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The quark mass gap in a magnetic field

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ABSTRACT

A magnetic field and the resulting Landau degeneracy enhance the infrared contributions to the quark mass gap. The gap does not grow arbitrarily, however, for models of asymptotic free interactions. For $B \to \infty$, the magnetic field decouples from the dimensionally reduced self-consistent equations, so that the gap behaves as $\sim \Lambda_{\rm QCD}$ (or less), instead of $\sim \sqrt{|eB|}$. On the other hand, the number of participants to the chiral condensate keeps increasing as $\sim |eB|$ so that $|\langle \bar{\psi}\psi \rangle| \sim |eB|\Lambda_{\rm QCD}$. After the mass gap stops developing, nothing tempers the growth of screening effects as $B \to \infty$. These features are utilized to interpret the reduction of critical temperatures for the chiral and deconfinement phase transitions at finite B, recently found on the lattice. The structures of mesons are analyzed and light mesons are identified. Applications for cold, dense quark matter are also briefly discussed.

1. Introduction

In past decades, systems in a magnetic field (B) have been useful laboratories to test theoretical ideas. A famous example is a system of cold atoms, in which a magnetic field controls the strength of the interactions. In QCD, similar utilities are also expected for the lattice Monte Carlo simulation at finite B [1–4]. In particular, we can study the nonperturbative gluon dynamics through polarization effects, controlling quark dynamics by a magnetic field. Such information may help the studies of cold, dense quark matter [5].

It seems that lattice studies already confirmed some of the theoretical ideas. At $T = 0$, a magnetic field enhances the size of the chiral condensate due to magnetic catalysis [6,7]. A key feature of this phenomenon is the effective dimensional reduction. For $B \neq 0$, the phase space for the low energy particles and anti-particles is $\sim |eB| \int \mathrm{d}p_\parallel$, increasing the number of participants to the formation of the chiral condensate. This should be contrast to the $B = 0$ case, where phase space quickly decreases as $\sim \int |p|^2\, \mathrm{d}|p|$ in the infrared region. In this case, due to the small number of participants, the system needs sufficiently strong attractive forces to form chiral condensates.

On the other hand, some surprises have been provided as well [4]. While B increases the chiral condensate below the (pseudo-)critical temperatures for the chiral restoration (T_χ) and deconfinement ($T_{\rm D}$), those temperatures themselves decrease. This might contradict with our intuitions, if we think that a larger chiral condensate should generate a greater quark mass gap. Such thinking would suggest that (i) a larger quark mass gap should suppress thermal quark fluctuations, leading to increasing T_χ, and (ii) a larger quark mass gap suppresses quark loops, so that the results should approach to the pure gauge results, leading to increasing $T_{\rm D}$.

To resolve this apparent contradiction, we shall argue that the quark mass gap at $T = 0$ can stay around $\sim \Lambda_{\rm QCD}$ (or less) for large B. Then we can imagine that the gap at $T < T_{\chi,\rm D}$ also stays around $\sim \Lambda_{\rm QCD}$, without strongly suppressing thermal quark fluctuations and quark loops. If this is the case, the decreasing of critical temperatures would not be so unnatural.

In addition, the aforementioned behavior of the quark mass gap does not contradict with the growing behavior of the chiral condensate, but instead naturally explains its B-dependence at $T = 0$. In fact, the lattice results in [4] showed the behavior $\langle \bar{\psi}\psi \rangle^B_{T=0} \sim |eB|\Lambda_{\rm QCD}$, for $|eB| \geqslant 0.3\ \mathrm{GeV}^2 \gg \Lambda^2_{\rm QCD}$ ($\simeq 0.04\ \mathrm{GeV}^2$). Noting that the relation under the Landau quantization,

$$\langle \bar{\psi}\psi \rangle_{\rm 4D} \sim |eB| \times \langle \bar{\psi}\psi \rangle_{\rm 2D}, \tag{1}$$

we can see that $\langle \bar{\psi}\psi \rangle_{\rm 2D}$ or the quark mass gap must be nearly B-independent and $O(\Lambda_{\rm QCD})$.

In this work we will carry out all the calculations in the large $N_{\rm c}$ limit.[1] The use of the large $N_{\rm c}$ is motivated by at least three reasons: (i) At large $N_{\rm c}$, gluons are not screened, so the *nonperturbative* forces (i.e. the forces in the infrared) are stronger than the $N_{\rm c} = 3$ case. Such forces can be used to set the *upper bound* of the

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[1] The large $N_{\rm c}$ limit in a magnetic field was also studied in Ref. [9] from a different perspective from ours.

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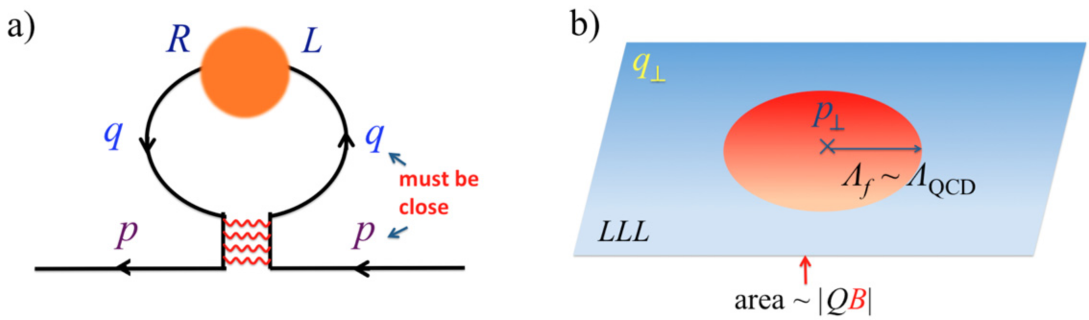

Fig. 1. (a) The Schwinger–Dyson equation at large N_c for the model in Eq. (4). (b) The distribution of states in the lowest Landau level for fixed p_z. A state with momentum $p_\perp$ can strongly couple to states within a domain of $|q_\perp - p_\perp| \leqslant \Lambda_f \sim \Lambda_{\rm QCD}$.

quark mass gap. (ii) The large N_c limit has captured many qualitative aspects of the confined phase at $B = 0$. Therefore it is worth thinking and testing this approximation in the confined phase at finite B, since its validity and invalidity are not evident *a priori*. (iii) It is easy to imagine how the $1/N_c$ corrections *qualitatively* modify the large N_c results, and such corrections just provide welcomed effects for our scenario (see below).

We will use the large N_c limit to just claim that the quark mass gap does not grow much beyond $\Lambda_{\rm QCD}$. To explain the reduction of the critical temperatures, in addition we have to argue the $1/N_c$ corrections. The quark loops as the $1/N_c$ corrections screen the nonperturbative forces. As B increases, the screening effects become larger because more low energy particles can participate to the gluon polarization, due to the enhanced Landau degeneracy $\sim |eB|$ in the lowest Landau level (LLL) [8]. If the quark mass gap stops growing as suggested in our scenario, there is nothing to suppress the growth of the screening effects as B increases.[2] Therefore the nonperturbative forces are reduced at large B, and such reduction should lower the critical temperatures for given B. In addition, hadronic fluctuations as the $1/N_c$ corrections also grow as B increases, helping the chiral symmetry to restore [10].[3]

We will argue that the demanded (nearly) B-independent gap of $O(\Lambda_{\rm QCD})$ can be derived, provided that it is dominantly created by the nonperturbative force mediated by the IR gluons. In particular, both the IR enhancement (that is more drastic than the perturbative $1/p^2$ case) and the UV suppression of the gluon exchanges are crucial for our discussions. Since we will deal with the LLL which is essentially soft physics in the present Letter, IR enhanced gluon is a key feature in this study (see [12] for a review). If we include only the perturbative $1/p^2$ force, the gap is much smaller than $\Lambda_{\rm QCD}$ and depends on B at most logarithmically. Similar arguments have been used in studies of the quark mass function at finite quark density [13,14].

To illustrate our points, we first consider the NJL model which does not have the abovementioned properties. For $|eB| \to \infty$, the gap equation within the LLL approximation is

$$M_{\rm NJL}(B) = G\,{\rm tr}\,S(x,x) \to G\frac{|eB|}{2\pi}\int\frac{{\rm d}q_z}{2\pi}\frac{M_{\rm NJL}(B)}{\sqrt{q_z^2+M_{\rm NJL}^2(B)}}f(q_z,B;\Lambda), \tag{2}$$

where $f(q_z,B;\Lambda)$ is some UV regulator function. The contact interaction couples all states in the LLL so that the Landau degeneracy factor $|eB|$ for the LLL appears. The intrinsic property of the model is that the chiral condensate has the same B-dependence as the mass gap:

$$\langle\bar\psi\psi\rangle^B_{\rm NJL} \simeq -\frac{1}{G}M_{\rm NJL}(B). \tag{3}$$

Depending on the regularization schemes, $M_{\rm NJL}(B)$ can be $\sim |eB|^{1/2}$ (proper time regularization [15]), or $\sim \Lambda$ (four momentum cutoff [10]), or else. Each scheme has its own problems. For schemes predicting the growing behavior of the chiral condensate [16], the quark mass gap also develops as B increases. Then at finite temperature, thermal quark contributions are largely reduced so that the increasing chiral restoration temperature is naturally expected. On the other hand, if the mass gap approaches to constant, the chiral condensate also does, contradicting with the lattice results. Therefore, as far as the relation like (3) is retained, it seems that we have to abandon either the increasing chiral condensate or the reduction of critical temperatures.

This dilemma can be bypassed if we use the gluon exchange type interactions with the IR enhancement and UV suppression. To emphasize the point, we consider a simple model for the gluon exchange with these features (for the moment we ignore spinor structures),

$$D(q) = G\theta\left(\Lambda_f^2 - \vec q^{\,2}\right) \quad (\Lambda_f \sim \Lambda_{\rm QCD}) \tag{4}$$

which was proposed in [14]. In this model, the quark mass function appears to be momentum dependent. For $B \to \infty$, the Schwinger–Dyson equation at large N_c (Fig. 1a) is

$$M(p;B) \simeq G\int\frac{{\rm d}^4q}{(2\pi)^4}\,{\rm tr}\,S^{LLL}_{\rm 2D}(q_z)\theta\left(\Lambda_f^2 - |\vec p - \vec q|^2\right), \tag{5}$$

where B is used to reduce the quark propagator to the $(1+1)$-dimensional one and to separate higher Landau levels from the LLL. In contrast to the previous case, the factor $|eB|$ does not appear in front of the integral. This is because the interaction (4) does not couple all the states in the LLL, but couples the states having similar momenta (Fig. 1b). This feature makes the gap B-independent. In fact, carrying out the integral over the transverse momenta, we get

$$M(p;B) \simeq \frac{G}{2\pi}\int\frac{{\rm d}q_z}{2\pi}\frac{M(q;B)}{\sqrt{q_z^2+M^2(q;B)}} \times\theta\left(\Lambda_f^2 - |p_z - q_z|^2\right)F(p_z - q_z), \tag{6}$$

where $F(k_z) = \sqrt{\Lambda_f^2 - |k_z|^2}$. Note that the equation does not have any explicit B dependence, so the mass gap is solely determined by the scale Λ_f, i.e. $M(p;B) = M_{\Lambda_f}(p)$.

[2] This suggests that even at $T = 0$, the nonperturbative gluons will be screened out at some critical value of B such that the screening mass, $m_D \sim g_s|eB|^{1/2} \sim N_c^{-1/2}|eB|^{1/2}$, becomes comparable to $\Lambda_{\rm QCD}$ (see also Section 4).

[3] For hadronic fluctuations at small $|eB|$, see Ref. [11], where chiral perturbation theory should be at work.

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