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Evaporation of (quantum) black holes and energy conservation

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ABSTRACT

We consider Hawking radiation as due to a tunneling process in a black hole were quantum corrections, derived from Quantum Einstein Gravity, are taken into account. The consequent derivation, satisfying conservation laws, leads to a deviation from an exact thermal spectrum. This has consequences for the *information loss paradox* since the non-thermal radiation is shown to carry information out of the black hole. Under the appropriate approximation, a quantum corrected temperature is assigned to the black hole. The evolution of the quantum black hole as it evaporates is then described by taking into account the full implications of energy conservation as well as the backscattered radiation. It is shown that, as a critical mass of the order of Planck's mass is reached, the evaporation process decelerates abruptly while the black hole mass decays towards this critical mass.

1. Introduction

Based on results of quantum field theory on a fixed curved background (Schwarzschild's solution) Hawking showed in 1975 [1] that black holes radiate a thermal spectrum of particles and derived an exact expression for their entropy. Only recently [2] Hawking radiation has been derived taking into account the backreaction effect of the radiation on the black hole thanks to the requirement of energy conservation. Moreover, the method proposed in [2] corresponds with the heuristic picture most commonly proposed of pair creation near the horizon of the black hole and the corresponding tunneling of particles.

One of the most interesting features of the tunneling method is that it shows that new terms appear in the distribution function which deviate it from pure thermal emission, i.e. the standard Boltzmann distribution. Since the claim of information loss in black holes [3] has as one of its pillars that black holes have an exact thermal spectrum, it seems that the deviation from thermality could have consequences for the *information loss paradox*, i.e., the radiation could allow the information to escape the black hole.

Of course, this picture is incomplete since, in order to describe the last stages of black hole evaporation, one should take into account quantum gravity effects. A step in this direction was taken by Bonanno and Reuter in [4] by introducing an effective quantum spacetime for spherically symmetric black holes based on the Quantum Einstein Gravity approach. They did this by using the idea of the Wilsonian renormalization group [5] in order to study quantum effects in the Schwarzschild spacetime. Specifically, they obtained a *renormalization group improvement* of the Schwarzschild metric based upon a scale dependent Newton constant G obtained from the exact renormalization group equation for gravity [6] describing the scale dependence of the effective average action [7,8]. Later, in [9], the same authors described the strict thermal evolution of the improved black hole by estimating Hawking's energy flux directly from Stefan–Boltzmann's law.

Our aim in this Letter is to analyze the evaporation of a quantum black hole (specifically, the solution found in [4]) thanks to the consideration of a tunneling process in its horizon and, consequently, satisfying energy conservation. This has to allow us to find the quantum corrections to the temperature of the quantum black hole under the appropriate approximations. On the other hand, our study of the evolution of the evaporating quantum black hole satisfying energy conservation will take into account the effect of the non-negligible backscattered radiation. This analysis is intended to shed some light into the escape of information from black holes throughout their complete evaporation process as well as into the study of the lasts stages of their evaporation.

The Letter has been divided as follows. Section 2 introduces the solution for the quantum black hole (the *improved Schwarzschild spacetime*) and its main properties. In Section 3 we summarize black hole radiation according to the tunneling method in an extended improved Schwarzschild spacetime. In Section 4 we consider the backscattering of the emitted radiation taking into account energy conservation. This allows us, in Section 5, to evaluate the luminosity of a quantum black hole when energy conservation

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is imposed and to compare it with the standard 'thermal' result. The evolution of an evaporating quantum black hole fulfilling energy conservation is treated in Section 6. In Section 7 we analyze the escape of information throughout the evaporation process. Finally the results are discussed in Section 8.

2. Improved Schwarzschild solution

The *renormalization group improved* Schwarzschild solution found by Bonanno and Reuter [4] can be written as

$$ds^2 = -\left(1 - \frac{2G(R)M}{R}\right) dt_S^2 + \left(1 - \frac{2G(R)M}{R}\right)^{-1} dR^2 + R^2\, d\Omega^2, \tag{2.1}$$

where

$$G(R) = \frac{G_0 R^3}{R^3 + \tilde{\omega} G_0 (R + \gamma G_0 M)}, \tag{2.2}$$

G_0 is Newton's universal gravitational constant, M is the mass measured by an observer at infinity and $\tilde{\omega}$ and γ are constants coming from the non-perturbative renormalization group theory and from an appropriate "cutoff identification", respectively. Despite the preferred value for γ is $\gamma = 9/2$, it is argued [4,9] that the qualitative properties of this solution are fairly insensitive to the precise value of this constant. In fact, the important differences appear only near the singularity. For instance, for the value $\gamma = 9/2$ the usual singularity in the classical Schwarzschild solution does not exist in the improved solution while if, in order to simplify the calculations, one chooses $\gamma = 0$ there is still a scalar curvature singularity at $R = 0$, even if it has a milder character than in the classical case [10]. On the other hand, $\tilde{\omega}$ can be found by comparison with the standard perturbative quantization of Einstein's gravity (see [11] and references therein). It can be deduced that its precise value is $\tilde{\omega} = 167/30\pi$, but again the properties of the solution do not rely on its precise value as long as it is strictly positive. A relevant fact with regard to $\tilde{\omega}$ is that it carries the quantum modifications. In effect, if we make explicit Planck's constant in (2.2), it can be considered that $\tilde{\omega} = 167\hbar/30\pi$ and, thus, $\tilde{\omega} = 0$ would turn off the quantum corrections.

The horizons in this solution can be found by solving

$$1 - \frac{2G(R)M}{R} = 0.$$

The number of positive real solutions to this equation correspond to the positive real solutions of a cubic equation and depends on the sign of its discriminant or, equivalently, on whether the mass is bigger, equal or smaller than a critical value M_{cr}. In general, the critical value takes the form

$$M_{cr} = a(\gamma)\sqrt{\frac{\tilde{\omega}}{G_0}} = a(\gamma)\sqrt{\tilde{\omega}}\, m_p \sim \sqrt{\tilde{\omega}}\, m_p, \tag{2.3}$$

where m_p is Planck's mass and the function $a(\gamma)$ has, in general, an involved expression that, for reasonable values of γ satisfies $a(\gamma) \sim 1$. In particular, the preferred value $\gamma = 9/2$ provide us with

$$M_{cr} = \frac{1}{24}\sqrt{\frac{1}{2}\left(2819 + 85\sqrt{1105}\right)}\sqrt{\frac{\tilde{\omega}}{G_0}} \simeq 2.21\sqrt{\tilde{\omega}}\, m_p \simeq 2.94 m_p,$$

while the value $\gamma = 0$ implies

$$M_{cr} = \sqrt{\frac{\tilde{\omega}}{G_0}} \simeq 1.33 m_p.$$

If $M > M_{cr}$ then the equation has two positive real solutions $\{R_-, R_+\}$ satisfying $R_- < R_+$. The inner solution R_- represents a novelty with regard to the classical solution, while the outer solution R_+ can be considered as the *improved Schwarzschild horizon*, i.e., the Schwarzschild horizon when the quantum modifications are taken into account. The 'improvement' in this horizon can be made apparent for masses much bigger than Planck's mass if one expands R_+ in terms of m_p/M obtaining

$$R_+ \simeq 2G_0 M\left[1 - \frac{(2+\gamma)}{8}\tilde{\omega}\left(\frac{m_p}{M}\right)^2\right].$$

On the other hand, if $M = M_{cr}$ then there is only one positive real solution to the cubic equation, whereas if $M < M_{cr}$ the equation has not positive real solutions.

3. Tunneling

Let us now consider Hawking radiation coming out from an improved black hole satisfying $M > M_{cr}$ thanks to the tunneling process occurring through the outer horizon R_+. First, we will rewrite the improved Schwarzschild's solution in Painlevé-like coordinates [12] so as to have coordinates which are not singular at the horizon. In order to do this it suffices to introduce a new coordinate t replacing the Schwarzschild-like time t_S such that $t = t_S + h(R)$ and fix $h(R)$ by demanding the constant time slices to be flat. In this way one gets:

$$ds^2 = -\left(1 - \frac{2G(R)M}{R}\right) dt^2 + 2\sqrt{\frac{2G(R)M}{R}}\, dt\, dR + dR^2 + R^2\, d\Omega^2, \tag{3.1}$$

where R can now take the values $0 < R < \infty$. In these coordinates the radial null geodesics describing the evolution of *test* massless particles are given by

$$\frac{dR}{dt} = \pm 1 - \sqrt{\frac{2G(R)M}{R}} \tag{3.2}$$

with the upper (lower) sign corresponding to outgoing (ingoing, respectively) geodesics. Since the coefficients of the metric do not depend on t there is a killing vector $\partial/\partial t$ which is straightforwardly found to be timelike for $R > R_+$, lightlike for $R = R_+$ (the event horizon) and spacelike for $R_- < R < R_+$.[1] The possibility of tunneling is based on the fact that the killing vector is spacelike beneath the event horizon, what allows the existence of negative energy states. Pair production can occur either just inside the horizon with a positive energy particle tunneling out or just outside the event horizon with a negative energy particle tunneling in.

In [13,2] it was found that, when a self-gravitating shell of energy E travels in a spacetime characterized by an ADM mass M, the geometry outside the shell is also characterized by M, but energy conservation implies that the geometry inside the shell is characterized by $M - E$. It was also found that the shell then moves on the geodesics given by the interior line element. In this way, according to (3.2), one expects a shell of energy E to satisfy the evolution equation

[1] For the sake of completeness, let us comment that the killing vector is also lightlike for $R = R_-$ and timelike for $R < R_-$.