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Why is the top-quark much heavier than other fermions?

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ABSTRACT

The recent ATLAS and CMS experiments show the first observations of a new particle in the search for the Standard Model Higgs boson at the LHC. We revisit the theoretical inconsistency of the fundamental high-energy cutoff with the parity-violating gauge symmetry of local quantum field theory for the Standard Model. This inconsistency suggests high-dimensional operators of fermion interactions, which are attributed to the quantum gravity. In this Letter, recalling the minimal dynamical symmetry breaking mechanism, we show that it is energetically favorable for the top-quark to acquire its mass via spontaneous symmetry breaking, whereas other fermions acquire their masses via explicit symmetry breaking.

1. Introduction

Since its appearance, the Standard Model for elementary particle physics has always been extremely peculiar. The parity-violating gauge couplings, the hierarchy of fermion masses, and flavor mixing have been at the center of a conceptual elaboration and an intensive experimental analysis that have played a major role in donating to mankind the beauty of the Standard Model (SM) for particle physics. Chiral gauge symmetries on the one hand and spontaneous/explicit breakings of these symmetries on the other play essential roles in understanding the parity-violating gauge couplings and mass spectra of fermions in the Standard Model. The Nambu–Jona-Lasinio model [1] for high energies and its effective counterpart for low energies, the Higgs model [2], provides an elegant description for the electroweak-breaking-scale, intermediate gauge boson masses and their relations. Nevertheless, we are still searching for a physical mechanism to explain the hierarchy of the fermion masses and the flavor mixing angles. Much theoretical effort has been made on this central issue since the Standard Model was established. Likewise, after a great experimental effort for many years, the ATLAS [3] and CMS [4] experiments have recently shown the first observations of a new particle in the search for the Standard Model Higgs boson at the LHC. This far-reaching result begins to shed light on this most elusive and fascinating arena of fundamental particle physics.

Assuming *the dynamical symmetry breaking* to dynamically generate the electroweak scale is theoretically rather attractive. Much effort has been made in this direction since the Standard Model was born. Apart from their theoretical problems, such as the fine-tuning and hierarchy problems, the models of the dynamical symmetry breaking are still far from quantitatively explaining experimental results, particularly given the recent discovery of the 125 GeV scalar particle [3,4]. Nevertheless, this newly discovered particle spurs it is a impetus and worthwhile for us to revisit the theoretical issue of the dynamical symmetry breaking to find any dynamical or symmetrical aspect that requires further studies to satisfactorily explain the experimental results.

When the top-quark mass m_t was discovered to be greater than $\sim 10^2$ GeV, several authors [5–8] in 1989 suggested that the symmetry breakdown of the Standard Model could be a dynamical mechanism of the Nambu–Jona-Lasinio or BCS type that intimately involves the top-quark at a high-energy scale Λ. This dynamical mechanism leads to the formation of a low-energy $\bar{t}t$-condensate, which is responsible for the top-quark, $W^\pm$ and Z° gauge bosons masses, and a composite particle of the Higgs type. Since then, many models based on this idea have been proposed and studied [9]. For our discussions on this idea, we will adopt the model for the minimal dynamical symmetry breaking via an effective four-fermion operator of the Nambu–Jona-Lasinio type

$$L = L_{\text{kinetic}} + G\big(\bar{\Psi}_L^{ia} t_{Ra}\big)\big(\bar{t}_R^b \Psi_{Lib}\big), \tag{1}$$

which was studied by Bardeen, Hill and Lindner (BHL) [8] in the context of a well-defined quantum field theory at the high-energy scale Λ; the coupling G is on the order of $1/\Lambda^2$.

To achieve the low-energy electroweak scale for the top-quark mass m_t by the renormalization group equations [6,8,10], this model (1) requires $\Lambda/m_t \gg 1$ with a drastically unnatural fine-tuning, which is known as the gauge hierarchy problem, and the top-quark mass m_t is determined by the infra red quasi-fixed point [10]. To have a natural scheme incorporating the effective four-fermion operator of the Nambu–Jona-Lasinio type (1), some strong

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technicolor (TC) dynamics at the $\sim$ TeV scale were invoked [11]; this scheme is preferentially coupled to the third quark family of top and bottom quarks. In addition, from the phenomenological point view, the newly discovered 125 GeV particle does not seems to be the neutral $\bar{t}t$-composite scalar that is significantly heavier than the top-quark mass [8]. The possibility of the 125 GeV particle being a light pseudoscalar, such as the top-pion [10], seems unlikely because the loop-suppressed couplings of light pseudoscalars to the SM gauge bosons are too small to generate the observed signal [12].

These discussions indicate that much effort is still required to study the issue of the minimal dynamical symmetry breaking that preferentially associated with the top-quark (the top-Higgs system) in the theoretical aspects of dynamics or/and symmetry (see for example [13]) to discover if the issue agree with experiments. In this Letter, we will focus on the theoretical question of why the dynamical symmetry breaking is minimally or preferentially associated with the top-quark. We assume that the high-dimensional operators of all fermion fields could be attributed to the new dynamics, such as the quantum gravity at the Planck scale $\Lambda_{\rm pl}$. For example, the four-fermion operator in the Einstein–Cartan theory can be obtained by integrating over static torsion fields at the Planck scale. It is conceivable that the new dynamics at the scale Λ should be on an equal footing with all the fermions in the Standard Model because the scale Λ is much larger than the masses of all the fermions. This finding raises a neutral question: why should the new dynamics preferentially act on the top-quark alone? It is the aim of this Letter to understand, from the dynamical point of view, a compelling possible answer to this question. Within the context of the Standard Model and dynamical symmetry breaking, we attempt to show that the minimal dynamical symmetry breaking (1) for the top-quark, by which this particle acquires its dynamical mass, is an energetically favorable configuration (the ground state) of the quantum field theory with the high-dimension operators of all the fermion fields at the cutoff Λ.

To explain why the top-quark is much heavier than the other fermions, some discussions of the origins of fermion masses are required; the top-quark mass is attributed to the dynamical symmetry breaking, whereas the other fermion masses are attributed to explicit symmetry breakings. In addition to the broken phase where the dynamical symmetry breaking occurs, we will discuss the symmetric phase for strong couplings where the dynamics of high-dimension operators of fermion fields form the massive composite states of three fermions the preserve the chiral gauge symmetries of the Standard Model. This dynamical feature gives a possibility to solve the fine-tuning problem and may hint at the composite scalar mass. The natural units $\hbar = c = 1$ are adopted, unless otherwise specified.

2. Dynamical symmetry breaking of the third quark family

To simplify the discussions and calculations, we first consider the third quark family only, the left-handed doublet $\Psi_L = (t_L, b_L)$ and the right-handed singlet $\psi_R = t_R, b_R$, and generalize the BHL proposal (1) as follows:

$$L = L_{\rm kinetic} + G\big(\bar{\Psi}_L^{ia}\psi_{Rja}\big)\big(\bar{\psi}_R^{jb}\Psi_{Lib}\big)$$
$$= L_{\rm kinetic} + G\big(\bar{\Psi}_L^{ia}t_{Ra}\big)\big(\bar{t}_R^{b}\Psi_{Lib}\big) + G\big(\bar{\Psi}_L^{ia}b_{Ra}\big)\big(\bar{b}_R^{b}\Psi_{Lib}\big), \qquad (2)$$

where a, b and i, j are, respectively, the color and flavor indexes of the top and bottom quarks. The fermion fields in $L_{\rm kinetic}$ are supposed to be massless. This Lagrangian has not only an $SU_c(3) \times SU_L(2) \times U_Y(1)$ gauge symmetry of the Standard Model but also a global $SU_L(2) \times U_R(1)$ flavor symmetry.

Following the BHL calculations, we have the gap equations for the induced top- and bottom-quark masses $m_t = -G\langle\bar{t}t\rangle$ and $m_b = -G\langle\bar{b}b\rangle$:

$$\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} = 2GN_c\frac{i}{(2\pi)^4}\int d^4l\,\big(l^2 - m^2\big)^{-1}\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}, \qquad (3)$$

where $m = m_t = m_b$. The result of evaluating Eq. (3) with a momentum-space cutoff Λ is

$$G^{-1} = \frac{N_c}{8\pi^2}\big[\Lambda^2 - m^2\ln\big(\Lambda^2/m^2\big)\big]. \qquad (4)$$

In addition to the trivial solution $m = 0$, the gap equation (3) has a nontrivial solution $m \neq 0$ for a sufficiently strong coupling, $G \geqslant G_c = 8\pi^2/(N_c\Lambda^2)$, where G_c is the "critical" coupling constant. The nontrivial solution $m_t = m_b = m \neq 0$ to the gap equation (4) is valid for both the t-channel and the b-channel. The gap equation (4) can be written as

$$\frac{1}{G_c} - \frac{1}{G} = (1/G_c)(m/\Lambda)^2\ln(\Lambda/m)^2$$
$$= \big(N_c m^2/8\pi^2\big)\ln(\Lambda/m)^2 > 0. \qquad (5)$$

When $m \ll \Lambda$, the four-fermion coupling $G = G(\Lambda, m_t) \to G_c$.

As will be shown below, the energetically favorable configuration of this defined quantum field theory (2) should be the configuration $(m_t = m \neq 0, m_b = 0)$, rather than the configuration $(m_t = m_b = m \neq 0)$. To calculate all possible bubble diagrams contributing to the vacuum energy of such defined quantum field theory (2), we must identify all the elementary and composite modes when the gap equation (4) is satisfied for $G \gtrsim G_c$ as well as the two-point Green functions of these modes.

Following the BHL calculations [8], we use the four-fermion interacting vertexes $G(\bar{\Psi}_L^{ia}t_{Ra})(\bar{t}_R^b\Psi_{Lib})$ (t-channel) and $G(\bar{\Psi}_L^{ia}b_{Ra}) \times (\bar{b}_R^b\Psi_{Lib})$ (b-channel) in the effective Lagrangian (2) to calculate the gap equation (3) or (4) for $m_t = m_b = m \neq 0$. In addition, we obtain the following: (1) the inverse propagators of the top t and bottom b quarks:

$$\Gamma_{t,b}^{-1}\big(p^2, m_{t,b}\big) = \big(\gamma_\mu p^\mu - m_{t,b}\big); \qquad (6)$$

(2) two composite scalar modes for the t- and b-channels and the inverse propagators of these modes

$$\Gamma_S^{-1}\big(p^2, m_{t,b}\big)$$
$$= 2N_c\big(p^2 - m_{t,b}^2\big)(4\pi)^{-2}\int_0^1 dx\,\ln\big\{\Lambda^2/\big[m_{t,b}^2 - x(1-x)p^2\big]\big\}; \qquad (7)$$

(3) two composite neutral-pseudoscalar modes for the t- and b-channels and the inverse propagators of these modes:

$$\Gamma_P^{-1}\big(p^2, m_{t,b}\big)$$
$$= 2N_c p^2(4\pi)^{-2}\int_0^1 dx\,\ln\big\{\Lambda^2/\big[m_{t,b}^2 - x(1-x)p^2\big]\big\}; \qquad (8)$$

(4) two composite charged-pseudoscalar modes for the t- and b-channels and the inverse propagators of these modes

$$\Gamma_F^{-1}\big(p^2, m_{t,b}\big)$$
$$= 8N_c p^2(4\pi)^{-2}\int_0^1 dx\,(1-x)\ln\big\{\Lambda^2/\big[m_{b,t}^2 x + m_{t,b}^2(1-x)$$
$$- x(1-x)p^2\big]\big\}. \qquad (9)$$