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Gauge threshold corrections and field redefinitions

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ABSTRACT

We review the argument for field redefinitions arising from threshold corrections to heterotic string gauge couplings, and the relation between the linear and the chiral multiplet. In the type IIB case we argue that the necessity for moduli mixing at one-loop order has not been clearly established, since this is based on extending the background field expansion way beyond its regime of validity. We also resolve some issues related to the form of non-perturbative terms resulting from gaugino condensation. This enables us to estimate the effective cutoff in the field theory by evaluating the non-perturbative superpotential by two different methods, and find that it is around the Kaluza–Klein scale, as one might have expected on general grounds of self-consistency.

1. Introduction

Background field methods in string theory (see for example [1–3]) are different from what one encounters in say, the quantum mechanical study of the behavior of an atom in an external magnetic field. In the latter case the background magnetic field is truly external to the system under study. In field theory applied say to condensed matter physics or atomic physics, where one is studying not the theory of the entire universe but some system within it, the concept of an external background makes sense. However when one studies theories such as the standard model coupled to gravity, which purports to be an effective theory of the entire universe, strictly speaking there is no meaning to the concept of an external background.

Of course the standard model is usually formulated in a particular metric background — namely the flat one. Here the reasoning is that for small standard model field energy densities, the Einstein equations are solved by the Minkowski metric. In principle it can be studied in a different metric background for example a cosmological background (FRW, de Sitter, etc.). However this smooth gravitational background field assumption is certainly expected to break down close to the Planck scale ($M_P \equiv 1/\sqrt{8\pi G_{Newton}}$). At such high energies one expects a significant contribution from virtual quantum gravity processes (such as the creation and annihilation of blackholes, wormholes, etc.) and the entire framework will break down.

String theory on the other hand is supposed to be an UV completion of field theory (or at least of a class of field theories hopefully including the standard model). The theory is not supposed to have any free parameters and is defined purely in terms of a fundamental dimensional constant — the string scale $l_{string} \equiv \sqrt{2\pi\alpha'} = 1/M_{string}$. If string theory were four-dimensional, M_{string} would essentially be the same as M_P. However in all tractable string theoretic constructions, there is an internal six-dimensional space with a volume which is typically large compared to l^6_{string}, i.e. $\mathrm{Vol} = \mathcal{V} l^6_{string}$ often with $\mathcal{V} \gg 1$. In this case there is a significant difference between the two scales and $M_S \simeq M_P/\sqrt{\mathcal{V}} \ll M_P$. There is also an additional scale, the Kaluza–Klein scale $M_{KK} = M_{string}/\mathcal{V}^{1/6} = M_P/\mathcal{V}^{2/3}$. For large $\mathcal{V}$ we thus have a hierarchy of scales $M_{KK} \ll M_{string} \ll M_P$. Four-dimensional field theory is strictly valid only below M_{KK}. Above this scale the theory is essentially ten-dimensional but remains a field theory. However above the scale M_{string}, the theory *cannot* be described by point like field theoretic degrees of freedom. The field theoretic description necessarily breaks down.

Consider first the case of strings propagating in a general metric background. In this case the world sheet theory is formulated as a generalized two-dimensional sigma model, and one derives consistency conditions (beta function equations) for the propagation of strings, in an expansion in the squared string length scale — the so-called α' expansion. For energy scales that are well below the string scale, this is a valid expansion and one can get useful information about the low energy limit of string theory in this way. However this expansion obviously breaks down at the string scale. In fact this is highlighted by the fact that the derivative expansion (as with generic higher derivative theories) has ghosts. These

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however appear at the string scale and are merely a sign that the theory has been pushed beyond its regime of validity. Thus any argument that is made about the interaction vertices of the theory — that is derived from the α' expansion — is invalid when the momentum flowing through those vertices is greater than the string scale. At such energies the low energy point field theory needs to be replaced by string (field?) theory.

The same is true for open string background calculations. Here one turns on a gauge field strength (magnetic or electric) background that is slowly varying (if not constant), to derive a low energy effective action in an α' expansion. Much important work has been done by using this technique. However for the most part this work has been used only to get an effective field theory valid below the string scale. In particular it does not make sense to consider the behavior of a term like $\frac{1}{g^2(\mu^2)}\mathrm{tr}(F_{uv}F^{uv})$ when the momentum flowing through this operator is greater than the string scale. The representation in terms of such a local operator simply breaks down at these energies. In particular this means that any argument which purports to have a low (around say few TeV) string scale and gauge coupling unification at some much higher scale (such as the standard GUT scale of 10^{16} GeV), cannot possibly make sense.

In this work we will address the question of string threshold corrections to gauge couplings by first reviewing the literature. The issue came up with Kaplunovsky's calculation of these effects in the context of the heterotic string [4]. The question that arose was how to account for moduli dependent corrections that could not be written as harmonic functions of the moduli as would be required in the usual chiral field formulation of the effective supergravity coming from string theory. The resolution in terms of the linear and chiral multiplet duality was discussed in [5,6] (BGG) and is reviewed in section two, where we also point out that the argument fails when the modulus in question has a non-linear superpotential term in the relevant chiral superfield. In the next section we review the arguments of Kaplunovsky and Louis which compared their field theoretic formula for the gauge coupling function with the corresponding string theory calculation. It should be stressed that this only involved momentum scales below the string scale (as we will make clear below). By contrast the work of Refs. [7–9], as well as the earlier work of [10–12] in connection with low scale strings, is essentially based on using the effective field theory and background string theory approaches well above the string scale. The point is that in identifying the infra-red region of the string theory integrand the upper (UV) cutoff is taken beyond the string scale. This is essentially the meaning of taking the infra-red region of the one-loop string integral all the way up to and beyond the string scale. The argument for the necessity of certain field redefinitions that result from this comparison are then very sensitive to exactly where the cutoffs are located, and can be changed by appropriately choosing the cutoffs at scales below the string scale. Finally we resolve a long standing puzzle regarding two different derivations of the non-perturbative (gaugino condensate) term in the superpotential.

2. General framework

2.1. Linear-chiral duality with $\partial_S W = 0$

As mentioned earlier the linear multiplet — chiral multiple duality has been discussed for example in [5] and [6] (BGG). We essentially follow the discussion of BGG except that the Kähler supergravity framework is replaced by the standard (minimal) one. We begin with the following action (with $\kappa = M_P^{-1} = 1$, $d^8z = d^4x\,d^4\theta$, $d^6z = d^4x\,d^2\theta$) for chiral superfields Φ (having superpotential W and Kähler potential K) coupled to supergravity and gauge fields with prepotential V and gauge field strength $\mathcal{W}$

$$\mathcal{A} = -3\int d^8z\,\mathbf{E}\exp\left[-\frac{1}{3}K(\Phi,\bar{\Phi};V)\right] + \left(\int d^8z\,\frac{\mathbf{E}}{2R}\left[W(\Phi) + \frac{1}{4}f(\Phi)\mathcal{W}\mathcal{W}\right] + h.c.\right). \tag{1}$$

Here $\mathbf{E}$ is the full superspace superdeterminant and R is the chiral superspace curvature. Note that the gauge coupling function f is in general a *holomorphic* gauge invariant function of the chiral superfields Φ. However threshold effects in string theory appeared to give non-holomorphic moduli dependent corrections to the gauge coupling function. The resolution lay in the introduction of the linear multiplet formulation of the gauge coupling function. The key observation here is that string theory moduli and the dilaton naturally arise in the string theory context as (components of) linear multiplets. This is because axionic partners of the scalar moduli are in fact second rank tensor fields. Thus for instance the axio-dilaton S, which is often identified in 4D as a chiral scalar, has its origins in a multiplet containing an antisymmetric second rank tensor $b_{\mu\nu}$, and thus naturally belongs to a linear multiplet.

Let us first focus on this case where in the usual formulation the gauge coupling function is given by $f = kS$ where S is the dilaton chiral superfield i.e. $\bar{\nabla}^{\dot{\alpha}}S = 0$. Let $\{\phi\}$ denote all the other chiral superfields in the theory. We take (for the moment) the superpotential W to be independent of S as is the case in perturbative string theory (except in IIB where it can be linear in S in the presence of internal fluxes). Let U be an unconstrained real superfield and modify the above action to the following form (for simplicity we ignore chiral fields which are charged under the gauge group),

$$\mathcal{A} = -3\int d^8z\,\mathbf{E}\exp\left[-\frac{1}{3}K(\phi,\bar{\phi},U)\right]\left(F(\phi,\bar{\phi},U) + U(S+\bar{S})\right) + \left(\int d^8z\,\frac{\mathbf{E}}{2R}\left[W(\phi) + \frac{1}{4}kS\mathcal{W}\mathcal{W}\right] + h.c.\right). \tag{2}$$

Here a trace over the gauge group is implicit in the gauge kinetic term, F is a real function of the chiral fields ϕ and the real field U, which will be determined by a normalization condition below. U is introduced as the linear superspace dual of the chiral superfield S (for more details see [5,6]) and as we will see this field becomes useful in interpreting threshold corrections in string theory. Note that we have also modified the Kähler potential to include dependence on U. Now we may eliminate the chiral superfield S in favor of the real superfield U, by using the equation of motion coming from taking the δS variation of this action to get[1];

$$-3\left(-\frac{1}{4}\bar{\nabla}^2 + 2R\right)\left(Ue^{-K/3}\right) + \frac{k}{4}\mathcal{W}^2 = 0. \tag{3}$$

Note that this equation and its conjugate are now effectively constraints on the initially unconstrained superfield U. In the absence of the gauge field kinetic term and K this is essentially the linear superfield constraint. Here we have a modified linear superfield.

Substituting (3) into (2) we get the (modified) linear multiplet formulation of the above action

$$\mathcal{A}_{LMF} = -3\int d^8z\,\mathbf{E}e^{-K(\phi,\bar{\phi},U)/3}F(\phi,\bar{\phi},U) + \left(\int d^8z\,\frac{\mathbf{E}}{2R}W(\phi) + h.c.\right). \tag{4}$$

[1] In taking a variation w.r.t. a chiral field we need to set $\delta S = (-\frac{1}{4}\bar{\nabla}^2 + 2R)\delta\Sigma$ where Σ is an unconstrained superfield.