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Effect of quark masses on the QCD pressure in a strong magnetic background

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ABSTRACT

We compute the two-loop contribution to the QCD pressure in a strong magnetic background, for arbitrary quark masses. We show that, for very large fields, the chiral limit is trivial.

Large magnetic fields can be created not only in the core of magnetars [1] but also in current experiments at BNL/RHIC and CERN/LHC involving non-central heavy-ion collisions. The fields created in these collisions are possibly the largest magnetic fields produced since the primordial electroweak transition, reaching values perhaps as high as $B \sim 10^{19}$ Gauss ($eB \sim 6m_\pi^2$) for peripheral collisions at RHIC [2], and even much higher at the LHC thanks to the fluctuations in the distributions of protons inside the nuclei [3]. Such intense magnetic fields may dramatically affect the phases of strongly interacting matter, as is the case in more ordinary circumstances [4]. The mapping of the QCD phase diagram in the T–eB plane is still in its infancy (see e.g. [5] and references therein). There are clear indications that sufficiently large magnetic fields do modify the nature and behavior of the chiral and the deconfinement phase transitions [6–19]. New phases are also predicted [20–23], and it has even been suggested that the vacuum may turn into a superconducting medium via ρ-meson condensation [24].

While most of the analyses so far have relied on effective models, or calculations in the large N_c limit of QCD [25], first results from lattice QCD have been obtained recently [26–29]. This opens a new channel for comparison between analytical or semi-analytical techniques and numerical non-perturbative approaches. In this perspective, we note that the recently observed discrepancy between different lattice QCD results (for large [27] and physical [28,29] values of quark masses) is most likely related to quark mass effects. It is the purpose of this Letter to analyze the possible competition between mass and magnetic field corrections to the QCD pressure. More specifically, we compute the two-loop correction to the QCD pressure in a magnetic background field and for arbitrary quark masses. We indeed find a significant competition between the effects of quark masses and those of the magnetic background. In particular, for extremely intense magnetic fields, we show that the two-loop contribution to the pressure is trivial in the chiral limit.

We shall assume in our calculation a constant and uniform Abelian magnetic background, whose strength is large enough to produce interesting effects, i.e. $eB \gtrsim m_\pi^2$. We also consider the temperature to be large enough[1] that perturbation theory can be applied to the calculation of the pressure, albeit, admittedly, with *a priori* marginal accuracy. Nevertheless, the effect of the strong background magnetic field must be treated non-perturbatively: this is achieved by using the propagator that was obtained long ago by Schwinger [30], and that can be cast in a convenient form using

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[1] This high temperature assumption hinders the occurrence of non-perturbative phenomena that could be triggered by such extreme magnetic fields, namely ρ-meson condensation and formation of pion domain walls, or by confinement *per se*. Regarding the former, these interesting new phases are usually disfavored at high temperatures, as is the case with ordered phases in general, leaving the pressure unaffected in the regime of high temperatures we are interested in.

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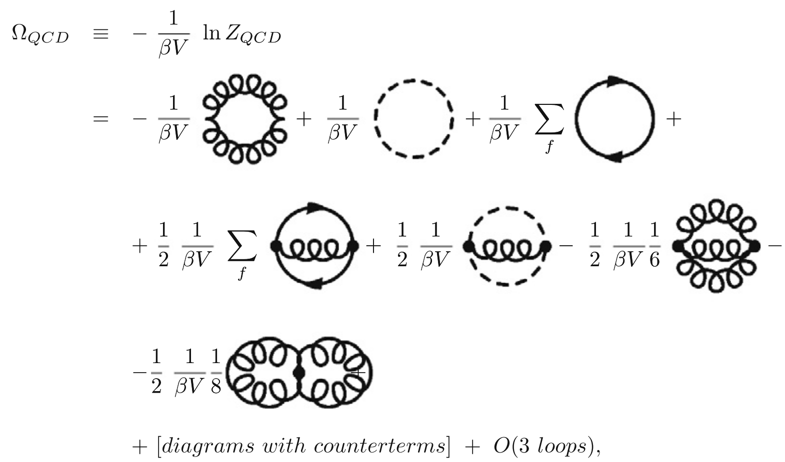

Fig. 1. (Color online.) Diagrammatic expansion of the thermodynamic potential of QCD. Here full lines are fermions, dressed by the magnetic field, curly lines are gluons and dashed lines represent ghosts (whose role is essentially to cancel the contribution of spurious degrees of freedom in the gluonic pressure). The exchange diagram is the first one in the third line.

Landau levels, as shown in Ref. [31] (see also Refs. [32–34]). Since we restrict our analysis to the case of very intense magnetic fields, the summation over the Landau levels is rapidly convergent, and the leading correction to the pressure is obtained from the lowest Landau level (we shall refer to the corresponding calculations as the lowest Landau level (LLL) approximation). The corresponding propagator for a fermion of a given flavor f and (absolute) electric charge q_f, in the presence of the classical field $A_{\rm cl} = (0, \vec{A})$ (with $\nabla \times \vec{A} = \vec{B} = B\hat{z}$) reads:

$$S_0^{\rm LLL}(x,y) = \exp\left\{\frac{iq}{2}\left[x^\mu - y^\mu\right]A_\mu^{\rm ext}(x+y)\right\} \times \int \frac{d^d P}{(2\pi)^d} e^{-iP\cdot(x-y)} i \exp\left(-\frac{\mathbf{p}_T^2}{|qB|}\right)\frac{1+i\gamma^1\gamma^2}{\mathbf{p}_L\cdot\gamma_L - m_f}, \tag{1}$$

where we have used a compact notation for the transverse ($\mathbf{p}_T = (p_1, p_2)$, $\gamma_T = (\gamma^1, \gamma^2)$) and longitudinal ($\mathbf{p}_L = (p_0, p_3)$, $\gamma_L = (\gamma^0, \gamma^3)$) quantities. This is equivalent to the result used in Ref. [33], obtained by constructing the projectors on the different Landau levels from the exact solution of the Dirac equation [35].[2]

The thermodynamic potential of QCD, up to two loops, is obtained form the standard diagrammatic expansion displayed in Fig. 1. The calculation is carried out in Feynman gauge.

The gluonic part is equivalent to the usual hot perturbative QCD result and is well known [36]:

$$\Omega^G_{QCD} = -2\left(N_c^2 - 1\right)\frac{\pi^2 T^4}{90} + \left(N_c^2 - 1\right)N_c g^2 T^4 \frac{1}{144}. \tag{2}$$

The one-loop contribution to the fermionic pressure has been considered in different contexts (usually, in effective field theories [7,9,8,15,18,37]) and computed from the direct knowledge of the Landau levels $E^2(n, p_3) = p_3^2 + m_f^2 + 2q_f Bn$ and their degeneracies $q_f B/(2\pi)$ for $n = 0$ and $q_f B/\pi$ for $n = 1, 2, \ldots$. The final exact result reads (see Ref. [18] for a discussion on the subtraction procedure)

$$\frac{P^F_{\rm free}}{N_c} = \sum_f \frac{(q_f B)^2}{2\pi^2}\left[\zeta'(-1, x_f) - \zeta'(-1, 0) + \frac{1}{2}\left(x_f - x_f^2\right)\ln x_f + \frac{x_f^2}{4}\right] + T\sum_{n,f}\frac{q_f B}{\pi}(1 - \delta_{n0}/2) \times \int \frac{dp_3}{2\pi}\left\{\ln\left(1 + e^{-\beta[E(n,p_3)-\mu_f]}\right) + \ln\left(1 + e^{-\beta[E(n,p_3)+\mu_f]}\right)\right\}, \tag{3}$$

where μ_f is the quark chemical potential (associated to baryon number conservation). In the limit of large magnetic field (i.e. $x_f \equiv m_f^2/(2q_f B) \to 0$), the expression above reduces to the LLL contribution

$$\frac{P^F_{\rm free}}{N_c} \overset{\text{large } B}{=} \sum_f \frac{(q_f B)^2}{2\pi^2}\left[x_f \ln\sqrt{x_f}\,\right] + T\sum_f \frac{q_f B}{2\pi}\int \frac{dp_3}{2\pi}\left\{\ln\left(1 + e^{-\beta[E(0,p_3)-\mu_f]}\right) + \ln\left(1 + e^{-\beta[E(0,p_3)+\mu_f]}\right)\right\}. \tag{4}$$

The exchange diagram (first one in the third line of Fig. 1) corresponds to the first nontrivial contribution. In terms of the propagators in coordinate space, this diagram is given by

$$\beta V g^2 N_c(\lambda_a\lambda_a)\int \frac{d^d x\, d^d y}{\beta V}\int \frac{d^d K}{(2\pi)^d}\frac{e^{-iK\cdot(y-x)}}{K^2} \times {\rm Tr}\left[\gamma_\mu S_0(x,y)\gamma^\mu S_0(y,x)\right], \tag{5}$$

where λ_a are Gell-Mann matrices, with $\lambda_a\lambda_a = (N_c^2 - 1)/2$, the trace Tr acts over Dirac indices and the 4-momentum is given in terms of the Matsubara frequencies ($\omega_l^B = 2l\pi T$) and of the 3-momentum $\mathbf{k}$ as: $K = (k^0 = i\omega_l^B, \mathbf{k})$. This is expression reduces to the usual one [36] in the absence of magnetic field, and for free Dirac propagators (with $P = (p^0 = i\omega_n^F + \mu_f, \mathbf{p})$ and $\omega_n^F = (2n+1)\pi T$). In the presence of a uniform and constant magnetic background ($\mathbf{B} = B\hat{z}$), however, the fermion propagator becomes dependent on x and y in a nontrivial way due to the Schwinger phase, as discussed previously.

A detailed analysis of this diagram shows that it can be cast in the following neat form [34]:

[2] A peculiarity of the LLL approximation should be noted: the propagator is a 4×4 matrix, but it describes only two physical propagating modes. Two eigenvalues of $S_0^{\rm LLL}$ indeed go to zero in the vicinity of the lowest Landau level pole ($p_0^2 = m_f^2 + p_3^2$). This complicates in particular the computation of the free pressure in the LLL approximation [18,34].