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Bounces, turnarounds and singularities in bimetric gravity

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ABSTRACT

In this Letter, we consider cosmological solutions of bimetric theory without assuming that only one metric is coupled to gravity. We conclude that any cosmology can be described by fixing the matter content of the space that we are not inhabiting. On the other hand, we show that some conclusions can still be extracted independently of the matter content filling both spaces. In particular, we can conclude the occurrence of some extremality events in one universe if we know that they take place in the other space.

1. Introduction

The theory of bimetric gravity assumes the existence of two metric tensors interacting with each other [1]. If these metric tensors have kinetic terms of the Einstein–Hilbert form and the equivalence principle is fulfilled, then the interaction between both metric fields would lead to an elegant modification of general relativity, being the existence of the other gravitational sector only measurable by its gravitational effects. One could wonder why this theory has not gained a renewed interest as soon as the impossibility of general relativity to describe our universe at astrophysical and cosmological scales (at least without introducing *ad hoc* new material components) has been suggested by several competing approaches (see for example [2–9]). The principal reason is that bimetric gravity generally presents a Boulware–Deser ghost [10], which implies an instability of the theory. Nevertheless, it has been recently shown that this undesired ghost can be discarded or controlled by considering particular interactions between the metrics [11,12] (see also [13] for a bigravity version of $f(R)$).

Thus, ghost-free bimetric cosmologies have been considered [14–16,20,21] and matched to the observational data with promising results [15,17]. However, most of these studies are restricted to considering only a particular class of models which assume that no material content is present in one of the spaces. In this Letter we explicitly consider the behavior of the theory in a cosmological scenario in the most general case, illustrating that, as it could be expected, the dynamics of our Universe would depend on the material content of the other space in this framework. As that hidden matter cannot be observed directly, this fact could be indicating a possible degeneracy of the theory. It seems that this degeneracy could only be cured if the matter content of both sectors is specified from the very beginning using some argument based on fundamental principles, or if localized solutions are also taken into account.

Due to the complexity of analyzing the general theory, one can consider whether at least some information about the cosmology of one sector can be extracted from the knowledge of the behavior of the other universe, even without specifying the matter content of the spaces. It is the main aim of the present Letter to show that this is indeed possible regarding the occurrence of extremality events, as bounces, turnarounds and singularities.

2. Cosmological solutions of the general theory

The action of the ghost-free bimetric gravity theory found in [11] has an interaction term which is a function of $\gamma=\sqrt{g^{-1}f}$. That action can be re-expressed as [18]

$$\begin{aligned} S = &-\frac{1}{16\pi G}\int d^4x\sqrt{-g}\left\{R(g)+2\Lambda\right\}+\int d^4x\sqrt{-g}L_{\rm m} \\ &-\frac{\kappa}{16\pi G}\int d^4x\sqrt{-f}\left\{\bar{R}(f)+2\bar{\Lambda}\right\}+\epsilon\int d^4x\sqrt{-f}\bar{L}_{\rm m} \\ &+\frac{m^2}{8\pi G}\int d^4x\sqrt{-g}L_{\rm int}(\gamma), \end{aligned} \tag{1}$$

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where the interaction Lagrangian is

$$L_{\rm int}=\beta_1 e_1(\gamma)+\beta_2 e_2(\gamma)+\beta_3 e_3(\gamma), \tag{2}$$

with

$$e_1(\gamma)=\mathrm{tr}[\gamma]; \tag{3}$$

$$e_2(\gamma)=\frac{1}{2}\left(\mathrm{tr}[\gamma]^2-\mathrm{tr}\left[\gamma^2\right]\right); \tag{4}$$

$$e_3(\gamma)=\frac{1}{6}\left(\mathrm{tr}[\gamma]^3-3\,\mathrm{tr}[\gamma]\,\mathrm{tr}\left[\gamma^2\right]+2\,\mathrm{tr}\left[\gamma^3\right]\right), \tag{5}$$

being elementary symmetric polynomials. It can be noted that the effective Newton constant for the f-space, $\epsilon G/\kappa$, would be equal to that of the g-space only if $\epsilon=\kappa$ [18]. Apart from the effective Newton constant, the theory is completely symmetric under the interchange of f and g due to the properties of the elementary symmetric polynomials [11,18].

If we consider a cosmological scenario, then, assuming that both metrics have the same sign of spatial curvature k, we can write

$$ds_g^2=-dt^2+a(t)^2\left[\frac{dr^2}{1-kr^2}+r^2\left(d\theta^2+\sin^2\theta\,d\phi^2\right)\right]. \tag{6}$$

and

$$ds_f^2=-N(t)^2\,dt^2+b(t)^2\left[\frac{dr^2}{1-kr^2}+r^2\left(d\theta^2+\sin^2\theta\,d\phi^2\right)\right], \tag{7}$$

where we are dismissing some special case solutions for particular values of the parameters[1] β_i [14]. The modified Friedman equations of both spaces can be obtained by brute force from the action (1) and metrics (6) and (7) [14,15], or noticing that this scenario can be described by the generalized Gordon ansatz [16]. These are

$$H_{\rm g}^2+\frac{k}{a^2}=\frac{m^2}{3}\rho+\frac{8\pi G}{3}\rho_{\rm m}+\frac{\Lambda}{3}, \tag{8}$$

and

$$H_{\rm f}^2+\frac{k}{b^2}=\frac{m^2}{3\kappa}\bar{\rho}+\frac{8\pi\epsilon G}{3\kappa}\bar{\rho}_{\rm m}+\frac{\bar{\Lambda}}{3}, \tag{9}$$

with

$$\rho=\frac{b}{a}\left(3\beta_1+3\beta_2\frac{b}{a}+\beta_3\frac{b^2}{a^2}\right), \tag{10}$$

for the g-space, and

$$\bar{\rho}=\frac{a}{b}\left(3\beta_3+3\beta_2\frac{a}{b}+\beta_1\frac{a^2}{b^2}\right), \tag{11}$$

for the f-space. We have defined the Hubble parameters as $H_{\rm g}=\dot{a}/a$ and $H_{\rm f}=\dot{b}/(Nb)$, where $\dot{}\equiv d/dt$. The appearance of the factor $1/N(t)$ in the Hubble parameter $H_{\rm f}$ can be expected by noting that metric (7) is not expressed in terms of the cosmic time of this space. We can define the cosmic time of the f-space as

$$\tau(t)=\int N(t)\,dt. \tag{12}$$

Thus, we are using the usual definition of the Hubble parameter also in the f-space, that is $H_{\rm f}=b'/b$ with $'\equiv d/d\tau$.

[1] Those solutions are not compatible with considering both metrics being diagonal in the same coordinate patch. That can be understood noting that the argument presented in [14] regarding the classification of solutions would be valid for any material content of both spaces since it is based on the symmetry of the spaces.

In view of action (1), one can note that due to the diffeomorphism invariance the matter stress energy tensor of both spaces is conserved. Thus, defining $w_{\rm m}(a)=p_{\rm m}/\rho_{\rm m}$ and $\bar{w}_{\rm m}(b)=\bar{p}_{\rm m}/\bar{\rho}_{\rm m}$, we have $\dot{\rho}_{\rm m}+3H_{\rm g}[1+w_{\rm m}(a)]\rho_{\rm m}=0$ and $\bar{\rho}'_{\rm m}+3H_{\rm f}[1+\bar{w}_{\rm m}(b)]\bar{\rho}_{\rm m}=0$, which can be integrated to obtain

$$\rho_{\rm m}(a)=\rho_{\rm m0}\exp\left[-3\int_{a_0}^{a}\left[1+w_{\rm m}(a)\right]\frac{da}{a}\right], \tag{13}$$

and

$$\bar{\rho}_{\rm m}(b)=\bar{\rho}_{\rm m0}\exp\left[-3\int_{b_0}^{b}\left[1+\bar{w}_{\rm m}(b)\right]\frac{db}{b}\right], \tag{14}$$

respectively. Taking into account the Bianchi identities, the stress energy tensor coming from the interaction term must be also conserved. This leads to [14–16]

$$\dot{b}(t)=N(t)\dot{a}(t). \tag{15}$$

Therefore, the Hubble parameter of the f-space can be expressed as $H_{\rm f}=\dot{a}/b$, which implies that the Friedmann equations of both spaces, (8) and (9), are coupled. This fact allowed the authors of Refs. [14] and [15] to solve the system (or to indicate how to obtain the solutions) in the particular case that no material content is considered in the f-space. In a similar way, we could, in principle, obtain the solution of the system (8) and (9), taking into account Eqs. (10), (11), (13), (14) and (15). In the first place, multiplying Eq. (9) by b^2/a^2, subtracting the resulting expression from Eq. (8), inserting Eqs. (10) and (11), and simplifying the result, we obtain the following algebraic equation:

$$c_4b^4+c_3ab^3-\frac{\bar{C}}{m^2}\bar{\rho}_{\rm m}ab^3+c_2a^2b^2+\frac{C}{m^2}\rho_{\rm m}a^3b$$
$$+c_1a^3b-c_0a^4=0, \tag{16}$$

where $c_4=\beta_3/3$, $c_3=\beta_2-\bar{\Lambda}/(3m^2)$, $c_2=\beta_1-\beta_3/\kappa$, $c_1=\Lambda/(3m^2)-\beta_2/\kappa$, $c_0=\beta_1/(3\kappa)$, $C=8\pi G/3$, $\bar{C}=8\pi G\epsilon/(3\kappa)$, and we are simplifying notation by assuming the dependence of both material energy densities in their corresponding scale factors, those are $\rho_{\rm m}(a)$ and $\bar{\rho}_{\rm m}(b)$. In the second place, considering particular forms for $w_{\rm m}(a)$ and $\bar{w}_{\rm m}(b)$ in Eqs. (13) and (14), and inserting the results in Eq. (16), the LHS of Eq. (16) can be considered as a polynomial on b. Thus, once $w_{\rm m}(a)$ and $\bar{w}_{\rm m}(b)$ are fixed, Eq. (16) can be solved to obtain b as a function of a, at least in principle (e.g. for $\bar{w}_{\rm m}=0$ we have a quartic equation which can be analytically solved [15]). In the third place, the obtained function $b(a)$ can be inserted in Eq. (10) and the result in Eq. (8), which could be integrated considering again Eq. (13). Moreover, up to now, most studies have paid attention only to the physics of one space (see Refs. [16,19] for two interesting exceptions[2]), probably because if no material content is considered in the other space it cannot describe an inhabited universe. Nevertheless, once we have the functions $a(t)$ and $b(a)$, it is straightforward to obtain $b(t)$. This scale factor can be more properly interpreted when it is expressed in terms of its cosmic time, $b(\tau)$, which can also be done easily using Eqs. (15) and (12). In summary, this procedure would allow us to know the evolution of both universes once $w_{\rm m}(a)$ and $\bar{w}_{\rm m}(b)$ are fixed.

On the other hand, it seems that one could describe any possible cosmology in the g-space, i.e. any possible combination of $a(t)$ and $w_{\rm m}(a)$, by assuming a different matter content in the

[2] Note that in Ref. [16] a different definition of $H_{\rm f}$ is used.

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