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Can the viscosity in astrophysical black hole accretion disks be close to its string theory bound?

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ABSTRACT

String theory and gauge/gravity duality suggest the lower bound of shear viscosity (η) to entropy density (s) for any matter to be $\sim \mu\hbar/4\pi k_B$, when $\hbar$ and k_B are reduced Planck and Boltzmann constants respectively and $\mu \leqslant 1$. Motivated by this, we explore η/s in black hole accretion flows, in order to understand if such exotic flows could be a natural site for the lowest η/s. Accretion flow plays an important role in black hole physics in identifying the existence of the underlying black hole. This is a rotating shear flow with insignificant molecular viscosity, which could however have a significant turbulent viscosity, generating transport, heat and hence entropy in the flow. However, in presence of strong magnetic field, magnetic stresses can help in transporting matter independent of viscosity, via celebrated Blandford–Payne mechanism. In such cases, energy and then entropy produces via Ohmic dissipation. In addition, certain optically thin, hot, accretion flows, of temperature $\gtrsim 10^9$ K, may be favourable for nuclear burning which could generate/absorb huge energy, much higher than that in a star. We find that η/s in accretion flows appears to be close to the lower bound suggested by theory, if they are embedded by strong magnetic field or producing nuclear energy, when the source of energy is not viscous effects. A lower bound on η/s also leads to an upper bound on the Reynolds number of the flow.

1. Introduction

In order to explain the Quark Gluon Plasma (QGP) observables at Relativistic Heavy Ion Collider (RHIC) having temperature (T) $\gtrsim 10^{12}$ K, it has become apparent that a nonzero shear viscosity, η, is needed. One way of characterizing η, which is a dimensionful number, is to take its ratio with the entropy density, s. It turns out that RHIC plasma has

$$\frac{\eta}{s} \sim 0.1\frac{\hbar}{k_B}. \tag{1}$$

Although η itself is large (in CGS units $\sim 10^{12}$ gm/cm/s), its ratio with s is small. Note that this ratio for a known fluid, say water, is about two orders of magnitude greater than the above value. Hydrodynamic simulations at RHIC and Large Hadron Collider (LHC) (which provide the environments created around a few microseconds after the big bang) as well suggest a small value for this ratio in the hot QGP state. Such a small value points towards strongly interacting matter. In fact, string theory arguments and the famous gauge/gravity duality in the form of the AdS/CFT correspondence suggest a lower bound [1] for η/s for any matter given by

$$\frac{\eta}{s} \geqslant \frac{\mu}{4\pi}\frac{\hbar}{k_B} \sim 0.08\mu\frac{\hbar}{k_B}, \tag{2}$$

where $\mu < 1$ [2] but nonzero [3,4]. For the present purpose, we assume that there is a bound and that $\mu \sim 1$.

Is there any natural site revealing an η/s close to above lower bound? Note that even if $\mu \ll 1$, still the same question remains. Such a value may be possible to arise naturally, if the temperature and/or density of the systems are/is the same as that in RHIC/LHC. In the search of such an exotic system, our story starts which could serve as a natural verification of gauge/gravity duality in the form of the AdS/CFT correspondence.

Observed data suggest that the accretion flows around certain black holes must be optically thin, geometrically thick and very hot. Such flows often exhibit outflows/jets in their hard spectral states. The ion temperature (T_i) in such flows/disks is as high as 7×10^{11} K. The supermassive black hole system at the centre of our galaxy, Sgr A* [5] (most of the temporal classes of) the X-ray binary Cyg X-1 [6] are some of the examples of this kind of hot accretion disks, which exhibit radiatively inefficient flows. They are particularly different from their radiatively efficient counter part, namely the Keplerian accretion disk, which is geometrically thin, optically thick, cooler, and of temperature $T \sim 10^7$ K [7]. The flow in certain temporal classes (and soft spectral states) of micro-quasar GRS 1915 + 105 [8] is an example of

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Keplerian disk. While the temperature of the latter cases could be same order in magnitude as that in the centre of a star, due to much low density they practically do not exhibit any thermonuclear burning. Hence any source of energy therein must be due to magnetic and viscous effects. On the other hand, the former cases may be favourable for thermonuclear reactions due to their very high temperature, even if their density is low [9,10]. Hence, the energy released/absorbed due to nuclear reactions in an optically thin, hot, accretion flow could be comparable to or even dominating over its viscous counter part [9–11]. Being very hot, such flows are also highly ionized and hence expected to be strongly magnetized (see e.g. [12]) rendering magnetic dissipation. In fact, the celebrated Blandford–Payne mechanism [13] is based on such a magnetized accretion disk in the Keplerian regime, which argues for angular momentum transport due to outflows/jets through the outgoing magnetic field lines, in absence of shear (turbulent) viscosity. Now an obvious question arises: what is the value of η/s in optically thin accretion flows (e.g. Advection Dominated Accretion Flow (ADAF) [14], General Advective Accretion Flow (GAAF) [15]) when the temperature is close to that of the QGP matter in RHIC? Primarily guess is that it is not too much different from its lower bound as in Eq. (1). Is this that simple to anticipate? Note that accretion flows with very small molecular viscosity (and then very large Reynolds number) exhibit turbulence and then turbulent viscosity. If the turbulent viscosity dominates, then the energy dissipated due to viscosity in disks is comparable/dominant to/over the magnetic/nuclear energy, as will be discussed in detail below. Note also that at low densities (zero chemical potential), lattice QCD calculations [16–18] suggest a crossover from the hadronic state to the QGP state at around $1.5 \times 2 \times 10^{12}$ K [19]. At such high temperatures, there is expected to be copious pion production in a strongly interacting system. This QGP state is thought to have existed a few microseconds after the bigbang and is being extensively studied at RHIC and LHC. Of course the density in early universe is large as well, unlike that in accretion flows, a large temperature is well enough to reveal such a phase which has been mimicked in RHIC/LHC. However, at a very large density (several orders of magnitude larger than that in accretion flows) the lattice QCD results are not valid (Zoltan Fodor, private communication).

Recently, Sinha and Mukhopadhyay [20] initiated to look at the above issue and argued that the Shakura–Sunyaev turbulent viscosity parameter α [7] should not be constant throughout the flow, and be decreasing with the increase of temperature and/or density of the flow. Based on the general relativistic model of a viscous ADAF, they showed that η/s for an astrophysical black hole, at finite α, is always several orders of magnitude higher than its value in a QGP fluid. Then they argued for the flow to become very weekly viscous (not exhibiting turbulence) close to a black hole, rendering a smaller η/s. The results appear to be independent of the choice of equation of state (EoS); whether of ideal gas or of QCD which is appropriate for flows with $T \gtrsim 10^{12}$ K and low density (close to zero chemical potential). However, they could not clarify the natural circumstances when α is small. Note that small α would reveal negligible dissipation of energy in the flow. Hence, question arises, what is the source of energy and entropy in the accretion flows, which is also very important in order to address η/s? In addition, how is the accretion possible in a very small-α flow? Moreover, as will be explained in the next section, above work did not model the entropy of the flow adequately, while generally interpreted the results correctly. Instead, that was the first step forward in the direction of evaluating η/s for any astrophysical flow, even though incomplete, which is a potential natural site for small η/s. In the present work, we have removed the above uncertainties lying in the previous work. Then we plan to establish the underlying physics giving rise to the astrophysical values of η/s. More precisely, we plan to address: (1) regime of accretion flows giving rise to η/s close to its theoretical lower bound, (2) observational implications of such regimes, (3) black hole sources presumably revealing lower bound of η/s, (4) constraining physical parameters (e.g. Reynolds number) of accretion flows based on η/s bound.

In the next section, we discuss the salient features of hot advective accretion flows and the importance of strong magnetic field and nuclear energy produced therein in order to obtain small η/s. Subsequently in Section 3 we analyse the solutions of the underlying equations and address the flow parameters making η/s to be close to the lower limit. Finally Section 4 summarizes our findings with discussion and implication.

2. Advective accretion flows around black holes

In order to understand the underlying physics, let us consider the equations describing optically thin, viscous, magnetized, advective accretion disk [21]. We describe the model in the pseudo-Newtonian framework with the Paczyński–Wiita potential [22,23]. This is particularly because, for the present purpose, the pure general relativistic results for a rotating black hole qualitatively do not reveal any new physics. Hence, the vertically averaged hydromagnetic equations of energy–momentum balance along with the equations of continuity, induction and divergence of magnetic field in the limit of very large conductivity are

$$\begin{aligned}
&\dot{M} = 4\pi x \Sigma \vartheta,\\
&\vartheta \frac{d\vartheta}{dx} + \frac{1}{\rho}\frac{dP}{dx} - \frac{\lambda^2}{x^3} + \frac{1}{2(x-1)^2} = 0,\\
&\vartheta \frac{d\lambda}{dx} = \frac{1}{x\Sigma}\frac{d}{dx}\left(x^2 W_{x\phi}\right) + \frac{hx}{\Sigma}\left(B_x\frac{dB_\phi}{dx} + B_z\frac{dB_\phi}{dz} + \frac{B_x B_\phi}{x}\right),\\
&\vartheta h T\frac{ds}{dx} = \frac{\vartheta h}{\Gamma_3 - 1}\left(\frac{dP}{dx} - \frac{\Gamma_1 P}{\rho}\frac{d\rho}{dx}\right) = Q^+ - Q^-,\\
&\frac{d}{dx}(xB_x) = 0,\\
&\frac{d}{dx}\left(\vartheta B_\phi - \frac{B_x\lambda}{x}\right) = 0,\\
&\frac{d}{dx}(x\vartheta B_z) = 0,
\end{aligned} \tag{3}$$

assuming that the variables do not vary significantly in the vertical direction such that $d/dz \to 1/z$, which is indeed true for the disk flows. Here $\dot{M}$ is the conserved mass accretion rate, Σ and ρ are the vertically integrated density and density of the flow respectively, ϑ is the radial velocity, P the total pressure, λ the specific angular momentum, $W_{x\phi}$ the vertically integrated shearing stress, h the half-thickness, s the entropy per unit volume, T the temperature of the flow, Q^+ and Q^- are the vertically integrated net energy released and absorbed rates per unit volume in/from the flow respectively, Γ_1, Γ_3 indicate the polytropic indices depending on the gas and radiation content in the flow (see, e.g., [15] for exact expressions) and B_x, B_ϕ and B_z are the components of magnetic field. Note that, the independent variable x is the radial coordinate of the flow expressed in the units of $2GM/c^2$, where G is the gravitation constant, M the mass of the black hole and c the speed of light. Accordingly, all the above variables are expressed in dimensionless units. For any other details, see the existing literature [15,24].

Let us now pay a special attention to the energy equation of the equation set (3). We can rewrite it as