



The covariant, time-dependent Aharonov–Bohm effect

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ABSTRACT

We discuss two possible covariant generalizations of the Aharonov–Bohm effect – one expression in terms of the space–time line integral of the four-vector potential and the other expression in terms of the space–time “area” integral of the electric and magnetic fields written in terms of the Faraday 2-form. These expressions allow one to calculate the Aharonov–Bohm effect for time-dependent situations. In particular, we use these expressions to study the case of an infinite solenoid with a time varying flux and find that the phase shift is zero due to a cancellation of the Aharonov–Bohm phase shift with a phase shift coming from the Lorentz force associated with the electric field, $\mathbf{E} = -\partial_t \mathbf{A}$, outside the solenoid. This result may already have been confirmed experimentally.

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1. Introduction

The Aharonov–Bohm (AB) effect [1,2] lies at the interface of gauge theories and quantum mechanics. In its best known form, the AB effect predicts a shift in the interference pattern of the quantum mechanical double-slit experiment which has a magnetic flux carrying solenoid placed between the slits. If a solenoid with a magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$ (where $\mathbf{A}$ is the electromagnetic vector potential) is placed between the two slits of a double-slit experiment the phase, α , of the wave-function of the electrons going through the slits and following some *path* to the screen will be shifted by an amount

$$\alpha_B = -\frac{e}{\hbar} \int_{\text{path}} \mathbf{A} \cdot d\mathbf{x} \quad (1)$$

where e is the charge of the electron. If one considers two electrons arriving at the screen via two separate paths, namely *path*₁ and *path*₂, one can reverse one of the paths and find that the phase difference between the two electrons at the screen is given by

$$\delta\alpha_B = \alpha_{B_1} - \alpha_{B_2} = \frac{e}{\hbar} \oint_{2-1} \mathbf{A} \cdot d\mathbf{x} = \frac{e}{\hbar} \int \mathbf{B} \cdot d\mathbf{S} = \frac{e}{\hbar} \Phi_0 \quad (2)$$

where the subscript 2 – 1 means the path going from the slits to the screen along *path*₂ and returning along *path*₁. We used

Stokes' theorem on the closed line integral and $\nabla \times \mathbf{A} = \mathbf{B}$. Finally, Φ_0 is the magnetic flux through the cross sectional area, $\mathbf{S}$, of the solenoid. This shift in the phase leads to a shift in the position, x , of the interference pattern maxima and minima on the screen by $\Delta x = \frac{L\lambda}{2\pi d} \delta\alpha_B$ where L is the distance to the screen, d is the distance between the slits and λ is the wavelength of the wave-function. This shift due to the magnetic AB effect has been measured [3,4]. Note, there is some unavoidable arbitrariness in the sign $\delta\alpha_B$ depending on the rotational sense of the closed loop going around the solenoid – going along *path*₁ to the screen and returning along *path*₂ versus going along *path*₂ to the screen and returning along *path*₁. However, the shift of the interference pattern is independent of this arbitrariness.

The importance of the magnetic AB effect of (2) is that it shows (to some degree) the physical nature of the vector potential, $\mathbf{A}$, since the electrons move in a region, outside the solenoid, where $\mathbf{B} = 0$ but $\mathbf{A} \neq 0$. However, although $\mathbf{A}$ is gauge variant under the gauge transformation $\mathbf{A} \rightarrow \mathbf{A} - \nabla \Lambda$, where $\Lambda(\mathbf{x}, t)$ is some arbitrary function, the final result in the phase difference, $\delta\alpha_B$, is gauge independent since it can be turned into a surface integral of the magnetic field, which is gauge invariant.

The electric version of the AB effect has been less discussed and investigated. It was experimentally observed relatively recently in [5]. Similar to the magnetic case above, one can show [1] that for an electron moving through some region of space with an electric scalar potential ϕ , it will have its phase shifted by an amount

$$\alpha_E = \frac{e}{\hbar} \int_{t_1}^{t_2} \phi dt \quad (3)$$

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where $\Delta t = t_2 - t_1$ is the time the electron spends in the potential. If one considers electrons moving along two different paths, $path_1$ and $path_2$, with different values of the potential, ϕ_1 and ϕ_2 , along the different paths, then the electrons will acquire a phase difference due to traveling in different potentials given by

$$\delta\alpha_E = \frac{e}{\hbar} \int_{t_1}^{t_2} \Delta\phi dt = \frac{e}{\hbar} \int_{t_1}^{t_2} \int \mathbf{E} \cdot d\mathbf{x} dt \quad (4)$$

where $\Delta\phi = \phi_2 - \phi_1 = -\int_2^1 \nabla\phi \cdot d\mathbf{x} = \int \mathbf{E} \cdot d\mathbf{x}$ is the potential difference between the two paths through which the electrons move. The last form of the electric phase shift in (4), i.e. $\frac{e}{\hbar} \int_{t_1}^{t_2} \int \mathbf{E} \cdot d\mathbf{x} dt$, appears similar to the last form of the magnetic phase shift in (2), i.e. $\frac{e}{\hbar} \int \mathbf{B} \cdot d\mathbf{S}$, in that both have the form $\delta(\text{Phase}) \propto (\text{Field}) \times (\text{Area})$ although for the electric phase shift the “area” has one space side and one time side while the magnetic phase shift has a conventional area having two space sides. One can flesh out this connection via the following heuristic argument: For a small distance, $\Delta\mathbf{x}$, between the two different potentials, ϕ_2 and ϕ_1 , one can write $\Delta\phi = \mathbf{E} \cdot \Delta\mathbf{x}$. Using this in the first expression in (4), one can write $\delta\alpha_E = \frac{e}{\hbar} (\mathbf{E} \cdot \Delta\mathbf{x}) \Delta t$, where again Δt is the time that the two electrons spend in their respective potentials. Now $\Delta t \propto L$ where L is the length of the region through which the electrons move where the potentials are ϕ_2 and ϕ_1 – more precisely $v_e \Delta t = L$ where v_e is the speed of the electrons as they move through these regions of constant scalar potential. Combining these results, we find $\delta\alpha_E \propto (\mathbf{E} \cdot \Delta\mathbf{x})L$, and $(\Delta\mathbf{x})L$ is the area between the two tubes of length L separated by a distance $\Delta\mathbf{x}$, i.e. $d(\text{Area}) = (\Delta\mathbf{x})L$. Thus, both magnetic and electric AB phase differences from (2) and (4) can be written in the form $\delta(\text{Phase}) \propto (\text{Field}) \times (\text{Area})$. Pictorially, one can see this (Area) as the area swept out by an imaginary string which connects the two electrons – the length of the string is $\Delta\mathbf{x}$ and the length swept out is L . Note that the phase difference in (4) is in addition to any phase difference due to the path length difference between $path_1$ and $path_2$. Also, as in the magnetic case (2), there is an unavoidable sign ambiguity in (4) depending whether one considers $\Delta\mathbf{x}$ as coming from a path going from ϕ_1 to ϕ_2 or, alternatively, a reversed path going from ϕ_2 to ϕ_1 .

The expressions (2) and (4) are written in three-vector form so they are not obviously covariant. In the next section, we examine two possible covariant generalizations of the AB phase differences (2) and (4) which allow one to examine time-dependent Aharonov–Bohm experiments.

2. Covariant expressions for the AB phase shift

The first covariant version of the AB phase differences generalizes the potential form of the phase difference given by the first expressions on the right hand side of (2) and (4)

$$\delta\alpha_{EB} = \frac{e}{\hbar} \oint A_\mu dx^\mu = \frac{e}{\hbar} \left[\int_{t_1}^{t_2} \Delta\phi dt - \oint \mathbf{A} \cdot d\mathbf{x} \right]. \quad (5)$$

This covariant expression for the AB phase shift was used in [1]. The closed loop integral in the four-vector expression, $\oint A_\mu dx^\mu$, is not a closed time loop but is to be taken in the sense that the two electrons both start at the space–time point $(t_i, \mathbf{x}_i)$, travel along two different paths, $path_1$ and $path_2$, and end up at the same space–time point $(t_f, \mathbf{x}_f)$ with $t_f > t_i$. One reverses the direction of one of the paths and in this way gets $\Delta\phi = \phi_2 - \phi_1$ in the time integral and one gets a closed loop for the spatial integral, i.e. $\oint \mathbf{A} \cdot d\mathbf{x}$.

The second covariant version of the AB phase difference generalizes the (Field) $\times$ (Area) form of the phase difference, i.e. the last two expressions for the magnetic and electric phase differences given in (2) and (4). This second covariant version of the AB phase is best expressed in the notation of differential forms and the wedge product.¹ This second proposed expression for the covariant AB phase is

$$\delta\alpha_{EB} = -\frac{e}{2\hbar} \int F_{\mu\nu} dx^\mu \wedge dx^\nu = \frac{e}{\hbar} \int F \quad (6)$$

where $F = -\frac{1}{2} F_{\mu\nu} dx^\mu \wedge dx^\nu$ is the Faraday 2-form, dx^μ and dx^ν are differential four-vectors, and $\wedge$ is the anti-symmetric wedge product [6]. The factor of $\frac{1}{2}$ accounts for the anti-symmetry of $F_{\mu\nu}$ and $dx^\mu \wedge dx^\nu$.

We now expand the Faraday 2-form out, and show that it reproduces the standard, static AB phase results (2) and (4),

$$\begin{aligned} F &= -\frac{1}{2} F_{\mu\nu} dx^\mu \wedge dx^\nu \\ &= (E_x dx + E_y dy + E_z dz) \wedge dt + B_x dy \wedge dz + B_y dz \wedge dx \\ &\quad + B_z dx \wedge dy. \end{aligned} \quad (7)$$

If the electric field is zero, i.e. $\mathbf{E} = 0$, then one has $F = B_x dy \wedge dz + B_y dz \wedge dx + B_z dx \wedge dy = \mathbf{B} \cdot d\mathbf{S}$ where the differential forms expression has been converted back to three-vector notation and $d\mathbf{S}$ is the differential area. Thus, the expression in (6) reduces to $\delta\alpha_{EB} = \frac{e}{\hbar} \int F = \frac{e}{\hbar} \int \mathbf{B} \cdot d\mathbf{S}$ which is equivalent to the three-vector expression (2).

If, on the other hand, the magnetic field is zero, i.e. $\mathbf{B} = 0$, and one has a time independent system (so that $\partial_t \mathbf{A} = 0$ and $\mathbf{E} = -\nabla\phi$), then the non-zero terms of the Faraday 2-form are $F = -\partial_x \phi dx \wedge dt - \partial_y \phi dy \wedge dt - \partial_z \phi dz \wedge dt$. Doing the spatial integral of this expression for the Faraday 2-form yields $-\int_2^1 \nabla\phi \cdot d\mathbf{x} = \phi_2 - \phi_1 = \Delta\phi$. Thus, under these conditions, the expression in (6) reduces to $\delta\alpha_{EB} = \frac{e}{\hbar} \int F = \frac{e}{\hbar} \int \Delta\phi dt$ which is equivalent to the first three-vector expression on the right hand side of (4).

In summary, in this section, we have constructed two covariant versions of the AB phase difference, (5) and (6). In the next section, we will discuss how one can experimentally test these covariant expressions for the AB phase difference, (5) or (6), in the time-dependent situation of an infinite solenoid with a time varying magnetic flux.

3. Solenoid with time varying flux

For static situations, both (5) and (6) reproduce the results for the magnetic and electric AB phase differences (2) and (4). However, for certain time-dependent situations, the two expressions both lead to the conclusion that there is an exact cancellation of the magnetic and electric AB phase shifts so that one finds no net phase shift differences coming from the time-dependent electromagnetic field. In particular, we have in mind the usual magnetic AB set-up of an infinite solenoid but with a time-dependent magnetic field and vector potential, i.e. $\mathbf{B}(t)$, and $\mathbf{A}(t)$. Note that for this situation the scalar potential is still zero, $\phi = 0$. At first, one might think that for this set-up the AB phase would simply be obtained by inserting $\mathbf{A}(t)$ into the first expression on the right hand side of (2), or inserting $\mathbf{B}(t)$ into the second expression on the right hand side of (2), giving the usual magnetic AB phase shift (2) but with the time dependence of the vector potential, i.e. $\delta\alpha_B \propto \Phi_0(t)$. This is in fact what previous work [7–9] on the time-dependent AB

¹ For our purposes the elementary and excellent introduction to differential forms given in [6] is all we will need.

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