Physics Letters B 722 (2013) 160–166

Contents lists available at SciVerse ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Spontaneous chiral symmetry breaking as condensation of dynamical chirality

Andrei Alexandru [a], Ivan Horváth [b,*]

[a] The George Washington University, Washington, DC, USA
[b] University of Kentucky, Lexington, KY, USA

ARTICLE INFO

Article history:
Received 14 November 2012
Received in revised form 25 March 2013
Accepted 27 March 2013
Available online 29 March 2013
Editor: B. Grinstein

ABSTRACT

The occurrence of spontaneous chiral symmetry breaking (SChSB) is equivalent to sufficient abundance of Dirac near-zeromodes. However, dynamical mechanism leading to breakdown of chiral symmetry should be naturally reflected in *chiral properties* of the modes. Here we offer such connection, presenting evidence that SChSB in QCD proceeds via the appearance of modes exhibiting dynamical tendency for local chiral polarization. These modes form a band of finite width Λ_{ch} (*chiral polarization scale*) around the surface of otherwise anti-polarized Dirac sea, and condense. Λ_{ch} characterizes the dynamics of the breaking phenomenon and can be converted to a quark mass scale, thus offering conceptual means to determine which quarks of nature are governed by broken chiral dynamics. It is proposed that, within the context of SU(3) gauge theories with fundamental Dirac quarks, mode condensation is equivalent to chiral polarization. This makes Λ_{ch} an "order parameter" of SChSB, albeit without local dynamical field representation away from chiral limit. Several uses of these features, both at zero and finite temperature, are discussed. Our initial estimates are $\Lambda_{ch} \approx 150$ MeV ($N_f = 0$), $\Lambda_{ch} \approx 80$ MeV ($N_f = 2+1$, physical point), and that the strange quark is too heavy to be crucially influenced by broken chiral symmetry.

1. Introduction and conclusions

Chiral symmetry and its conjectured dynamical breaking pattern in multi-flavor massless QCD is a crucial ingredient in the current understanding of low energy hadronic physics. While SChSB scenario is widely accepted, and there is no first-principles evidence suggesting otherwise, it is not known how strong interactions induce its vacuum to become a non-symmetric state. To help the identification of the corresponding mechanism, it is desirable to search for dynamical circumstances accompanying the phenomenon, i.e. to find dynamical features that the eventual explanation needs to incorporate. Low-lying Dirac modes are a suitable place to look for such signatures since they encode the nature of quark propagation in the chiral regime as well as the condensate itself. Indeed, the most direct expression of this connection is the Banks–Casher relation, revealing that the symmetry breakdown is equivalent to sufficient accumulation of Dirac near-zeromodes [1]. However, being entirely generic, the Banks–Casher relation doesn't shed light on dynamical specifics of SChSB.

The premise of this work is that dynamical features of the theory relevant to chiral symmetry breaking should be imprinted in *chiral properties* of Dirac eigenmodes. While there cannot be any average preference for left or right, QCD dynamics induces specific chiral properties in the eigenmodes locally. The most basic of these describe whether values $\psi(x) = \psi_L(x) + \psi_R(x)$ tend to involve asymmetric participation of left–right subspaces (chiral polarization) as opposed to equal participation (chiral anti-polarization). Building on the earlier local chirality approach of Ref. [2], proper *dynamical* quantifiers of this type have recently been constructed [3].

Dynamical nature of these new polarization measures stems from the fact that they are defined relative to the case of statistically independent left–right components, and thus represent (uniquely constructed) correlations. In this work we will only be concerned with the overall dynamical tendency described by the correlation coefficient of chiral polarization C_A. Let $\mathcal{P}(\psi_L, \psi_R)$ be the probability distribution defined by the collection of values comprising given mode(s), and $\mathcal{P}^u(\psi_L, \psi_R)$ the associated distribution of statistically independent components. If Γ_A is the probability that a sample chosen from $\mathcal{P}$ is more polarized than the sample chosen from $\mathcal{P}^u$, then $C_A \equiv 2\Gamma_A - 1 \in [-1, 1]$. Thus, the enhancement of polarization relative to statistical independence ($\Gamma_A > 1/2$) is associated with correlation ($C_A > 0$) while its suppression ($\Gamma_A < 1/2$) with anti-correlation ($C_A < 0$).

The claims in this work are mostly based on the proposed behavior of the QCD average for C_A in modes at eigenvalue λ. In particular, for quark setups relevant to real world, such as $N_f = 2+1$ at zero temperature, we conclude the behavior shown in Fig. 1 (middle) at generic quark masses: there is a low-lying band of chirally polarized modes separated from the rest of anti-polarized

* Corresponding author.
E-mail address: horvath@pa.uky.edu (I. Horváth).

0370-2693/$ – see front matter
http://dx.doi.org/10.1016/j.physletb.2013.03.041

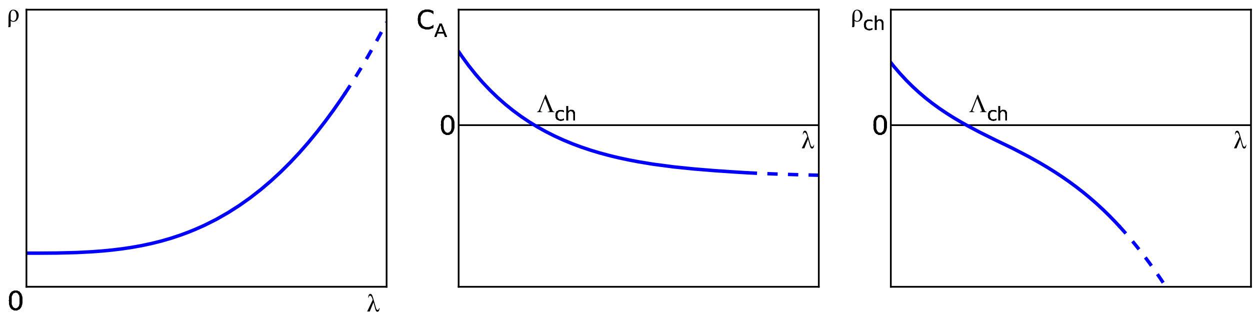

Fig. 1. Schematic behavior of $\rho(\lambda)$, $C_A(\lambda)$, $\rho_{ch}(\lambda)$ in theory with Dirac mode condensation.

bulk. The suggestion is that this applies in the infinite volume and also when light quark masses are asymptotically small, with *chiral polarization scale* Λ_{ch}, introduced in Ref. [3], remaining strictly positive.[1] Since broken chiral dynamics of quarks is dominated by near-zeromodes, we are proposing that this dynamics (1) is associated with chirally polarized (correlated) modes, and that (2) the mechanism generating the polarized band involves a dynamical scale Λ_{ch}. These definite properties need to be respected by viable models of SChSB, exemplifying a "bottom-up" approach to QCD vacuum structure [4].

Thus, our Conjecture 1 implies that the chirality-related dynamical feature of SChSB in QCD is that its condensing modes are locally polarized. How important is this in the context of the mechanism generating broken chiral dynamics? One way to approach this is to consider a wider range of theories, governed by the same gauge interaction, and ask whether the presence of broken chiral dynamics is equivalent to chiral polarization, i.e. whether SChSB occurs only in conjunction with chiral polarization and vice versa. Indeed, if such correspondence holds, then chiral polarization is intimately tied to the nature of the interaction. For purposes of this discussion, we will use the context of SU(3) gauge theories with arbitrary number N_f of Dirac fermions in the fundamental representation, and at arbitrary temperatures. Our Conjecture 2 implies that the above equivalence holds.

Accepting the relevance of chiral polarization elevates Λ_{ch} into a scale associated with SChSB. This input can be used to define the scale of SChSB in a more conventional language of quark mass. Resulting chiral polarization mass scale m_{ch} represents maximal valence mass at which spectral components of the associated scalar bilinear favor chiral polarization over chiral anti-polarization on average. From chiral standpoint, quark dynamics at lower mass is similar to massless one, while that at higher mass turns qualitatively different. Moreover, Λ_{ch} and m_{ch} are meaningful even when all quarks are massive, and some of them can still be driven by broken-like dynamics if their mass is lower than theory's m_{ch}. Thus, one utility of the above dynamical insight is a possibility of objective labeling nature's quarks as "light" or "heavy". These dynamical features also put a novel angle on the characterization of strong dynamics at finite temperature, as we discuss in some detail.

The proposed connection between SChSB and dynamical chirality is associated with the following narrative. Viewing Dirac eigenmodes as scale-dependent probes of gauge field, their non-interacting baseline is a perfectly anti-polarized state. Indeed, left and right components of free eigenmodes have identical magnitudes, independently of λ in this scale-invariant situation, and there is no mode condensation. Turning on the interaction among gluons (pure glue QCD) works against anti-polarization. This effect is scale dependent, due to running coupling, with modes in the infrared being affected more than those in the ultraviolet. At low energy the interaction becomes sufficiently strong for the chiral behavior of modes to undergo a qualitative change: scale Λ_{ch} is dynamically generated via appearance of chirally polarized, condensing Dirac modes, while logarithmically violated scale invariance remains in the ultraviolet. The above two cases (free and pure glue) represent chiral extremes, with other gauge-quark setups interpolating between them. Indeed, both light dynamical quarks and the temperature tend to reduce the effective gauge coupling at low energy, and thus possibly destroy chiral polarization relative to pure glue. Our conclusion in most general form is that the Dirac mode condensation and chiral polarization are present/absent simultaneously: chiral polarization scale Λ_{ch} is an indicator of mode condensation and, when some of the quarks are massless, an "order parameter" of SChSB.

2. Definitions and claims

Here we specify our main conclusions more precisely. The setup and notation of continuum Euclidean QCD is used for simplicity, but it should be understood that the concepts below have well-defined meaning acquired via lattice regularization respecting chirality, e.g. lattice QCD with overlap fermions [5]. Thus, the spectrum of continuum massless Dirac operator in a given gauge background is located on imaginary axis, and the zero-temperature theory with N_f flavors of quarks can be labeled by masses $M \equiv (m_1, m_2, \dots, m_{N_f})$. Ordering the eigenmodes $\psi_k(x)$ by magnitude of their eigenvalue $i\lambda_k$, the average correlation of chiral polarization for modes at $i\lambda$ reads

$$C_A(\lambda, M, V) \equiv \frac{\sum_k \langle \delta(\lambda-\lambda_k) C_{A,k} \rangle_{M,V}}{\sum_k \langle \delta(\lambda-\lambda_k)\rangle_{M,V}} = \frac{\rho_{ch}(\lambda, M, V)}{\rho(\lambda, M, V)} \tag{1}$$

Here $\langle \dots \rangle_{M,V}$ denotes QCD expectation value in 4-volume V, $C_{A,k}$ is the correlation associated with mode ψ_k, and we have introduced the spectral chiral polarization density

$$\rho_{ch}(\lambda, M, V) \equiv \frac{1}{V} \sum_k \big\langle \delta(\lambda-\lambda_k) C_{A,k} \big\rangle_{M,V} \tag{2}$$

in addition to spectral mode density $\rho(\lambda, M, V)$. Correlation $C_{A,k}$ can be evaluated relative to mode's own distribution $\mathcal{P}^u_k$ of independent left–right components, or relative to $\mathcal{P}^u_\lambda$ involving all modes at eigenvalue $i\lambda$. These definitions are expected to be equivalent in the infinite volume limit. To include setups at finite temperature T one simply replaces labels $V \to T, V_3$ and factors $1/V \to T/V_3$, where V_3 is a 3-volume.

There are two reasons for introducing ρ_{ch}. First, spectral contribution to fermionic bilinears is proportional to spectral mode density, and thus $\rho C_A = \rho_{ch}$ gives the proper spectral weight to

[1] Chiral polarization scale was denoted Λ_T in Ref. [3]. Here we switch to Λ_{ch} so that the label is not confused with temperature.