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# Dynamical dispersion relation for ELKO dark spinor fields

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ABSTRACT

An intrinsic mass generation mechanism for exotic ELKO dark matter fields is scrutinized, in the context of the very special relativity (VSR). Our results are reported on unraveling inequivalent spin structures that educe an additional term on the associated Dirac operator. Contrary to the spinor fields of mass dimension 3/2, this term is precluded to be absorbed as a shift of some gauge vector potential, regarding the equations for the dark spinor fields. It leads to some dynamical constraints that can be intrinsically converted into a dark spinor mass generation mechanism, with the encoded symmetries maintained by the VSR. The dynamical mass is embedded in the VSR framework through a natural coupling to the *kink* solution of a $\lambda\phi^4$ theory for a scalar field $\phi$. Our results evince the possibility of novel effective scenarios, derived from exotic couplings among dark spinor fields and scalar field topological solutions.



## 1. Introduction

The investigation on the nature of dark matter components, as well as the comprehension regarding all their intrinsic relations with the elements of the cosmic inventory, belongs to one of challenging current problems in theoretical physics [1–4]. A novel form of matter called ELKO, the acronym of *Eigenspinoren des Ladungskonjugationsoperators*, which designates the eigenspinors of the charge conjugation operator, seems to fulfill the requirements for a dark matter component, in the scope of the interplay among general relativity, astrophysics and particle physics [5–9,12–18]. These references for instance evince that ELKO spinor fields main interaction via the gravitational field makes them naturally dark, which enforces dark spinor fields investigation in a cosmological setting, where interesting solutions and also models where the spinor is coupled conformally to gravity are provided. Once embedded in the quantum field theory, besides leading to some non-local properties [19–22], the standard formulation of ELKO predicts modified dispersion relations. Furthermore, it allows for accomplishing dual-helicity mass dimension one eigenspinors of the spin-1/2 charge conjugation operator. The possibility of exotic interactions with the Higgs scalar field, and suppressed interactions with gauge fields, accredits such matter fields as potential candidates to describe dark matter [19–22]. At standard model (SM) energy scales, ELKO should behave as a representation of the Lorentz group through the setup of a preferred direction related to its wave equation [19–25]. It is recovered by the conjecture of the very special relativity (VSR) [26,27]. The Lorentz symmetries underlying the SM matter and gauge fields, as well as the algebraic structure underlying VSR [25] supported by the event space underlying dark matter and dark gauge fields, have been continuously evaluated, in order to describe the embedding of dark spinor fields into the SM [28,29].

The VSR operates at the Planck scale to reproduce the SM as an associated effective theory. It is supposed to be operative not solely at ultra-high energies, but also beyond SM energy scales, where dark matter interactions may eventually take place. In the context of elementary interactions between fermionic and gauge fields, as well as to preserve the intrinsic darkness with respect to the SM gauge fields [25], it is possible to construct a VSR invariant fermionic field with unitary mass dimension and spin-1/2: the ELKO.

To shed some primordial light on ELKO dark matter field properties, one may reckon that in spacetimes with non-trivial topology there ought to be an additional degree of freedom for fermionic particles [5,30–34]. Such novel element emerges when, for instance, SM spinor fields are parallelly transported: a complementary one-form field $\xi^{-1}(x)\,d\xi(x)$ is accrued on the Dirac operator, educed by the non-trivial topology [5,30–34]. Here $d$ denotes the exterior derivative operator and $\xi$ is a scalar field. When SM Dirac spinor fields are taken into account, the vector gauge field $V$ term is affected by the transformation $V \mapsto V + \frac{1}{2\pi i}\xi^{-1}\,d\xi$, which indeed corresponds to the addition of an gauge potential extra term. Such an exotic term may be therefore absorbed by an external abelian gauge potential, representing an element of

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the cohomology group $H^1(M, \mathbb{Z}_2)$ [31–35], in the context of the (exotic) Dirac equation. Notwithstanding, concerning exotic ELKO dark spinor fields the possibility of such an intrinsic coupling to SM gauge fields is null. Since gauge fields interactions with ELKO are suppressed [19–22], ELKO fields are able to probe purely the spacetime topology [5]. It consistently reinforces the above mentioned darkness of such fields. This Letter is organized as follows: in the next section, the exotic structure underlying ELKO dark spinor fields [5] is briefly revisited. In Section 3 we identify some similarities between the exotic Dirac operator and the covariant derivative embedded into the framework of the VSR, so that the Klein–Gordon equation for a massive particle can be reproduced. By identifying the VSR preferential direction with a dynamical dependence on the *kink* solution of a $\lambda\phi^4$ theory for a scalar field $\phi$, we show that an effective mass for the ELKO spinor can be naturally obtained. It evinces the possibility of novel scenarios for the mechanism of dynamical mass generation, as well as for exotic couplings with scalar field topological solutions. In Section 4 we conclude and provide novel perspectives on the potentially relevant and prominent results addressed in this Letter.

## 2. Exotic ELKO dark spinor fields

ELKO spinor fields $\lambda(\mathbf{p})$ are defined as eigenspinors of the charge conjugation operator $C = \left(\begin{smallmatrix} \mathbb{O} & i\Theta \\ -i\Theta & \mathbb{O}\end{smallmatrix}\right)K$, in the precise sense that $C\lambda(\mathbf{p}) = \pm\lambda(\mathbf{p})$, where, given the rotation generators $\mathcal{J}$, the Wigner's spin-1/2 time reversal operator $\Theta$ satisfies $\Theta\mathcal{J}\Theta^{-1} = -\mathcal{J}^*$. The operator $K$ is responsible to $\mathbb{C}$-conjugate spinor fields appearing on the right. The plus [minus] sign stands for self-conjugate [anti-self-conjugate] spinor fields $\lambda^S(\mathbf{p})$ [$\lambda^A(\mathbf{p})$]. The complete form of ELKO can be explicitly obtained [19–22] through the solution of the equation of helicity $(\sigma\cdot\widehat{\mathbf{p}})\lambda^{A/S}_{\pm}(\mathbf{p}) = \pm\lambda^{A/S}_{\pm}(\mathbf{p})$, where $\widehat{\mathbf{p}} = \mathbf{p}/\|\mathbf{p}\|$. The four boosted spinor fields are[1]

$$\lambda^{S/A}_{\{\mp,\pm\}}(\mathbf{p}) = \sqrt{\frac{E+m}{2m}}\left(1 \mp \frac{p}{E+m}\right)\lambda^{S/A}_{\{\mp,\pm\}}(\mathbf{0}),$$
$$\text{where } \lambda^{S/A}_{\{\mp,\pm\}}(\mathbf{0}) = \begin{pmatrix} \pm i\Theta[\phi^{\pm}(\mathbf{0})]^* \\ \phi^{\pm}(\mathbf{0})\end{pmatrix}.$$

The (Weyl) spinors fields $\Theta[\phi^{\pm}(\mathbf{0})]^*$ and $\phi^{\pm}(\mathbf{0})$ have opposite helicities.

Prior results moreover evince that ELKO can be expressed through a linear combination of Dirac particle and antiparticle fields [5,15–22], and the prescription $p_\mu \mapsto i\nabla_\mu$ holds for ELKO: $\lambda^{S/A}(x) = \lambda^{S/A}(\mathbf{p})\exp(\epsilon^{S/A} i p_\mu x^\mu)$, where $\epsilon^S = -1$ and $\epsilon^A = +1$ [19].

Besides the standard ELKO spinor fields $\lambda(x)$, one can get a second type of ELKO, denoted hereupon by $\mathring{\lambda}(x)$, associated to an inequivalent spin structure, that reflects a modification of the covariant derivative [31–34]:

$$\mathring{\nabla}_X\mathring{\lambda}(x) = \nabla_X\mathring{\lambda}(x) - \frac{1}{2}\left[X\cdot\left(\xi^{-1}(x)\,d\xi(x)\right)\right]\mathring{\lambda}(x), \tag{1}$$

where $X$ denotes a vector field. The so-called exotic term in Eq. (1) is assumed to be re-scaled as $\frac{1}{2\pi i}(\xi^{-1}(x)\,d\xi(x))$, an integer of a Čech cohomology class [5,31–34]. The exotic Dirac operator can be written thereupon as

$$i\gamma^\mu\mathring{\nabla}_\mu = i\gamma^\mu\nabla_\mu + \frac{1}{2}\xi^{-1}(x)\,d\xi(x), \tag{2}$$

[1] The boosts presented here are Lorentz boosts, although SIM(2) VSR boosted ELKO can be derived as in [25]. For our aim in what follows Lorentz boosts suffice, and therefore shall be adopted.

and the exotic Dirac equation then becomes

$$\left(i\gamma^\mu\nabla_\mu + \left(\xi^{-1}(x)\,d\xi(x)\right)/2 \pm m\mathbb{I}\right)\psi(x) = 0,$$

where $\psi$ denotes a Dirac spinor field. As sustained in [30–34], one can express $\xi(x) = \exp(2i\theta(x)) \in \mathrm{U}(1)$, so that the exotic term yields $\xi^{-1}(x)\,d\xi(x) = 2i\gamma^\mu\partial_\mu\theta(x)$. One hence obtains the explicit form for the coupled system of equations for the exotic ELKO as

$$\left((i\gamma^\mu\nabla_\mu + i\gamma^\mu\partial_\mu\theta)\delta^\beta_\alpha \pm m\mathbb{I}\varepsilon^\beta_\alpha\right)\mathring{\lambda}^{S/A}_\beta(x) = 0,$$
$$\varepsilon^{\{-,+\}}_{\{+,-\}} := -1 \tag{3}$$

or, more explicitly — taking into account Eq. (2):

$$\begin{pmatrix} i\gamma^\mu\mathring{\nabla}_\mu & \mathbb{O} & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & i\gamma^\mu\mathring{\nabla}_\mu & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & i\gamma^\mu\mathring{\nabla}_\mu & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \mathbb{O} & i\gamma^\mu\mathring{\nabla}_\mu \end{pmatrix}\begin{pmatrix}\mathring{\lambda}^S_{\{-,+\}} \\ \mathring{\lambda}^S_{\{+,-\}} \\ \mathring{\lambda}^A_{\{-,+\}} \\ \mathring{\lambda}^A_{\{+,-\}}\end{pmatrix} - im\mathbb{I}\begin{pmatrix} -\mathring{\lambda}^S_{\{+,-\}} \\ \mathring{\lambda}^S_{\{-,+\}} \\ \mathring{\lambda}^A_{\{+,-\}} \\ -\mathring{\lambda}^A_{\{-,+\}}\end{pmatrix} = 0.$$

The exotic operator $i\gamma^\mu\nabla_\mu + i\gamma^\mu\partial_\mu\theta \pm m\mathbb{I}$ annihilates each of the four exotic Dirac spinor fields used to construct $\mathring{\lambda}^{S/A}_\beta(x)$, as prescribed by the standard Dirac dynamics. However, since the operator in (3) couples the $\{\pm,\mp\}$ degrees of freedom [5,19], the modified exotic Dirac operator does not annihilate ELKO fields. By observing the off-diagonal nature of the mass term in Eq. (3) one should notice the differences from a phenomenological off-diagonal Majorana mass term introduced in the context of the Dirac equation [5,19].

Since the prerogatives for the ELKO dynamics are established [5,19], we drive our attention to the procedure for obtaining the corresponding effective dispersion relation derived from the exotic Dirac operator. By analogy with the relativistic quantum mechanics terminology, we shall discuss whether the exotic Dirac operator can be considered as a square root of the Klein–Gordon operator — in the sense that $(i\gamma^\mu\nabla_\mu + i\gamma^\mu\partial_\mu\theta - m\mathbb{I})(i\gamma^\mu\nabla_\mu + i\gamma^\mu\partial_\mu\theta + m\mathbb{I}) = (g^{\mu\nu}\nabla_\mu\nabla_\nu + m^2)\mathbb{I}$. This feature must remain true for the 5LKO and its exotic partner:

$$\left((i\gamma^\mu\nabla_\mu + i\gamma^\mu\partial_\mu\theta)\delta^\beta_\alpha \pm m\mathbb{I}\varepsilon^\beta_\alpha\right)\left((i\gamma^\mu\nabla_\mu + i\gamma^\mu\partial_\mu\theta)\delta^\beta_\alpha \mp m\mathbb{I}\varepsilon^\beta_\alpha\right)$$
$$= \left(g^{\mu\nu}\nabla_\mu\nabla_\nu + m^2\right)\mathbb{I}\delta^\beta_\alpha, \tag{4}$$

since the introduction of an exotic spin structure does not modify the Klein–Gordon propagator fulfillment by dark spinor fields. The corresponding Klein–Gordon equation for the exotic ELKO field is hence provided by

$$\left(\Box + m^2 + g^{\mu\nu}\nabla_\mu\nabla_\nu\theta + \partial^\mu\theta\nabla_\mu + \partial^\mu\theta\partial_\mu\theta\right)\mathring{\lambda}(x)^{S/A}_{\{\pm,\mp\}} = 0.$$

This equation can reproduce the same Klein–Gordon propagator for standard and exotic ELKO as well. For the exotic case, it demands the constraint

$$\left(\Box\theta + \partial^\mu\theta\nabla_\mu + \partial^\mu\theta\partial_\mu\theta\right)\mathring{\lambda}^{S/A}_{\{\pm,\mp\}}(x) = 0 \tag{5}$$

which can be formulated without restricting the theory to any particular condition as those assumed in [5].