = 1)$ or down $(s = -1)$,

$$\frac{1}{m_s^*} = \frac{1}{m} + \frac{1}{4}(b_0 n + s b_1 W) \tag{3}$$

where m is the bare neutron mass, n stands for the neutron number density and W is the spin asymmetry density,

$$W = \sum_s \frac{s}{(2\pi)^3} \int d^3p \, f_s(T, p). \tag{4}$$

The expressions for n and the kinetic density K_s of particles with spin s, are,

$$n = \sum_s \frac{1}{(2\pi)^3} \int d^3p \, f_s(T, p) \tag{5}$$

and

$$K_s = \frac{1}{(2\pi)^3} \int d^3p \, p^2 f_s(T, p). \tag{6}$$

In Eqs. (4)–(6), we have used the statistical distribution function at temperature T,

$$f_s(T, p) = \big[1 + \exp \beta\big(\varepsilon_s(p) - \mu\big)\big]^{-1}, \tag{7}$$

where $\beta \equiv 1/T$, μ is the chemical potential and the particle spectra is,

$$\varepsilon_s(p) = \frac{p^2}{2m_s^*} + \frac{1}{8} v_s - \mu_N \chi s B, \tag{8}$$

which is obtained in a self-consistent way through the functional derivative $\varepsilon_s(p) = \delta\mathcal{E}/\delta f_s(T, p)$. In Eq. (8) we have used,

$$\begin{aligned} v_s = {} & a(n - sW) + \sum_{s'} \big(b_0 + ss' b_1\big) K_{s'} \\ & + \frac{\sigma}{3} t_3 (1 - x_3)\big(n^2 - W^2\big) n^{\sigma - 1}. \end{aligned} \tag{9}$$

The parameters a, b_0 and b_1 can be written in terms of the standard parameters of the Skyrme model,

$$\begin{aligned} a &= 2t_0(1 - x_0) + t_3(1 - x_3) n^\sigma / 3, \\ b_0 &= t_1(1 - x_1) + 3t_2(1 + x_2), \\ b_1 &= t_2(1 + x_2) - t_1(1 - x_1). \end{aligned}$$

Note that W, n and K are all functions of the temperature T, the magnetic field intensity B and the chemical potential μ. The equilibrium state for a system in the presence of a constant magnetic field is given by the stationary configuration of the thermodynamic potential per volume, $U/V = F/V - \mathbf{M} \cdot \mathbf{B}$ [12], where we have introduced the free energy per volume, $F/V = \mathcal{E}_{Skm} - TS/V$ and we have adopted $M = \mu_N \chi W$ for the magnetic moment of the system. Finally the entropy corresponds, in a quasi-particle picture, to

$$\begin{aligned} S/V = {} & -\sum_s \int \frac{d^3p}{(2\pi)^3} \big[f_s(T, p) \ln\big(f_s(T, p)\big) + \big(1 - f_s(T, p)\big) \\ & \times \ln\big(1 - f_s(T, p)\big)\big]. \end{aligned} \tag{10}$$

For given values of n, T and B we solve in a self-consistent way the set of Eqs. (3)–(9), obtaining the spin polarization W and the chemical potential μ. Under these conditions the system can be in any of the following global spin configurations: completely polarized state (CPS) or partially polarized state (PPS). In the former case the system is either spin up (CPS-U) with $W/n = 1$ or spin down (CPS-D) with $W/n = -1$, whereas for the latter instance one has $|W/n| < 1$.

As a final remark, note that throughout this article we use units such that $c = 1$, $\hbar = 1$.

3. Results and discussion

In this section we present the results obtained with the Skyrme model, for which the SLy4 parametrization is used. An explicit description of the model parameters can be found, for instance, in Ref. [15]. We extend our analysis to densities up to three times the normal nuclear density, temperatures below $T = 80$ MeV, and the magnetic field intensity within the range 10^{14} G $\leqslant B < 10^{19}$ G.

In practice we solve the set of Eqs. (3)–(9) for fixed values of n, T and B, and for each of the three spin configurations CPS-D, CPS-U, and PPS. From these results we select the equilibrium state, which corresponds to the minimum of the thermodynamic potential U. In fact, only a CPS-D or a PPS is obtained. The points where the system changes from one polarization state to other one define a instability surface in the (n, T, B)-space. It is worthwhile to mention that the aim of this contribution is to describe this instability, and we have not considered the possibility of a coexistence of phases. As we have just stated, we have analyzed field intensities from $B = 10^{14}$ G on. However, we do not show results for relatively low field intensities, because a PPS is favored for all the range of densities and temperatures examined. The instabilities start approximately at $B \simeq 3 \times 10^{17}$ G, and become very complex in a neighborhood of $B = 5 \times 10^{18}$ G. Finally, for extreme fields $B \sim 10^{19}$ G they disappear because the system is firmly established in a CPS-D.

To understand the meaning of the instabilities it is convenient to consider the spin asymmetry W in first place. In Fig. 1, we have plotted W as a function of the density for several values of