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Black holes in realistic branes: Black string-like objects?

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ABSTRACT

A realistic model describing a black string-like object in an expanding Universe is analyzed in the context of the McVittie's solution of the Einstein field equations. The bulk metric near the brane is provided analogously to previous solutions for black strings. In particular, we show that at least when the Hubble parameter on the brane is positive, a black string-like object seems to play a fundamental role in the braneworld scenario, generalizing the standard black strings in the context of a dynamical brane.

The search for solutions engendering realistic black holes on the brane, stable and presenting no naked singularity, is an object of current interest. Although an exact solution is known for a $(1+2)$-brane in a 4D bulk [1], such task urges to be evinced in the 5D scenario with a single extra dimension of infinite extent. Numerical simulations of relativistic static stars on the brane and the exact analysis of the collapse on the brane as well – based on the AdS/CFT correspondence [2] – appear as good efforts to address the issue [3,4]. There are arguments indicating that whichever the solutions are, they approach the Schwarzschild geometry at large distances [3].

On the other hand, black holes embedded in an expanding Universe can be described by McVittie's solutions [5]. In this cosmological scenario, a more realistic solution can be probed, providing an asymptotic Schwarzschild–de Sitter geometry on the brane. The legitimate black hole interpretation holds at least when the cosmological scenario is dominated at late times by a positive cosmological constant [6]. Nice features regarding the McVittie metric on the brane can be listed and potentially employed. For instance, they reduce to the standard homogeneous and isotropic FRW cosmology, and to a Schwarzschild or de Sitter–Schwarzschild black hole, in appropriate limits. The issue is fascinating, endowing realistic models for black holes in the Universe. Interesting overviews on the subject are given, e.g., in [6–8], providing a deep and modern approach.

In this Letter we are mainly concerned with the black string profile induced by the McVittie solution, by delving into the Taylor expansion outside a McVittie black hole metric on the brane along the extra dimension, where the corrections in the area of the associated 5D black string warped horizon arise. The issue induces interesting physical effects in the black string-like object, as we shall prove. The fine character of the expansion along the extra dimension is crucial to analyze the generalized black string associated to the McVittie's solution on the brane. The way how the dynamical content of the solution on the brane affects the pathological properties regarding the black string [9] shall be deeply reported.

In a braneworld with a single extra dimension of infinite extent, a vector field in the bulk decomposes into components in the brane and orthogonal to the brane, as (x^α, y). The bulk is endowed with a metric $\mathring{g}_{AB}\,dx^A\,dx^B = g_{\mu\nu}(x^\alpha, y)\,dx^\mu\,dx^\nu + dy^2$. The brane metric components $g_{\mu\nu}$ and the bulk metric are related by $\mathring{g}_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$, where n^σ are the components related to a time-like vector field, splitting the bulk in normal coordinates, and $g_{44} = 1$ and $g_{i4} = 0$. In addition, $\kappa_4^2 = \frac{1}{6}\lambda\kappa_5^4$ and $\Lambda_4 = \frac{\kappa_5^2}{2}(\Lambda_5 + \frac{1}{6}\kappa_5^2\lambda^2)$, where Λ_4 is the effective brane cosmological constant, κ_4 [κ_5] denotes the 4D [5D] gravitational coupling, and λ is the brane tension. Usually $\kappa_5 = 8\pi G_5$, where G_5 denotes the 5D gravitational coupling, related to the 4D gravitational constant G by $G_5 = G\ell_{\rm Planck}$ and $\ell_{\rm Planck} = \sqrt{G\hbar/c^3}$. The extrinsic curvature is $K_{\mu\nu} = \frac{1}{2}\pounds_n g_{\mu\nu}$ ($\pounds_n$ denotes the Lie derivative, which in Gaussian normal coordinates reads $\pounds_n = \partial/\partial y$). The junction condition determines the extrinsic curvature on the brane as

$$K_{\mu\nu} = -\frac{1}{2}\kappa_5^2\left[T_{\mu\nu} + \frac{1}{3}(\lambda - T)g_{\mu\nu}\right]. \tag{1}$$

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Here $T^{\mu\nu}$ is the energy-momentum tensor, and $T = T_\mu^\mu$. We also denote $K = K_\mu^\mu$ and $K^2 = K_{\alpha\beta}K^{\alpha\beta}$. Given the 5D Weyl tensor

$$C_{\mu\nu\sigma\rho} = {}^{(5)}R_{\mu\nu\sigma\rho} - \frac{2}{3}\left(\mathring{g}_{[\mu\sigma}\,{}^{(5)}R_{\nu]\rho} + \mathring{g}_{[\nu\rho}\,{}^{(5)}R_{\mu]\sigma}\right) - \frac{1}{6}{}^{(5)}R\left(\mathring{g}_{\mu[\sigma}\mathring{g}_{\nu\rho]}\right)$$

where ${}^{(5)}R_{\mu\nu\sigma\rho}$ denotes the components of the 5D Riemann tensor (${}^{(5)}R_{\mu\nu}$ and ${}^{(5)}R$ are the associated Ricci tensor and the scalar curvature), the symmetric and trace-free components are respectively the electric ($\mathcal{E}_{\mu\nu} = C_{\mu\nu\sigma\rho}n^\sigma n^\rho$) and magnetic ($\mathcal{B}_{\mu\nu\alpha} = g_\mu^\rho g_\nu^\sigma C_{\rho\sigma\alpha\beta}n^\beta$) Weyl tensor components. The effective field equations are complemented by a set of equations, obtained from the 5D Einstein equations and Bianchi equations [3,10,11], which are employed to calculate the terms of the Taylor expansion of the metric along the extra dimension, providing in particular the black string profile and some physical consequences, given by (hereon we denote $g_{\mu\nu}(x^\alpha, 0) = g_{\mu\nu}$):

$$\begin{aligned} g_{\mu\nu}&\left(x^\alpha, y\right) \\ &= g_{\mu\nu} - \kappa_5^2\left[T_{\mu\nu} + \frac{1}{3}(\lambda - T)g_{\mu\nu}\right]|y| \\ &+ \left[\frac{1}{4}\kappa_5^4\left(T_{\mu\alpha}T_\nu^\alpha - \mathcal{E}_{\mu\nu} + \frac{2}{3}(\lambda - T)T_{\mu\nu}\right)\right. \\ &+ \left.\left(\frac{1}{36}\kappa_5^4(\lambda - T)^2 - \frac{\Lambda_5}{6}\right)g_{\mu\nu}\right]y^2 \\ &+ \left[2K_{\mu\beta}K_\alpha^\beta K_\nu^\alpha - \left(\mathcal{E}_{\mu\alpha}K_\nu^\alpha + K_{\mu\alpha}\mathcal{E}_\nu^\alpha\right)\right. \\ &- \frac{1}{3}\Lambda_5 K_{\mu\nu} - \nabla^\alpha \mathcal{B}_{\alpha(\mu\nu)} + \frac{1}{6}\Lambda_5(K_{\mu\nu} - g_{\mu\nu}K) \\ &+ K^{\alpha\beta}R_{\mu\alpha\nu\beta} + 3K^\alpha{}_{(\mu}\mathcal{E}_{\nu)\alpha} - K\mathcal{E}_{\mu\nu} \\ &+ \left.(K_{\mu\alpha}K_{\nu\beta} - K_{\alpha\beta}K_{\mu\nu})K^{\alpha\beta} - \frac{\Lambda_5}{3}K_{\mu\nu}\right]\frac{|y|^3}{3!} + \cdots. \end{aligned} \tag{2}$$

Such expansion regards the metric on the bulk near the brane, and was analyzed in [3,12] only up to the second order. The fourth order expansion was derived in [3] for a particular case. In [13] the most complete fourth order expansion, also containing the additional terms coming from the variable brane tension, was accomplished. Due to the awkward expression therein, we insert above the expansion up to the third order. As a particular case, the black hole horizon evolution along the extra dimension (the warped horizon [14]) may be examined, by exploring the component $g_{\theta\theta}(x^\alpha, y)$ in (2). Indeed, any spherically symmetric metric associated to a black hole presents radial coordinate given by $\sqrt{g_{\theta\theta}(x,0)} = \mathrm{r}$. The black hole solution, namely, the black string solution *on the brane*, is regarded when $\sqrt{g_{\theta\theta}(x,0)} = R$, where R denotes the coordinate singularity. More precisely, the black string horizon for the Schwarzschild metric is defined when $\mathrm{r} = \frac{2GM}{c^2}$, obtained when the coefficient $(1 - \frac{2GM}{c^2\mathrm{r}}) = g_{\mathrm{rr}}$ of the term $d\mathrm{r}^2$ goes to infinity [15]. It corresponds to the black hole horizon on the brane. On the another hand, the radial coordinate r in spherical coordinates legitimately appears as the term $g_{\theta\theta}\,d\theta^2 = \mathrm{r}^2\,d\theta^2$ in the Schwarzschild metric. Our analysis of the term $g_{\theta\theta}(x^\alpha, y)$ by Eq. (2) holds for any value r and provides the bulk metric. In particular, the term originally coined "black string" corresponds to the Schwarzschild metric on the brane [14], defined by the black hole horizon evolution along the extra dimension into the bulk. Hence, the black string regards solely the so-called "warped horizon", which is $g_{\theta\theta}(x^\alpha, y)$, for the particular case where $\mathrm{r} = R$ is a coordinate singularity.

Now we argue whether such interpretation regarding black strings holds for the McVittie's solutions. In their simplest form they have zero spatial curvature in the asymptotically FRW region, but can be generalized [5,16]. The spatial curvature of the FRW geometry is not expected to appreciably alter the behavior of the metric near a mass source as long as the gravitational radius of the mass M whichever larger, be smaller than the radius of curvature. The metric is written in isotropic spherical coordinates [17] defined by $\mathrm{r} = r(1 + \frac{GM}{r})$, as [5,6]

$$ds^2 = -\left(\frac{1-\mu}{1+\mu}\right)^2 dt^2 + (1+\mu)^4 a^2(t)\left(dr^2 + r^2\,d\Omega^2\right), \tag{3}$$

where $a(t)$ is the asymptotic cosmological scale factor, $\mu = \frac{M}{2a(t)r}$, and M is the mass parameter of the source. Using spatial translations, $r = 0$ is chosen as the center of spherical symmetry. Here the asymptotically spatially flat FRW metric is considered, suggesting a cosmic scenario compatible to current cosmological data [18,19]. It is an exact solution of the field equations for an arbitrary mass M provided that $a(t)$ solves the Friedmann equation and

$$\rho(t) = \frac{3\dot{a}^2}{8\pi G a^2}, \tag{4}$$

which describes the matter energy density, with $H = \frac{\dot{a}}{a}$ being the Hubble parameter. The isotropic pressure associated to the fluid can be written as [20]

$$p = -\frac{1}{8\pi G}\left(3\frac{\dot{a}^2}{a^2} + 2\frac{1+\mu}{1-\mu}\left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right)\right), \tag{5}$$

having a homogeneous term proportional to H^2 and an inhomogeneous term as well, containing $\dot{H}$. The McVittie's solution has a curvature singularity at $\mu = 1$, since the Ricci scalar can be expressed in the form $R = 12H^2 + 6\dot{H}(\frac{1+\mu}{1-\mu})$; this singularity is interpreted as a cosmological big bang singularity [6].

McVittie's solution describes black holes embedded in expanding FRW Universes, when the Hubble parameter is positive. Some results advocate spherically symmetric solutions in asymptotically FRW cosmologies [21]. McVittie's solution is one sample among the geometries describing masses in FRW, where the mass parameter is a constant and the energy density is homogeneous. The inhomogeneous pressure is hence necessary. The initial big bang singularity is absent when $\dot{H} = 0$, and in fact the geometries (3) reduce to either the Schwarzschild or Schwarzschild–de Sitter solutions. In the case $a(t) = 1$ the McVittie's solution reduces to a black hole in flat space, and the metric (3) provides the Schwarzschild solution in isotropic coordinates.

A black string-like object associated to the McVittie's solution is led into the Schwarzschild and FRW ones as limiting cases. Therefore, we adopt an effective approach, studying the horizon variation Taylor expansion. The Weyl term on the brane is given by [3]

$$\begin{aligned} \mathcal{E}_{\theta\theta}&(r,t) \\ &= -\left[\rho^2\left(\frac{1}{6}\left(\frac{1+\mu}{1-\mu}\right)^2\left(2 - \frac{1+\mu}{1-\mu}\right)^2\right) + \frac{1}{4(1+\mu)^4 a^2}\right] \\ &\quad - \frac{\rho p}{4a^2}\left(1 + 4\mu + 5\mu^2 + 4\mu^3\right) \end{aligned} \tag{6}$$

for the McVittie's solution (3). By substituting the expressions (4), (5) for $\rho(t)$ and $p(t)$ above, as the black string horizon variation along the extra dimension is analyzed, the term $g_{\theta\theta}(x^\alpha, y)$ in (2) is given by