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Symmetry energy and pion production in the Boltzmann–Langevin approach

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ABSTRACT

Based on the improved isospin-dependent Boltzmann–Langevin model which incorporates the dynamical fluctuations, we study the π production in central heavy ion collisions at different incident energies from 250 to 1200A MeV. It is found that the π multiplicity is sensitive to the nuclear equation of state. At π subthreshold energy, the fluctuations have a larger effect on the π multiplicity. The π^-/π^+ ratios as a probe of nuclear symmetry energy are calculated with different stiffness of symmetry energy. The results favor a supersoft symmetry energy of the potential term in comparison with the FOPI data, which supports the one obtained by the usual Boltzmann–Uehling–Uhlenbeck model.

During the last few years, the study of nuclear symmetry energy $E_{sym}(\rho)$ has been a highly interesting subject. The constraining of $E_{sym}(\rho)$ is important for not only understanding of heavy-ion reactions [1] but also many issues in astrophysics [1,2]. Unfortunately, the form of $E_{sym}(\rho)$ is very controversial, especially at supra-saturation density. At sub-saturation density, constraints on the $E_{sym}(\rho)$ were obtained by analyzing the isospin diffusion data [3]. At supra-saturation density, the main difference of the $E_{sym}(\rho)$ forms predicted by some microscopical or phenomenological many-body approaches is the trend of the $E_{sym}(\rho)$ with the density. One is the $E_{sym}(\rho)$ increases continuously with the increasing density, and the other is the $E_{sym}(\rho)$ increases up to the ρ_0 and then decreases with the density at $\rho > \rho_0$, where ρ_0 is the saturation density of nuclear matter. Moreover, through comparing with the FOPI data [4], the calculational results of different transport models were opposite. For example, a very soft nuclear symmetry energy was suggested by the isospin-dependent Boltzmann–Uehling–Uhlenbeck model (IBUU04) [5], and a hard one was predicted by the stochastic mean-field approach (SMF) [6] and the improved isospin-dependent quantum molecular dynamics model (ImIQMD) [7]. Therefore, further investigations of nuclear symmetry energy by improving the theoretical models or proposing a new theoretical model are still necessary.

The emission of pion has been proposed for many years to investigate the nuclear equation of state (EoS) [8–10] and the $E_{sym}(\rho)$ [1,5,7,11,12] under extreme conditions. A larger uncertainty exists in constraining the nuclear EoS using the pion production. The calculations reported in Refs. [8,10] indicated that the pion production is sensitive to the nuclear EoS, but the one reported in Ref. [9] indicated the opposite result. The pion emission as a probe of the $E_{sym}(\rho)$ is motivated by the $\Delta(1232)$ resonance model [13] which predicts a primordial relation between π^-/π^+ ratio and N/Z, that is

$$\pi^-/\pi^+ = \left(5N^2 + NZ\right)/\left(5Z^2 + NZ\right) \approx (N/Z)^2, \tag{1}$$

where the N and Z are the neutron and proton numbers in the participant region of the reaction. The N/Z is determined by the $E_{sym}(\rho)$ through the dynamical isospin fractionation [12]. Therefore, one can use the π^-/π^+ ratio to measure the isospin asymmetry N/Z of the dense nuclear matter and then constrain the $E_{sym}(\rho)$. A larger uncertainty also exists in constraining the $E_{sym}(\rho)$ using the π^-/π^+ ratio. The IBUU04 model [5] predicted a very soft symmetry energy corresponds to a larger π^-/π^+ ratio. Inversely, a larger ratio for stiffer symmetry energy are suggested by the ImIQMD model [7] and the relativistic Boltzmann–Uehling–Uhlenbeck model (RBUU) which contains an isovector–vector ρ field and an isovector–scalar δ field [14]. The phenomenon that the π^- (π^+) multiplicity is slightly increasing (decreasing) with

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the increasing symmetry energy are observed in the RBUU model because of threshold effects [14,15].

As a kind of ensemble-averaged theory, the Boltzmann–Uehling–Uhlenbeck (BUU) model cannot describe the fluctuation phenomena in the nuclear collisions, which are responsible for the multifragmentation processes, correlations in light-particle emission and fluctuations of one-body observables [16]. To describe the fluctuation phenomena in the nuclear collisions, according to the fluctuation–dissipation theorem, a Boltzmann–Langevin model (BL) which incorporates fluctuations, which are initially produced during the early, most dissipative stage of the reaction, into the BUU equation was proposed [16]. In the BL model the fluctuations are projected on the momentum space, which provide the initial seed for density fluctuations in coordinate space. A transient behavior of the momentum distributions is found by the numerical simulations of the BL model, which is consistent with the expectations from the fluctuation–dissipation theorem [17]. It is worth noting that another method implementing fluctuations in the mean-field dynamics has been proposed in the framework of the SMF, in which the fluctuations are projected on the coordinate space [18]. Based on the SMF, the properties of fragmentation have been discussed by the Catania group [19].

The BL model has been successfully applied to describe the nuclear collisions at low energies [17]. Moreover, this model is successful in describing the multifragmentation [20] and the extended BL model by incorporating the isospin effect which is called the isospin-dependent Boltzmann–Langevin model (IBL) can reproduce the fragmentation cross sections [21]. Furthermore, the calculations of K^+ production cross sections at subthreshold energies in the ^{12}C + ^{12}C collisions indicated that the yields obtained in the BL model are very larger than those obtained in the BUU model [22]. The calculations of Ref. [10] indicated that the momentum-dependent nuclear interactions (MDI) have a larger effect on the π production. Therefore, it is very interesting and imperative to improve the IBL model [21] and to investigate meson production in the heavy ion collisions, especially near the meson threshold energy. In this Letter, the inelastic channels which mainly produce the π mesons and the MDI are incorporated in the IBL model [21] (named the ImIBL model). We investigate the π emission in the framework of the ImIBL model for the first time.

For brevity, we outline simply the theoretical framework used in the present work. The isospin-dependent Boltzmann–Langevin equation can be written as [17,20]

$$\left(\frac{\partial}{\partial t}+\frac{\mathbf{p}}{m}\cdot\nabla_r-\nabla_r U(\hat{f})\cdot\nabla_p\right)\hat{f}(\mathbf{r},\mathbf{p},t)$$
$$=K(\hat{f})+\delta K(\mathbf{r},\mathbf{p},t). \qquad (2)$$

The left-hand side describes the Vlasov propagation determined by the fluctuating nuclear mean-field $U(\hat{f})$. $K(\hat{f})$ is the collision term of the usual BUU form but expressed in terms of the fluctuating density. The fluctuating collision term $\delta K(\mathbf{r},\mathbf{p},t)$ that can be explained as a stochastic force acting on $\hat{f}$ is characterized by a correlation function [20],

$$\langle\delta K(\mathbf{r}_1,\mathbf{p}_1,t_1)\delta K(\mathbf{r}_2,\mathbf{p}_2,t_2)\rangle$$
$$=C(\mathbf{p}_1,\mathbf{p}_2)\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(t_1-t_2), \qquad (3)$$

where the angle brackets stand for a local average, performed over fluctuating densities generated during a short time interval δt. The reduced correlation function $C(\mathbf{p}_1,\mathbf{p}_2)$ can be expressed in the weak-coupling limit and determined by the one-body properties of the locally averaged distribution as indicated in Ref. [20]. The method of numerical simulations of Eq. (2) employed here is the projection method [16,17,20] which projects the fluctuations on a set of low order local multipole moments of the momentum distribution. Shown in Fig. 1 are the time evolution of the ensemble-averaged total quadrupole moment Q_{20}, $Q_{20}\pm\sigma_{20}$ and $Q_{20}\pm 2\sigma_{20}$ of the momentum distribution and the associated variance σ_{20} from central ^{40}Ca + ^{40}Ca collisions at 250A MeV in the ImIBL and BUU models, where $\sigma_{20}=\sqrt{\langle Q_{20}^2\rangle-\langle Q_{20}\rangle^2}$ is the standard deviation function. For a gaussian distribution, $Q_{20}\pm\sigma_{20}$ and $Q_{20}\pm 2\sigma_{20}$ correspond to 84.3 and 99.5 percent of the number of events respectively [17]. It is seen that, for the existence of fluctuations, the range of variation of Q_{20} value in the ImIBL model is larger and the ImIBL simulations exhibit a different behavior comparing with the usual BUU transport theories.

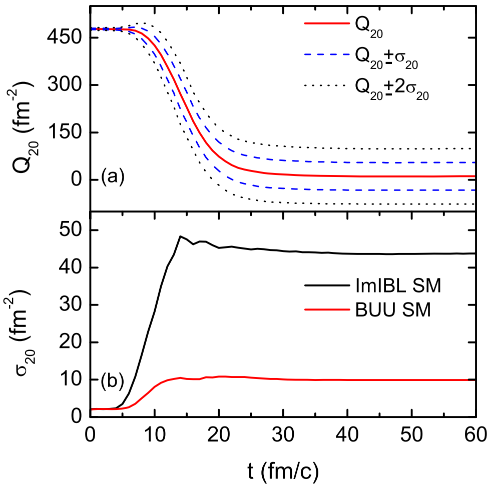

Fig. 1. (Color online.) Time evolution of the ensemble-averaged quadrupole moment Q_{20}, $Q_{20}\pm\sigma_{20}$ and $Q_{20}\pm 2\sigma_{20}$ of the momentum distribution in the ImIBL model (a) and the associated variance σ_{20} (b) for central ^{40}Ca + ^{40}Ca collisions at 250A MeV.

The isospin- and momentum-dependent single nucleon potential used in the ImIBL model reads

$$U_\tau(\rho,\delta,\mathbf{p})=\alpha\frac{\rho}{\rho_0}+\beta\left(\frac{\rho}{\rho_0}\right)^\gamma+E_{sym}^{loc}(\rho)\delta^2$$
$$+\frac{\partial E_{sym}^{loc}(\rho)}{\partial\rho}\rho\delta^2+E_{sym}^{loc}(\rho)\rho\frac{\partial\delta^2}{\partial\rho_\tau}+U_{MDI}, \qquad (4)$$

where $\delta=(\rho_n-\rho_p)/\rho$ is the isospin asymmetry, and ρ, ρ_n and ρ_p are the total, neutron and proton densities, respectively. The values of bulk parameters α, β, and γ taken here are −390 MeV, 320 MeV and 1.14 for the soft EOS plus MDI as SM and −130 MeV, 59 MeV and 2.09 for the hard EOS plus MDI as HM [23]. The compressibilities K are 200 and 380 MeV for the SM and HM, respectively. The E_{sym}^{loc} is the local part of the symmetry energy, which mimics the predictions by microscopical or phenomenological many-body theories. In this Letter, we take two forms as follows

$$E_{sym}^{loc}(\rho)=\frac{1}{2}C_{sym}\left(\frac{\rho}{\rho_0}\right)^{\gamma_s}, \qquad (5)$$

and

$$E_{sym}^{loc}(\rho)=a\left(\frac{\rho}{\rho_0}\right)+b\left(\frac{\rho}{\rho_0}\right)^2+c\left(\frac{\rho}{\rho_0}\right)^{5/3}, \qquad (6)$$

where $\gamma_s=0.5$, 1.0, and 2.0 correspond to the soft, linear and hard symmetry energy respectively. The coefficient values of C_{sym}, a, b and c are 29.4, 38.9, −18.4 and −3.8 MeV, respectively. Eq. (6) is directly deduced from Skyrme energy-density functional and gives