Physics Letters B 718 (2013) 1203–1207

Contents lists available at SciVerse ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Thermodynamics of fractal universe

Ahmad Sheykhi [a,b,*], Zeinab Teimoori [c], Bin Wang [d]

[a] Physics Department and Biruni Observatory, College of Sciences, Shiraz University, Shiraz 71454, Iran
[b] Research Institute for Astronomy and Astrophysics of Maragha (RIAAM), P.O. Box 55134-441, Maragha, Iran
[c] Department of Physics, Shahid Bahonar University, P.O. Box 76175, Kerman, Iran
[d] INPAC and Department of Physics, Shanghai Jiao Tong University, Shanghai 200240, China

ARTICLE INFO

Article history:
Received 15 December 2012
Received in revised form 29 December 2012
Accepted 31 December 2012
Available online 3 January 2013
Editor: M. Trodden

ABSTRACT

We investigate the thermodynamical properties of the apparent horizon in a fractal universe. We find that one can always rewrite the Friedmann equation of the fractal universe in the form of the entropy balance relation $\delta Q = T_h dS_h$, where δQ and T_h are the energy flux and Unruh temperature seen by an accelerated observer just inside the apparent horizon. We find that the entropy S_h consists two terms, the first one which obeys the usual area law and the second part which is the entropy production term due to nonequilibrium thermodynamics of fractal universe. This shows that in a fractal universe, a treatment with nonequilibrium thermodynamics of spacetime may be needed. We also study the generalized second law of thermodynamics in the framework of fractal universe. When the temperature of the apparent horizon and the matter fields inside the horizon are equal, i.e. $T = T_h$, the generalized second law of thermodynamics can be fulfilled provided the deceleration and the equation of state parameters ranges either as $-1 \leqslant q < 0$, $-1 \leqslant w < -1/3$ or as $q < -1$, $w < -1$ which are consistent with recent observations. We also find that for $T_h = bT$, with $b < 1$, the GSL of thermodynamics can be secured in a fractal universe by suitably choosing the fractal parameter β.
© 2013 Elsevier B.V. All rights reserved.

1. Introduction

Nowadays, it is a general belief that there is a deep connection between thermodynamics and gravity. The story started with the discovery of black holes thermodynamics in 1970s by Hawking and Bekenstein [1–4]. According to their discovery, a black hole can be regarded as a thermodynamic system, with temperature and entropy proportional to its surface gravity and horizon area, respectively. After that, people were speculating that maybe there is a direct connection between thermodynamics and Einstein equation, a hyperbolic second order partial differential equation for the spacetime metric. In 1995, Jacobson [5] was indeed able to derive the Einstein equation from the requirement that the Clausius relation $\delta Q = T\delta S$ holds for all local acceleration horizons through each spacetime point, where δS is one-quarter the horizon area change in Planck units and δQ and T are the energy flux across the horizon and the Unruh temperature seen by an accelerating observer just inside the horizon. Jacobson's derivation of the Einstein field equation from thermodynamics opened a new window for understanding the thermodynamic nature of gravity. After Jacobson, a lot of works have been done to disclose the profound connection between gravity and thermodynamics. It was shown that the gravitational field equations in a wide range of theories, can be rewritten in the form of the first law of thermodynamics and vice versa [6–14]. The studies were also generalized to the cosmological setup, where it was shown that the differential form of the Friedmann equation in the Friedmann–Robertson–Walker (FRW) universe can be transformed to the first law of thermodynamics on the apparent horizon [15–26].

On the other side, the second law of black hole mechanics expresses that the total area of the event horizon of any collection of classical black holes can never decrease, even if they collide and swallow each other. This is remarkably similar to the second law of thermodynamics where the area is playing the role of entropy. Note that the second law of black hole thermodynamics can be violated if one takes into account the quantum effect, such as the Hawking radiation. To overcome this difficulty, Bekenstein [2,4] introduced the so-called total entropy S_{tot} which is defined as

$$S_{\text{tot}} = S_h + S_m, \tag{1}$$

where S_h and S_m are, respectively, the black hole entropy and the entropy of the surrounding matter. According to Bekenstein's argument, in general, the total entropy should be a nondecreasing

* Corresponding author at: Physics Department and Biruni Observatory, College of Sciences, Shiraz University, Shiraz 71454, Iran.
E-mail addresses: sheykhi@uk.ac.ir (A. Sheykhi), wang-b@sjtu.edu.cn (B. Wang).

0370-2693/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.physletb.2012.12.072

function. This statement is known as the generalized second law (GSL) of thermodynamics,

$$\triangle S_{tot} \geqslant 0. \tag{2}$$

Besides, if thermodynamical interpretation of gravity near the apparent horizon is a generic feature, one needs to verify whether the results may hold not only for more general spacetimes but also for the other principles of thermodynamics, especially for the GSL of thermodynamics. The GSL of thermodynamics is a universal principle governing the evolution of the universe. It was argued that in the accelerating universe the GSL is valid provided the boundary of the universe is chosen the apparent horizon [27–31].

In this Letter, we would like to extend the study to the fractal universe. Fractal cosmology was recently proposed by Calcagni [32,33] for a power-counting renormalizable field theory living in a fractal spacetime. It is interesting to see whether the Friedmann equation of a fractal universe can be written in the form of the first law of thermodynamics. As we will see, in a fractal universe, the Friedmann equation can be transformed to Clausius relation, but a treatment with nonequilibrium thermodynamics of spacetime is needed.

In the next section we review the basic equations in the framework of fractal cosmology. In Section 3, we show that the Friedmann equation of a fractal universe can be written in the form of the fundamental relation $\delta Q = T_h\, dS_h$, where δQ and T_h are, respectively, the energy flux and Unruh temperature seen by an accelerated observer just inside the apparent horizon. In Section 4, we check the validity of the GSL of thermodynamics for a fractal cosmology. The last section is devoted to some concluding remarks.

2. Fractal universe

The total action of Einstein gravity in a fractal spacetime is given by [32,33]

$$S = S_G + S_m, \tag{3}$$

where the gravitational part of the action is given by

$$S_G = \frac{1}{16\pi G}\int d\varrho(x)\,\sqrt{-g}\big(R - 2\Lambda - \omega\partial_\mu\upsilon\partial^\mu\upsilon\big), \tag{4}$$

and the matter part of the action is

$$S_m = \int d\varrho(x)\,\sqrt{-g}\mathcal{L}_m. \tag{5}$$

Here g is the determinant of the dimensionless metric $g_{\mu\nu}$, Λ and R are, respectively, the cosmological constant and Ricci scalar. υ is the fractional function and ω is the fractal parameter. The standard measure d^4x replaced with a Lebesgue–Stieltjes measure $d\varrho(x)$. The derivation of the Einstein equations goes almost like in scalar-tensor models. Taking the variation of the action (3) with respect to the FRW metric $g_{\mu\nu}$, one can obtain the Friedmann equations in a fractal universe as [33]

$$H^2 + \frac{k}{a^2} + H\frac{\dot{\upsilon}}{\upsilon} - \frac{\omega}{6}\dot{\upsilon}^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}, \tag{6}$$

$$\dot{H} + H^2 - H\frac{\dot{\upsilon}}{\upsilon} + \frac{\omega}{3}\dot{\upsilon}^2 - \frac{1}{2}\frac{\Box\upsilon}{\upsilon} = -\frac{8\pi G}{6}(\rho + 3p) + \frac{\Lambda}{3}, \tag{7}$$

where $H = \dot{a}/a$ is the Hubble parameter, ρ and p are the total energy density and pressure of the ideal fluid composing the universe, respectively. The curvature constant $k = 0, 1, -1$ corresponding to a flat, closed and open universe, respectively. The continuity equation in a fractal universe takes the form [33]

$$\dot{\rho} + \left(3H + \frac{\dot{\upsilon}}{\upsilon}\right)(\rho + p) = 0. \tag{8}$$

It is clear that for $\upsilon = 1$, the standard Friedmann equations are recovered. We further assume that only the time direction is fractal, while spatial slices have usual geometry. Indeed, in the framework of fractal cosmology, classically fractals can be timelike $[\upsilon = \upsilon(t)]$ or even spacelike $[\upsilon = \upsilon(x)]$ (see Ref. [33] for details). These two cases lead to different classical physics, but at quantum level all configurations should be taken into account, so there is no quantum analogue of space or timelike fractals. In this Letter we take a timelike fractal. Thus, those parameters that depend on time change and those parts that related to x remain fixed.

Assuming a timelike fractal profile $\upsilon = t^{-\beta}$ [33], where $\beta = 4(1-\alpha)$ is the fractal dimension, the Friedmann equations (6) and (7) in the absence of the cosmological constant can be written as

$$H^2 + \frac{k}{a^2} - \frac{\beta}{t}H - \frac{\omega\beta^2}{6t^{2(\beta+1)}} = \frac{8\pi G}{3}\rho, \tag{9}$$

$$\dot{H} + H^2 - \frac{\beta}{2t}H + \frac{\beta(\beta+1)}{2t^2} + \frac{\omega\beta^2}{3t^{2(\beta+1)}} = -\frac{8\pi G}{6}(\rho + 3p), \tag{10}$$

while, the continuity equation (8) takes the form

$$\dot{\rho} + \left(3H - \frac{\beta}{t}\right)(\rho + p) = 0. \tag{11}$$

From the definition of the fractional integral [33,34], we know that α ranges as $0 < \alpha \leqslant 1$. Thus for $\alpha = 1$, we obtain $\beta = 0$ which physically means that the universe does not have any fractal structure and one can recover the well-known Friedmann equations in standard cosmology. As one can see from Friedmann equations (9) and (10), we have no limit $t \to 0$ for a timelike fractal profile, since in this case the Friedmann equations diverge unless $\beta = 0$. This implies that at the early stages of the universe, we could not have the timelike fractal structure.

In the remaining part of this Letter we show that the differential form of the Friedmann equation (9) can be written in the form of the fundamental relation $\delta Q = T_h\, dS_h$, where S_h is the entropy associated with the apparent horizon. We also investigate the validity of the GSL of thermodynamics for the fractal universe surrounded by the apparent horizon.

3. First law of thermodynamics in fractal cosmology

For a homogenous and isotropic FRW universe the line elements can be written

$$ds^2 = h_{\mu\nu}\, dx^\mu\, dx^\nu + \tilde{r}^2\big(d\theta^2 + \sin^2\theta\, d\phi^2\big), \tag{12}$$

where $\tilde{r} = a(t)r$, $x^0 = t$, $x^1 = r$, and $h_{\mu\nu} = \mathrm{diag}(-1, a^2/(1-kr^2))$ is the two-dimensional metric. The dynamical apparent horizon, a marginally trapped surface with vanishing expansion, is determined by the relation $h^{\mu\nu}\partial_\mu\tilde{r}\partial_\nu\tilde{r} = 0$. Straightforward calculation gives the apparent horizon radius for the FRW universe as [30]

$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + k/a^2}}. \tag{13}$$

The associated temperature T with the apparent horizon is given by

Download English Version:

https://daneshyari.com/en/article/8189529

Download Persian Version:

https://daneshyari.com/article/8189529

Daneshyari.com