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Testing scaling laws for the elastic scattering of protons

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ABSTRACT

Theoretical proposals of scaling laws for the differential elastic scattering cross sections of protons are confronted with experimental data over a wide energy range. Different combinations of the transferred momentum and energy resulting from the solution of the definite partial differential equation are attempted as scaling variables. Reasonable scaling of the differential cross sections in the diffraction cone has been shown for one of these variables. The violation of the geometrical scaling is ascribed to the increase of the proton blackness with energy. The origin of high-t region violations of scaling laws is discussed.

The differential cross section for elastic scattering of particles $d\sigma(s,t)/dt$ is the only measurable characteristics of this process. At any fixed energy s, one presents a one-dimensional plot of its dependence on the transferred momentum t. However, the possibility that the differential cross sections might be described as functions of a single scaling variable representing a definite combination of energy and transferred momentum has been discussed [8,11]. No rigorous proof of this assumption has been proposed. Recently this property was obtained [16,17] from the solution of the partial differential equation for the imaginary part $\mathrm{Im}\,A(s,t)$ of the elastic scattering amplitude. The equation has been derived by equating the two expressions for the ratio of the real to imaginary parts of the amplitude $\rho(s,t)$. They were known from the local dispersion relations [20,10,18,19] with the s-derivative and from the linear t-approximation [8,21] with the t-derivative. These expressions are, correspondingly,

$$\rho(s,t)=\frac{\pi}{2}\left[\frac{\partial \ln \mathrm{Im}\,A(s,t)}{\partial \ln s}-1\right] \tag{1}$$

and

$$\rho(s,t)=\rho(s,0)\left[1+\frac{\partial \ln \mathrm{Im}\,A(s,t)}{\partial \ln|t|}\right]. \tag{2}$$

Therefrom the following partial differential equation is valid

$$p-f(x)q=1+f(x), \tag{3}$$

where $p=\partial u/\partial x$; $q=\partial u/\partial y$; $u=\ln \mathrm{Im}\,A(s,t)$; $f(x)=2\rho(s,0)/\pi\approx d\ln\sigma_t/dx$; $x=\ln s$; $y=\ln|t|$; σ_t is the total cross section. The variables s and $|t|$ should be considered as scaled by the corresponding constant factors s_0^{-1} and $|t_0|^{-1}$.

Eq. (3) can be rewritten as

$$\frac{\partial u}{\partial \ln\sigma_t}-\frac{\partial u}{\partial \ln t}=1+\frac{d\ln s}{d\ln\sigma_t}. \tag{4}$$

The general solution of Eq. (4) reveals the scaling law

$$\frac{t}{s}\,\mathrm{Im}\,A(s,t)=\phi(t\sigma_t). \tag{5}$$

For the differential cross section it looks like

$$t^2 d\sigma/dt=\phi^2(t\sigma_t), \tag{6}$$

if the real part of the amplitude is neglected compared to the imaginary part. Thus the scaling law is predicted not for the differential cross section itself but for its product to t^2. Let us note that the often used ratio (see, e.g., [9]) of $d\sigma/dt$ to $d\sigma/dt|_{t=0}\propto\sigma_t^2$ is also a scaling function at the $t\sigma_t$-scale. However, expression (6) is more suitable for comparison with experiment.

The scaling law with the $t\sigma_t$-scale is known as "geometrical scaling". Different aspects of its violation are often discussed. Here we contribute another view of this problem.

The geometrical scaling violation is seen in Fig. 1 in the diffraction cone especially for the TOTEM data at 7 TeV. With the common approximation $d\sigma/dt\propto\exp(Bt)$ in the diffraction cone one gets that the maximum of the function $t^2d\sigma/dt$ displayed in Fig. 1 should be positioned at $t_m\sigma_t=2\sigma_t/B=16\pi(1-\exp(-\Omega(s)))$. It is important that it depends only on the opacity of protons $\Omega(s)$

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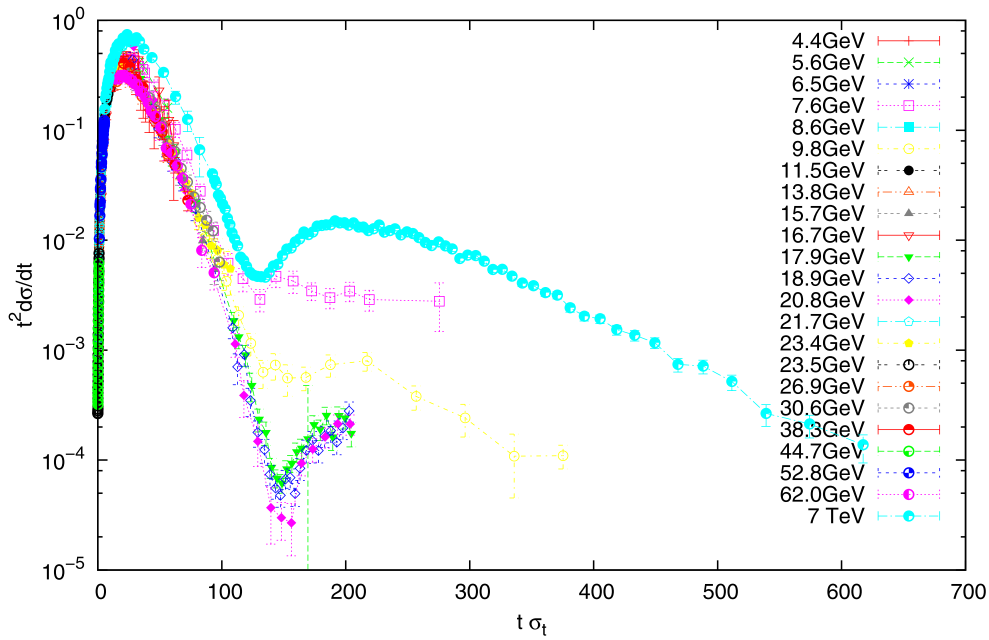

Fig. 1. (Color online.) The values of $t^2 d\sigma/dt$ for pp-scattering at energies $\sqrt{s}$ from 4.4 GeV to 7 TeV as functions of $t\sigma_t$ with σ_t provided by the corresponding experiment. The scale on the abscissa axis is defined by $t_0 = -1$ GeV^2 and σ_t in GeV^{-2}. The data are from [1,2,6,5].

(see the Table 1 in the review paper [14,12]) but not on their radii. The shift of the maximum is completely determined by the energy increase of the opacity. The relation of the opacity $\Omega(s, \mathbf{b})$ at any given impact parameter $\mathbf{b}$ with the amplitude looks like

$$A(s, t = -q^2) = 2is \int d^2 b e^{i\mathbf{qb}} \left(1 - e^{-\Omega(s,\mathbf{b})}\right). \tag{7}$$

The scaling is violated due to the stronger energy dependence of σ_t compared to the diffraction cone slope B observed in experiment. Their ratio is approximately proportional to the ratio of the elastic to total cross sections because

$$\frac{\sigma_t(1 + \rho^2(s, t = 0))}{16\pi B} \approx \frac{\sigma_{el}}{\sigma_t} \tag{8}$$

which, in its turn, is defined by the blackness of protons. Their energy dependencies coincide only if the opacity saturates.

Thus this simple geometrical scaling is not fulfilled at very high energies, even at low transferred momenta. We show below how this problem can be cured. Moreover, the scaling is much more strongly violated outside the diffraction region. It is also discussed in what follows.

In attempts to cure these problems we turn to the assumptions used in the derivation of the scaling law (6). The neglect by the real part of the amplitude in (6) is the most evident one. Its contribution is easily estimated using Eqs. (1), (2). One gets

$$t^2 d\sigma/dt = \phi^2(t\sigma_t)\left[1 + \left(d\ln\phi/d\ln(t\sigma_t)\right)^2 \rho^2(s, t = 0)\right]. \tag{9}$$

The second term violates scaling – albeit it is very small because of smallness of $\rho(s, t = 0)$ and does not pose any problem.

Another approximation is involved in the relation (1). It was guessed as the extension to non-zero transferred momenta of the first term in the series expansion of the exact expression for $\rho(s, 0)$ which looks like

$$\rho(s,0) \approx \frac{1}{\sigma_t}\left[\tan\left(\frac{\pi}{2}\frac{d}{d\ln s}\right)\right]\sigma_t$$
$$= \frac{1}{\sigma_t}\left[\frac{\pi}{2}\frac{d}{d\ln s} + \frac{1}{3}\left(\frac{\pi}{2}\right)^3 \frac{d^3}{d\ln s^3} + \cdots\right]\sigma_t. \tag{10}$$

The terms with higher derivatives s were neglected. This assumption is quite reasonable because their contribution seems negligible for experimentally measured energy dependence of σ_t and to any analytical fits.

More serious questions arise concerning Eq. (2). It looks as if only the first term in the t-expansion of $\rho(s, t)$ is taken into account in this relation. It could be satisfactory in the diffraction cone where $\operatorname{Im} A(s, t) \propto \exp(Bt/2)$. Let us note here that according to (2) $\rho(s, t)$ should become ever smaller in the diffraction cone crossing zero at $t = t_m$ and be negative at larger $|t|$. Moreover, even dealing within a linear approximation, one gets negative values of ρ in the region directly attached to the diffraction cone (known as the Orear region by the name of its discoverer) if $\rho(s, t)$ is approximated by its constant average value there [13].

The behavior of $\rho(s, t)$ may become there strongly non-linear in t [13]. The solution of the unitarity equation for the imaginary part of the amplitude in the Orear region [3,4] (see also the review paper [14,12]) $u \propto -[2B \ln(4\pi B/\sigma_t f_\rho)|t|]^{0.5}$ is quite complicated and does not seem to satisfy the scaling law. Here, $f_\rho = 1 + \rho(s, 0)\rho_l$ with ρ_l denoting the average value of ρ in this region. If ρ_l is replaced by the non-averaged $\rho(s, t)$ and such u inserted in Eq. (2), then the derivative of the imaginary part naturally produces the derivative of $\rho(t)$. The resulting differential equation for $\rho(t)$ was solved. The strongly non-linear t-dependence with large negative values of ρ in the Orear region was obtained. The more rigorous approach was also attempted.

These indications suit quite well the results of the fit in the Orear region at 7 TeV [15] where the negative and quite large values of $\rho \approx -2.1$ had to be chosen in that region. No such tendency is provided directly by Eq. (2).

The violation of the simple geometrical scaling law (6) is clearly seen in Fig. 1. In the diffraction cone it is rather well satisfied at most energies except the highest one of 7 TeV. In the Orear region there is no scaling even at lower energies. We ascribe it to necessary modifications of Eq. (2). Until now we are unable to propose any admissible generalization of Eq. (2) at large t.

Nevertheless, we try to modify it at small t in such a way to get better scaling inside the diffraction cone even at 7 TeV compared