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# A bi-invariant Einstein–Hilbert action for the non-geometric string

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ABSTRACT

Inspired by recent studies on string theory with non-geometric fluxes, we develop a differential geometry calculus combining usual diffeomorphisms with what we call $\beta$-diffeomorphisms. This allows us to construct a manifestly bi-invariant Einstein–Hilbert type action for the graviton, the dilaton and a dynamical (quasi-)symplectic structure. The equations of motion of this symplectic gravity theory, further generalizations and the relation to the usual form of the string effective action are discussed. The Seiberg–Witten limit, known for open strings to relate commutative with non-commutative theories, makes an interesting appearance.

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## 1. Introduction

String theory is expected to be a consistent theory of quantum gravity. In this respect, it is interesting to note that a generic feature of all known string theories is that besides the graviton, there exist two additional massless excitations, the Kalb–Ramond field $B_{\mu\nu}$ and the dilaton $\phi$. At leading order, the dynamics of these fields is governed by the extension of the Einstein–Hilbert action

$$S=-\frac{1}{2\kappa^2}\int d^n x\,\sqrt{-G}e^{-2\phi}\left(R-\frac{1}{12}H^2+4(\partial\phi)^2\right), \tag{1}$$

which has two types of local symmetries. Namely, it is invariant under diffeomorphisms of the space–time coordinates, and under gauge transformation of the Kalb–Ramond field. Note also, this action is a valid approximation for solutions with large radii.

Employing T-duality [1], methods of generalized geometry [2–4] and double field theory [5–8], it has become clear during the last years that also a non-geometric frame exists, where the degrees of freedom are described by a metric on the co-tangent bundle, by a dilaton and by a (quasi-)symplectic structure $\beta^{ab}$. The latter gives rise to so-called non-geometric $Q$- and $R$-fluxes. In particular, the $R$-flux has been argued to be related to a non-associative structure [9–13]. However, in contrast to the well-established non-commutative behavior of open strings [14], the generalization to closed strings is more complex, as in a gravitational theory the non-commutativity parameter is expected to be dynamical.

Since in the non-geometric frame, apart from the dilaton, one deals with just a metric and a (quasi-)symplectic structure, it is natural to expect that both local symmetries of the string action can be given a description in terms of a (generalized) differential geometry. Starting from so-called double field theory, this question has already been approached in an interesting way in [15,16] (see also [17]). However, the action derived in [15,16] is not manifestly invariant under both local symmetries. It is the objective of this Letter to construct such a manifestly bi-invariant action for the non-geometric string. The appropriate mathematical framework for this turns out to be the theory of Lie and Courant algebroids [18, 19], which we will mention only briefly. More details on the underlying mathematical structure of Lie algebroids and the details of the computations will appear in [20].

Here, we present the main steps of a construction of an Einstein–Hilbert type action, which is manifestly invariant under both usual diffeomorphisms and what we call $\beta$-diffeomorphisms. This bi-invariant action turns out to be closely related to the action derived for non-geometric fluxes using double field theory [15,16]. Remarkably, relations familiar from the Seiberg–Witten limit for D-branes in a two-form background also appear in this closed-string framework.

## 2. $\beta$-diffeomorphisms

As mentioned in the introduction, in addition to the dilaton, we consider the co-tangent bundle $T^{*}M$ of a manifold with metric

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$\hat{g} = \hat{g}^{ab} e_a \otimes e_b$ and an invertible anti-symmetric bi-vector $\hat{\beta} = \frac{1}{2}\hat{\beta}^{ab} e_a \wedge e_b = \hat{\beta}^{ab} e_a \otimes e_b$, where our notation is $e_a = \partial_a$ and $e^a = dx^a$. Note that $\hat{\beta}$ can be thought of as a (quasi-)symplectic structure giving rise to a (quasi-)Poisson structure $\{f, g\} = \hat{\beta}^{ab}\partial_a f \partial_b g$, with Jacobi identity $\mathrm{Jac}(f, g, h) = \hat{\Theta}^{abc}\partial_a f \partial_b g \partial_c h$. The $R$-flux is defined as $\hat{\Theta}^{abc} = 3\hat{\beta}^{[a|m}\partial_m \hat{\beta}^{|bc]}$, where the (anti-)symmetrization of indices contains a factor of $(1/n!)$. Moreover, $\hat{\beta}$ provides a natural (anchor) map $\beta^\sharp : \mathrm{T}^\star\mathrm{M} \to \mathrm{TM}$ via $\beta^\sharp e^a = \hat{\beta}^{am} e_m$. As we will see, it is essential that $\hat{\beta}$ is invertible, which is however the generic situation. On the other hand, that means we can only describe backgrounds for which that requirement is satisfied.

Compared to the standard differential geometry calculus, here, not only the tangent bundle but also the co-tangent bundle plays an important role. This suggests that the former principle of diffeomorphism covariance of gravity, the equivalence principle, should be extended by a second class of diffeomorphisms. Recall, that in the former case, infinitesimal diffeomorphisms $x^a \to x^a + \xi^a(x)$ are given by the Lie derivative $\delta_\xi X = L_\xi X$, which acts as the Lie bracket on vector fields and as the anti-commutator of the insertion map and the exterior differential on forms. For the second class, that is infinitesimal transformations parametrized by the components of a one-form $\hat{\xi} = \hat{\xi}_a dx^a$, we note the following. The bracket, generalizing the commutator of vector fields to forms, is the so-called Koszul bracket defined as

$$[\hat{\xi}, \eta]_{\mathrm{K}} = L_{\beta^\sharp \hat{\xi}}\, \eta - \iota_{\beta^\sharp \eta}\, d\hat{\xi}, \tag{2}$$

where $\iota$ denotes the insertion map. In addition, let us define the action of a one-form on a function $\phi$ by the anchor map:

$$dx^a(\phi) := \beta^\sharp\big(dx^a\big)(\phi) = \hat{\beta}^{am}\partial_m \phi =: D^a\phi. \tag{3}$$

Now, we can proceed as in ordinary differential geometry and define tensors by their infinitesimal transformation properties. In particular, a scalar field $\phi$ is called a $\beta$-scalar if it transforms as

$$\hat{\delta}_{\hat{\xi}}\phi = \mathcal{L}_{\hat{\xi}}\phi = \hat{\xi}(\phi) = \hat{\xi}_m D^m \phi, \tag{4}$$

and a one-form $\eta$ is a $\beta$-one-form if

$$\begin{aligned} \hat{\delta}_{\hat{\xi}}\eta &= \mathcal{L}_{\hat{\xi}}\eta = [\hat{\xi}, \eta]_{\mathrm{K}} \\ &= \big(\hat{\xi}_m D^m \eta_a - \eta_m D^m \hat{\xi}_a + \hat{\xi}_m \eta_n \hat{Q}_a{}^{mn}\big) e^a, \end{aligned} \tag{5}$$

with $\hat{Q}_c{}^{ab} = \partial_c \hat{\beta}^{ab}$. The transformation properties of general $\beta$-tensors are then determined by requiring the Leibniz rule of $\delta_{\hat{\xi}}$ for tensor products and contractions, which implies for instance that a $\beta$-vector field $X = X^a e_a$ transforms as

$$\begin{aligned} \hat{\delta}_{\hat{\xi}} X &= \mathcal{L}_{\hat{\xi}} X \\ &= \big(\hat{\xi}_m D^m X^a + X^m D^a \hat{\xi}_m - X^m \hat{\xi}_n \hat{Q}_m{}^{na}\big) e_a. \end{aligned} \tag{6}$$

To continue, we have to fix the nature of the metric $\hat{g}^{ab}$ and the anti-symmetric bi-vector $\hat{\beta}^{ab}$. The former should be a tensor with respect to both diffeomorphisms and $\beta$-diffeomorphisms, while we require the latter only to be a tensor under diffeomorphisms. As will become clear below, it should transform under $\beta$-diffeomorphisms non-covariantly

$$\begin{aligned} \hat{\delta}_{\hat{\xi}}\hat{\beta} &:= \mathcal{L}_{\hat{\xi}}\hat{\beta} + \hat{\beta}^{am}\hat{\beta}^{bn}(\partial_m \hat{\xi}_n - \partial_n \hat{\xi}_m) e_a \otimes e_b \\ &= \hat{\xi}_m \hat{\Theta}^{mab} e_a \otimes e_b. \end{aligned} \tag{7}$$

Moreover, the variation with respect to $\hat{\xi}$ should commute with partial derivatives, i.e. $[\hat{\delta}_{\hat{\xi}}, \partial_a] = 0$. The Lie brackets of infinitesimal diffeomorphisms and $\beta$-diffeomorphisms are

$$\begin{aligned} &[\delta_{\xi_1}, \delta_{\xi_2}] = \delta_{[\xi_1, \xi_2]}, \\ &[\hat{\delta}_{\hat{\xi}}, \delta_\eta] = \delta_{\mathcal{L}_{\hat{\xi}}\eta}, \\ &[\hat{\delta}_{\hat{\xi}_1}, \hat{\delta}_{\hat{\xi}_2}] = \hat{\delta}_{[\hat{\xi}_1, \hat{\xi}_2]_{\mathrm{K}}} + \delta_{(\iota_{\hat{\xi}_1}\iota_{\hat{\xi}_2}\hat{\Theta})}. \end{aligned} \tag{8}$$

Ordinary differential geometry is based on the covariantization of the partial derivative of tensors, however, because of

$$\hat{\delta}_{\hat{\xi}}(\partial_a \phi) = \mathcal{L}_{\hat{\xi}}(\partial_a \phi) + \big(D^m \phi\big)(\partial_a \hat{\xi}_m - \partial_m \hat{\xi}_a), \tag{9}$$

under a $\beta$-diffeomorphism the partial derivative of a scalar does not transform as a $\beta$-vector. But, on the other hand, we have defined the transformation of $\hat{\beta}$ in Eq. (7) such that the derivative $D^a\phi$ transforms precisely as a $\beta$-vector, i.e. $\hat{\delta}_{\hat{\xi}}(D^a\phi) = \mathcal{L}_{\hat{\xi}}(D^a\phi)$. Finally, using one of the Bianchi identities derived in [19,15], we find that the $R$-flux is also a $\beta$-tensor, that is $\hat{\delta}_{\hat{\xi}}\hat{\Theta}^{abc} = \mathcal{L}_{\hat{\xi}}\hat{\Theta}^{abc}$.

## 3. Covariant derivative, torsion and curvature

As established in the last section, the role played by $\partial_a$ in usual gravity theories is now taken by the derivative $D^a$. Following the same steps as in standard differential geometry, we then define the covariantization of $D^a$ as

$$\hat{\nabla}^a X^b = D^a X^b - \hat{\Gamma}_c{}^{ab} X^c, \tag{10}$$

and the action on forms reads $\hat{\nabla}^a \eta_b = D^a \eta_b + \hat{\Gamma}_b{}^{ac}\eta_c$. Demanding that the covariant derivative is a $\beta$-tensor requires that the $\beta$-connection cancels the anomalous transformation of the first term, leading to

$$\hat{\Delta}_{\hat{\xi}}\big(\hat{\Gamma}_c{}^{ab}\big) = D^a\big(D^b \hat{\xi}_c - \hat{\xi}_m \hat{Q}_c{}^{mb}\big), \tag{11}$$

with $\hat{\Delta}_{\hat{\xi}} = \delta_{\hat{\xi}} - \mathcal{L}_{\hat{\xi}}$. Under usual diffeomorphisms, $\hat{\Gamma}_c{}^{ab}$ needs to transform anomalously as

$$\Delta_\xi\big(\hat{\Gamma}_c{}^{ab}\big) = -D^a\big(\partial_c \xi^b\big). \tag{12}$$

Taking the commutator of two covariant derivatives defines the $\beta$-torsion

$$\big[\hat{\nabla}^a, \hat{\nabla}^b\big]\phi = -\hat{T}_c{}^{ab} D^c \phi, \tag{13}$$

which can be expressed as

$$\hat{T}_c{}^{ab} = \hat{\Gamma}_c{}^{ab} - \hat{\Gamma}_c{}^{ba} - \hat{\mathcal{Q}}_c{}^{ab}, \tag{14}$$

with $\hat{\mathcal{Q}}_c{}^{ab} = \hat{Q}_c{}^{ab} + \hat{\Theta}^{abm}\hat{\beta}_{mc}$. By construction, this is a usual tensor and a $\beta$-tensor. The curvature is defined by

$$\big[\hat{\nabla}^a, \hat{\nabla}^b\big] X^c = -\hat{R}_m{}^{cab} X^m - \hat{T}_m{}^{ab}\hat{\nabla}^m X^c, \tag{15}$$

leading to

$$\begin{aligned} \hat{R}_m{}^{cab} = {}& D^a \hat{\Gamma}_m{}^{bc} - D^b \hat{\Gamma}_m{}^{ac} + \hat{\Gamma}_n{}^{bc}\hat{\Gamma}_m{}^{an} - \hat{\Gamma}_n{}^{ac}\hat{\Gamma}_m{}^{bn} \\ & - \hat{\mathcal{Q}}_n{}^{ab}\hat{\Gamma}_m{}^{nc}. \end{aligned} \tag{16}$$

The metric-compatible and torsion-free Levi-Civita connection takes the form

$$\hat{\Gamma}_c{}^{ab} = \tilde{\Gamma}_c{}^{ab} - \hat{g}_{cq}\hat{g}^{p(a|}\hat{\mathcal{Q}}_p{}^{|b)q} + \frac{1}{2}\hat{\mathcal{Q}}_c{}^{ab}, \tag{17}$$

with

$$\tilde{\Gamma}_c{}^{ab} = \frac{1}{2}\hat{g}_{cp}\big(D^a \hat{g}^{bp} + D^b \hat{g}^{ap} - D^p \hat{g}^{ab}\big). \tag{18}$$

Note that one can check explicitly that (17) has the right anomalous transformation behavior under diffeomorphisms (12) and $\beta$-diffeomorphisms (11).

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