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Rotating regular black holes

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ABSTRACT

The formation of spacetime singularities is a quite common phenomenon in General Relativity and it is regulated by specific theorems. It is widely believed that spacetime singularities do not exist in Nature, but that they represent a limitation of the classical theory. While we do not yet have any solid theory of quantum gravity, toy models of black hole solutions without singularities have been proposed. So far, there are only non-rotating regular black holes in the literature. These metrics can be hardly tested by astrophysical observations, as the black hole spin plays a fundamental role in any astrophysical process. In this Letter, we apply the Newman–Janis algorithm to the Hayward and to the Bardeen black hole metrics. In both cases, we obtain a family of rotating solutions. Every solution corresponds to a different matter configuration. Each family has one solution with special properties, which can be written in Kerr-like form in Boyer–Lindquist coordinates. These special solutions are of Petrov type D, they are singularity free, but they violate the weak energy condition for a non-vanishing spin and their curvature invariants have different values at $r = 0$ depending on the way one approaches the origin. We propose a natural prescription to have rotating solutions with a minimal violation of the weak energy condition and without the questionable property of the curvature invariants at the origin.

1. Introduction

Under the main assumptions of the validity of the strong energy condition and of the existence of global hyperbolicity, in General Relativity collapsing matter forms spacetime singularities [1]. At a singularity, predictability is lost and standard physics breaks down. In analogy with the appearance of divergent quantities in other classical theories, it is widely believed that even spacetime singularities are a symptom of the limitations of General Relativity and that they must be solved in a theory of quantum gravity. While quantum gravity effects are traditionally thought to show up at the Planck scale, $L_{\rm Pl} \sim 10^{-33}$ cm, making experimental and observational tests likely impossible, more recent studies have put forward a different idea [2,3]: $L_{\rm Pl}$ would be the quantum gravity scale for a system of a few particles, while the quantum gravity scale for systems with many constituents would be its gravitational radius. In these frameworks, even astrophysical black holes (BHs) of tens or millions Solar masses may be intrinsically quantum objects, macroscopically different from the Kerr BHs predicted in General Relativity.

While we do not yet have any mature and reliable candidate for a quantum theory of gravity, more phenomenological approaches have tried to somehow solve these singularities and study possible implications. In this context, an important line of research is represented by the work on the so-called regular BH solutions [4–7]. These spacetimes have an event horizon and no pathological features like singularities or regions with closed timelike curves. Of course, their metric is not a solution of Einstein's vacuum equations, but they can be introduced either with some exotic field, usually some form of non-linear electrodynamics, or modifications to gravity. They can avoid the singularity theorems because they meet the weak energy condition, but not the strong one.

The purpose of the present Letter is to construct rotating regular BH solutions. This is a necessary step to test these metrics with astrophysical observations [8–10]. The spin enters as the current-dipole moment of the gravitational field of a compact object and it is thus the leading order correction to the mass-monopole term. It is not possible to constrain deviations from classical predictions without an independent estimate of the spin. However, exact rotating BH solutions different from the classical Kerr–Newman metric are very hard to find. In most cases, including all the regular BH metrics currently available in the literature, we know only the non-rotating solution. In a few cases, we have an approximated solution valid in the slow-rotation limit [11], which is also not very useful for tests. A rotating solution in the Einstein–Gauss–Bonnet-dilaton

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gravity has been recently found numerically in Ref. [12], while proposals for some rotating quantum BHs have been suggested in [13,14].

2. Newman–Janis algorithm

Our strategy is to use the Newman–Janis transformation [15] (for more details, see Ref. [16]). Roughly speaking, the algorithm starts with a non-rotating spacetime and, at the end of the procedure, the spacetime has an asymptotic notion of angular momentum. The starting point is a spherically symmetric spacetime

$$ds^2 = f(r)\,dt^2 - \frac{dr^2}{f(r)} - h(r)\big(d\theta^2 + \sin^2\theta\, d\phi^2\big). \tag{1}$$

The *first step* of the algorithm is a transformation to get null coordinates $\{u, r, \theta, \phi\}$, where

$$du = dt - dr/f(r). \tag{2}$$

The *second step* is to find a null tetrad $Z^\mu_\alpha = (l^\mu, n^\mu, m^\mu, \bar{m}^\mu)$ for the inverse matrix in null coordinates

$$g^{\mu\nu} = l^\mu n^\nu + l^\nu n^\mu - m^\mu \bar{m}^\nu - m^\nu \bar{m}^\mu, \tag{3}$$

where the tetrad vectors satisfy the relations

$$\begin{aligned} &l_\mu l^\mu = m_\mu m^\mu = n_\mu n^\mu = l_\mu m^\mu = n_\mu m^\mu = 0, \\ &l_\mu n^\mu = -m_\mu \bar{m}^\mu = 1, \end{aligned} \tag{4}$$

and $\bar{x}$ is the complex conjugate of the general quantity x. One finds

$$\begin{aligned} &l^\mu = \delta^\mu_r, \qquad n^\mu = \delta^\mu_u - \frac{f(r)}{2}\delta^\mu_r, \\ &m^\mu = \frac{1}{\sqrt{2h(r)}}\left(\delta^\mu_\theta + \frac{i}{\sin\theta}\delta^\mu_\phi\right). \end{aligned} \tag{5}$$

The *third step* of the procedure is the combination of two operations. A complex transformation in the $r - u$ plane as follows

$$r \to r' = r + ia\cos\theta, \qquad u \to u' = u - ia\cos\theta, \tag{6}$$

together with a complexification of the functions $f(r)$ and $h(r)$ of the metric. The new tetrad vectors are

$$\begin{aligned} &l'^\mu = \delta^\mu_r, \qquad n'^\mu = \delta^\mu_u - \frac{\tilde{f}(r')}{2}\delta^\mu_r, \\ &m'^\mu = \frac{1}{\sqrt{2\tilde{h}(r')}}\left(ia\sin\theta\big(\delta^\mu_u - \delta^\mu_r\big) + \delta^\mu_\theta + \frac{i}{\sin\theta}\delta^\mu_\phi\right), \end{aligned} \tag{7}$$

where $\tilde{f}(r')$ and $\tilde{h}(r')$ are real functions on the complex domain. This step of the procedure is in principle completely arbitrary. In fact, in the original paper, Newman and Janis could not give a true explanation of the procedure if not that it works for the Kerr metric with a particular choice of the complexifications. The situation improved with Drake and Szekeres in [16], where the authors proved that the only Petrov D spacetime generated by the Newman–Janis algorithm with a vanishing Ricci scalar is the Kerr–Newman solution. Using the new tetrad in Eq. (3), we find the new inverse metric and then the metric. The non-vanishing coefficients of $g_{\mu\nu}$ are

$$\begin{aligned} &g_{uu} = \tilde{f}(r,\theta), \qquad g_{ur} = g_{ru} = 1, \\ &g_{u\phi} = g_{\phi u} = a\sin^2\theta\big(1 - \tilde{f}(r,\theta)\big), \\ &g_{r\phi} = g_{\phi r} = a\sin^2\theta, \qquad g_{\theta\theta} = -\tilde{h}(r,\theta), \\ &g_{\phi\phi} = -\sin^2\theta\big[\tilde{h}(r,\theta) + a^2\sin^2\theta\big(2 - \tilde{f}(r,\theta)\big)\big]. \end{aligned} \tag{8}$$

The *fourth and last step* of the algorithm is a change of coordinates. In some cases, we can write the metric in the Boyer–Lindquist form, in which the only non-vanishing off-diagonal term is $g_{t\phi}$. This requires a coordinate transformation of the form

$$du = dt' + F(r)\,dr, \qquad d\phi = d\phi' + G(r)\,dr, \tag{9}$$

where

$$\begin{aligned} &F(r) = \frac{\tilde{h}(r,\theta) + a^2\sin^2\theta}{\tilde{f}(r,\theta)\tilde{h}(r,\theta) + a^2\sin^2\theta}, \\ &G(r) = \frac{a}{\tilde{f}(r,\theta)\tilde{h}(r,\theta) + a^2\sin^2\theta}. \end{aligned} \tag{10}$$

This transformation is possible only when F and G depend on the coordinate r only. In general, however, the expressions on the right hand sides of (10) depend also on θ, and we cannot perform a global transformation of the form (9). If the transformation (9) is allowed and we go to Boyer–Lindquist coordinates, the non-vanishing metric coefficients of the rotating BH metric are:

$$\begin{aligned} &g_{tt} = \tilde{f}(r,\theta), \qquad g_{t\phi} = g_{\phi t} = a\sin^2\theta\big(1 - \tilde{f}(r,\theta)\big), \\ &g_{rr} = -\frac{\tilde{h}(r,\theta)}{\tilde{h}(r,\theta)\tilde{f}(r,\theta) + a^2\sin^2\theta}, \\ &g_{\theta\theta} = -\tilde{h}(r,\theta), \\ &g_{\phi\phi} = -\sin^2\theta\big[\tilde{h}(r,\theta) + a^2\sin^2\theta\big(2 - \tilde{f}(r,\theta)\big)\big]. \end{aligned} \tag{11}$$

In the case of the Schwarzschild solution, we have $f(r) = 1 - 2M/r$ and $h(r) = r^2$. In the Newman–Janis algorithm, we have to choose a complexification of the $1/r$ and of the r^2 term. In general, this prescription is not unique. However, since we know what the Kerr solution is, we know that if we take the following complexification

$$\frac{1}{r} \to \frac{1}{2}\left(\frac{1}{r'} + \frac{1}{\bar{r}'}\right), \qquad r^2 \to r'\bar{r}', \tag{12}$$

then this trick works well. The functions $f(r)$ and $h(r)$ become

$$f(r) \to \tilde{f}(r,\theta) = 1 - \frac{2Mr}{\Sigma}, \qquad h(r) \to \tilde{h}(r,\theta) = \Sigma, \tag{13}$$

where $\Sigma = r^2 + a^2\cos^2\theta$. In this case, the functions F and G in Eq. (10) depend on r only and we find the Kerr solution in Boyer–Lindquist coordinates

$$\begin{aligned} ds^2 = &\left(1 - \frac{2Mr}{\Sigma}\right)dt^2 + \frac{4aMr\sin^2\theta}{\Sigma}\,dt\,d\phi - \frac{\Sigma}{\Delta}\,dr^2 - \Sigma\,d\theta^2 \\ &- \sin^2\theta\left(r^2 + a^2 + \frac{2a^2Mr\sin^2\theta}{\Sigma}\right)d\phi^2, \end{aligned} \tag{14}$$

where $\Delta = r^2 - 2Mr + a^2$.

3. Hayward black hole

As first example of regular black hole, we consider the Hayward metric, whose analytic expression is quite simple [6]. The line element is given by Eq. (1), with the following $f(r)$

$$f(r) = 1 - \frac{2m}{r}, \quad m = m(r) = M\frac{r^3}{r^3 + g^3}, \tag{15}$$

where M is the BH mass and g is some real positive constant measuring the deviations from the classical Kerr metric. Let us note