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Notes on teleportation in an expanding space

Jun Feng [a,*], Wen-Li Yang [b], Yao-Zhong Zhang [c], Heng Fan [a]

[a] Beijing National Laboratory for Condensed Matter Physics, Institute of Physics, Chinese Academy of Sciences, Beijing 100190, PR China
[b] Institute of Modern Physics, Northwest University, Xian 710069, PR China
[c] School of Mathematics and Physics, The University of Queensland, Brisbane, QLD 4072, Australia

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ABSTRACT

We investigate the quantum teleportation between a conformal detector Alice and an inertial detector Bob in de Sitter space in two schemes, (i) one uses free scalar modes and (ii) one utilizes cavity to store qubit. We show that the fidelity of the teleportation is degraded for Bob in both cases. While the fidelity-loss is due to the Gibbons–Hawking effect associated with his cosmological horizon in the scheme (i), the entanglement decreases in the scheme (ii) because the ability to entangle the cavities is reduced by the spacetime curvature. With a cutoff at Planck-scale, comparing with the standard Bunch–Davies choice, we also show that the possible Planckian physics cause extra modifications to the fidelity of the teleportation protocol in both schemes.

One of the main challenges in modern physics is to find a complete theory of quantum gravity which merges quantum mechanics and general relativity into a unified framework. Evidences from some candidate theories (e.g. string theory) have shown [1] that typical quantum gravitational phenomena should be a unitary process. This however conflicts with the semiclassical analysis that predict the information loss during the process. To resolve this paradox, it has become increasingly clear [2] that non-locality, the basic feature of quantum information theory, should be employed. This indeed allows one to understand some quantum gravitational effects in a quantum-information framework. Recently, a new fast growing field called Relativistic Quantum Information (RQI) (see Ref. [3] for a review) has shed new light on this issue. The insight from RQI is the novel observer-dependent character of quantum correlations like entanglement. For a bipartite entangled system in flat space, this means [4] an accelerated observer would experience decrement of quantum entanglement he shares initially with an inertial partner due to the celebrated Unruh effect [5]. Such kind of environmental decoherence has later been generalized to curved background [6]. For a static observer nearby the black hole, a degradation of quantum correlations provoked by Hawking radiation from event horizon would be detected. The entanglement produced in the formation of a black hole has also been studied and provides a quantum information resource between the field modes falling into the black hole and those radiated to infinity. By imposing proper final-state boundary conditions at the singularity [7], this non-locality could transmit information outside the event horizon via a teleportation-like process and restore the unitarity of black hole evaporation process. On the other hand, it has been emphasized [8] that even the standard teleportation protocol [9] is highly non-trivial in the RQI framework. In flat space, as a result of entanglement degradation, the fidelity of a teleportation process would also suffer a reduction for an observer with uniform acceleration. Moreover, quantum teleportation process in a black hole background was also investigated [10] and it was shown that the fidelity is considerably reduced for the fixed observer near the horizon. An analogues experiment using sonic black hole is proposed [11] to test this phenomenon in a suitable laboratory setting.

In this Letter, we investigate a quantum teleportation process in de Sitter space, which idealizes the inflation epoch of early universe and plays a fundamental role in quantum gravity theory (see Ref. [12] for a review). We propose a protocol to teleport an unknown qubit $|\psi\rangle$ from a conformal observer Alice to her inertial partner Bob while they initially share a Bell state. More specifically, we investigate two schemes of our protocol, (i) one using free scalar modes from which a qubit can be truncated [13], and (ii) another scheme utilizing moving cavities to store the field modes related with respective observers.

In the scheme (i), unlike the standard teleportation protocol, we will show that the decoherence, provoked by the Gibbons–Hawking effect associated with Bob's cosmological horizon [14], would reduce the fidelity of the teleportation process. While de Sitter space provide the best scenario to the so-called trans-Planck problem [15], we also discuss its influence on our teleportation scheme from the existence of some fundamental scales (like Planckian or even stringy). With a cutoff on physical momentum

* Corresponding author.
E-mail addresses: tsunfeng@iphy.ac.cn (J. Feng), hfan@iphy.ac.cn (H. Fan).

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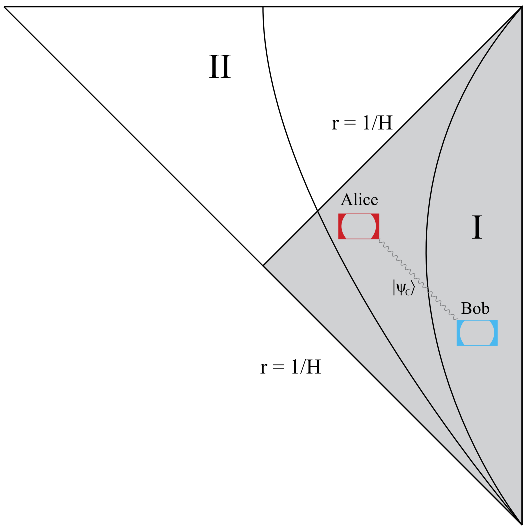

Fig. 1. The Carter–Penrose diagram of de Sitter space. A teleportation scheme between the conformal observer Alice and her inertial partner Bob has been illustrated. Both observers share a Bell state when they coincide initially. Since the information loss associated with Bob's cosmological horizon at $r = 1/H$ (or the entanglement decrease during entangling cavities), the fidelity of teleporting a qubit $|\psi_C\rangle$ would be suppressed.

of field mode at Planck-scale, comparing with the standard Bunch–Davies choice, we calculate the extra modifications to the fidelity of the teleportation process from the possible high-energy new physics.

In the scheme (ii), since the field modes are localized in cavities by the reflecting mirrors, the entanglement between the cavities could be protected from the degradation once it has been prepared [16]. However, as we will show that, even using of localized cavity states could save the protocol from the Gibbons–Hawking radiation, the ability to entangle the different cavity mode is still decreased by the curvature of spacetime, that results a similar qualitatively dependence as in (i) between the fidelity and the Hubble parameter H.

To proceed, we first recall the thermal feature of quantum field theory in de Sitter space. Consider the mode expansion of a free scalar field in de Sitter space

$$\phi(x) = \sum_k \left[a_k\phi_k(x) + a^\dagger_{-k}\phi^*_{-k}(x)\right] \tag{1}$$

The vacuum state, which respects the spacetime isometries, is defined by $a_k|vac\rangle = 0$. To specify the mode functions $\phi_k(x)$, the coordinate systems affiliated to different observers should be employed prior to solve the field equation.

A conformal observer in de Sitter space adopts the planar coordinates which reduce the spacetime metric to

$$ds^2 = \frac{1}{(H\eta)^2}\left(d\eta^2 - d\rho^2 - \rho^2\, d\Omega^2\right) \tag{2}$$

where $\eta = -e^{-Ht}/H$ is conformal time, and the coordinates cover the upper right triangle of the Carter–Penrose diagram (both regions I and II), as depicted in Fig. 1.

Since the space undergoes an accelerated expansion, it follows that the wavelength of field mode could become arbitrarily small if one goes backwards in η long enough, where any distinction between de Sitter space and Minkowski space could be safely ignored. Therefore, an essentially unique Bunch–Davies vacuum $a_k(\eta)|0,\eta\rangle = 0$ could be defined, by requesting it approaching the conformal vacuum of Minkowski space in the limit $\eta \to -\infty$.

However, the existence of the fundamental scales, where the quantum gravitational effects become unignorable, prevents us from following a mode back unlimited (or equivalently, to the arbitrary short distance) [15]. Imposing a reasonable cutoff on physical momentum as $p = ka(\eta) = \Lambda$, the latest time with quantum gravity dominant is $\eta_0 = -\frac{\Lambda}{Hk}$, where Λ refers to the Planck energy scale. This results a modified vacuum state of the conformal observer as $a_k(\eta_0)|0,\eta_0\rangle = 0$, which is in general different from the Bunch–Davies choice and could be formally realized as a squeezed state

$$|0,\eta_0\rangle = S\left|0^\infty\right\rangle \tag{3}$$

where the superscript ∞ indicates the Bunch–Davies choice and throughout. Without a complete theory of quantum gravity, this new vacuum of conformal observer can provide a typical signature of Planck-scale physics. For instance, it was shown (see [17] and the references therein) that the inflation power spectrum $P(k) \sim \langle|\phi_k|^2\rangle$ with respect to the new vacuum would be modified as $\Delta P(k)/P(k) = \frac{H}{\Lambda}\sin\frac{2\Lambda}{H}$, which is expected to be observed in the WMAP or Planck satellite experiments.

More ambitious view is that above argument indeed provides a one-parameter family of vacua with the Λ predicted by various quantum gravity theories, e.g. an energy scale interpolated between Planckian and stringy scales. Equivalently, this leads the so-called α-vacua which have been known for a long time [18].

Introducing the new mode basis related to the Bunch–Davies one by the Mottola–Allen (MA) transformation

$$\phi^\alpha_k(\eta,\vec{x}) = N_\alpha\left[\phi^\infty_k(\eta,\vec{x}) + e^\alpha\phi^{\infty *}_{-k}(\eta,\vec{x})\right] \tag{4}$$

where α is an arbitrary complex number with $\mathrm{Re}\,\alpha < 0$, $N_\alpha = 1/\sqrt{1-e^{\alpha+\alpha^*}}$. The one-parameter family of vacua is defined as $a^\alpha_k|0^\alpha\rangle = 0$, where

$$a^\alpha_k = N_\alpha\left[a^\infty_k - e^{\alpha^*}a^{\infty\dagger}_{-k}\right] \tag{5}$$

are the corresponding annihilation operators. These α-vacua preserve all $SO(1,4)$ de Sitter isometries, and clearly include the Bunch–Davies vacuum as one element since $a^\alpha_k \to a^\infty_k$ if $\mathrm{Re}\,\alpha \to -\infty$. Moreover, the condition (3) can now be explicitly resolved as

$$\left|0^\alpha_k\right\rangle = \exp\left[\alpha\left(a^{\infty\dagger}_k a^{\infty\dagger}_{-k} - a^\infty_{-k}a^\infty_k\right)\right]\left|0^\infty_k\right\rangle \tag{6}$$

In a realistic model, the value of α could be strictly constrained. First, for a theory consistent with CPT-invariance, α should be real. Therefore, we henceforth adopt $\alpha = \mathrm{Re}\,\alpha$ for simplicity. On the other hand, rather than the Bunch–Davies choice, if a non-trivial α-state ($\alpha \neq -\infty$) is chosen as an alternative initial state of inflation, the modified power spectrum of inflationary perturbations requires that [15] $e^\alpha \sim \frac{H}{\Lambda}$.

It was shown [14] that the vacuum state defined by the conformal observer would be nonempty in the view of a static observer. While the Bunch–Davies vacuum appears thermal with the Gibbons–Hawking temperature $T = H/2\pi$, it is clear that these α-vacua would exhibit non-thermal feature encoding the quantum gravitational corrections.

In terms of the static coordinates, de Sitter metric becomes

$$ds^2 = \left(1 - r^2H^2\right)dt^2 - \left(1 - r^2H^2\right)^{-1}dr^2 - r^2\, d\Omega^2 \tag{7}$$

where t is the cosmic time. The coordinates only cover the region I in Fig. 1, half of the planar coordinates dose. The hypersurface on $r = 1/H$ is a cosmological horizon for an observer situated at $r = 0$. Since the existence of the Killing vector ∂_t, a static vacuum $|0^S\rangle$ could be defined unambiguously. To analyze the thermality of this vacuum, we employ the particular useful Painlevé coordinates [19], which reduce the metric (2) into