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Absorption of massless scalar wave by high-dimensional Lovelock black hole

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ABSTRACT

In this Letter we investigate the absorption of a massless scalar field propagating in a static and spherically symmetric five-dimensional (5D) Lovelock black hole spacetime. We give out the analytical and numerical results of partial and total absorption cross sections of the massless scalar wave in detail. we find that when the wavelength of the incoming wave is much smaller than the black hole horizon, the absorption cross section oscillates and tends to the geometry-optical limit of $\sigma_{abs}^{hf} = \pi b_c^2$ which is similar to the electromagnetic wave, Fermion wave by the Schwarzschild black hole, massless scalar wave in Reissner–Nordström black hole and the minimally-coupled massless scalar wave by stationary black hole spacetimes. On the other hand, we investigate effects of two parameters α and Λ of the 5D Lovelock black hole on the absorption cross sections. We also find that higher value of the coupling constant α corresponding to lower total absorption cross section and higher value of Λ corresponding to higher total absorption cross section.

It is well known that general relativity and quantum mechanics are incompatible in their current form. However, after Hawking found that black holes can emit, as well as scatter, absorb and radiation, and that the evaporation rate is proportional to the total absorption cross section. A lot of scholars are interest in the absorption of quantum fields by black hole since 1970s. By using numerical methods, Sanchez [1,2] found that the absorption cross section of massless scalar wave exhibits oscillation around the geometry-optical limit characteristic of diffraction patterns by Schwarzschild black hole. Unruh [3] showed that the scattering cross section for the fermion is exactly 1/8 of that for the scalar wave in the low-energy limit. By numerically solving the single-particle Dirac equation in Painlevé–Gullstrand coordinates, Chris Doran et al. [4] studied the absorption of massive spin-half particle by a small Schwarzschild black hole and they found oscillations around the classical limit whose precise form depends on the particle mass. Crispino et al. [5] have computed numerically the absorption cross section of electromagnetic waves for arbitrary frequencies and have found that its high-frequency behavior is very similar to that for the massless scalar field by the Schwarzschild black hole. In last several years, Oliveira et al. [6] extended to study the absorption of planar in a draining bathtub, the absorption cross section of sound waves with arbitrary frequencies in the canonical acoustic hole spacetime [7] and electromagnetic absorption cross section from Reissner–Nordström black holes [8]. Recently, absorption cross section (or gray body factors) has been of interest in the context of higher-dimensional using standard field theory in curved spacetimes [9–11] and effective string model [12].

More recently Yves Décanini et al. [13,14] used Regge pole techniques and the complex angular momentum approach to investigate the absorption problem for a massless scalar field propagating in static and spherically symmetric black holes of arbitrary dimension endowed with a photon sphere. They obtained an elegant and powerful resummation of the absorption cross section and extracted all the physical information encoded in the sum over the partial wave contributions. On the other hand Lovelock [15,16] extended the Einstein tensor, which is the only symmetric and conserved tensor depending on the metric and its derivatives up to the second order, to the most general tensor. Therefore, the Lovelock theory is the most natural extension of general relativity in higher-dimensional spacetimes. Since the Lovelock theory represents a very interesting scenario to study how the physics of gravity results corrected at short distance due to the presence of higher order curvature terms in the action. C. Garraffo et al. [17–19] gave a black hole solution of this theory, and discussed how short distance corrections to black hole physics substantially change the qualitative features. And M. Aiello et al. [20] presented an exact five-dimensional charged black hole solution in Lovelock gravity coupled to Born–Infeld electrodynamics. In this Letter, following the Décanini's numerical methods, we mainly focus on the massless scalar wave absorption in 5D Lovelock black hole spacetimes. At first we will give a brief review on the Lovelock black

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hole solution, then we analyze the absorption cross section of massless scalar wave by the five-dimensional Lovelock black hole. Then, we numerically stimulate the absorption cross section for fixed different parameters of 5D Lovelock black hole in detail.

The Lovelock Lagrangian density in D dimensions is [15,16]

$$L=\sum_{k=0}^{N}\alpha_k\lambda^{2(k-1)}L_k, \tag{1}$$

where $N=\frac{D}{2}-1$ (for even D) and $N=\frac{D-1}{2}$ (for odd D). In Eq. (1), α_k and λ are constants which represent the coupling of the terms in the whole Lagrangian and give the proper dimensions.

In Eq. (1) L_k is

$$L_k=\frac{1}{2^k}\sqrt{-g}\delta^{i_1\ldots i_{2k}}_{j_1\ldots j_{2k}}R^{j_1j_2}_{i_1i_2}\cdots R^{j_{2k-1}j_{2k}}_{i_{2k-1}i_{2k}}, \tag{2}$$

where $R^{\mu}{}_{\nu\rho\gamma}$ is the Riemann tensor in D dimensions, $R^{\mu\nu}{}_{\rho\sigma}=g^{\nu\delta}R^{\mu}{}_{\delta\rho\sigma}$, g is the determinant of the metric $g_{\mu\nu}$ and $\delta^{i_1\ldots i_{2k}}_{j_1\ldots j_{2k}}$ is the generalized Kronecker delta of order $2k$ [21].

The Lagrangian up to order 2 are given by [22,23]

$$L_0=\sqrt{-g}, \tag{3}$$

$$L_1=\frac{1}{2}\sqrt{-g}\delta^{i_1i_2}_{j_1j_2}R^{j_1j_2}_{i_1i_2}=\sqrt{-g}R, \tag{4}$$

$$\begin{aligned}L_2&=\frac{1}{4}\sqrt{-g}\delta^{i_1i_2i_3i_4}_{j_1j_2j_3j_4}R^{j_1j_2}_{i_1i_2}R^{j_3j_4}_{i_3i_4}\\&=\sqrt{-g}\left(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}-4R_{\mu\nu}R^{\mu\nu}+R^2\right),\end{aligned} \tag{5}$$

where we recognize the usual Lagrangian for the cosmological term, the Einstein–Hilbert Lagrangian and the Lanczos Lagrangian [22,23], respectively.

For $D=5$ dimensions, the Lovelock Lagrangian is a linear combination of the Einstein–Hilbert and Lanczos Lagrangian.

So the geometric action is written as

$$S=\int L\,d^Dx. \tag{6}$$

Here we only consider the spacetime in five dimensions, so the Lagrangian is a linear combination of the Einstein–Hilbert and the Lanczos ones, and the Lovelock tensor results

$$\begin{aligned}\mathcal{G}_{\mu\nu}&=R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}+\Lambda g_{\mu\nu}\\&\quad-\alpha\left\{\frac{1}{2}g_{\mu\nu}\left(R_{\rho\delta\gamma\lambda}R^{\rho\delta\gamma\lambda}-4R_{\rho\delta}R^{\rho\delta}+R^2\right)-2RR_{\mu\nu}\right.\\&\quad\left.+4R_{\mu\rho}R^{\rho}_{\nu}+4R_{\rho\delta}R^{\rho\delta}_{\mu\nu}-2R_{\mu\rho\delta\gamma}R^{\rho\delta\gamma}_{\nu}\right\}.\end{aligned} \tag{7}$$

The five-dimensional Lovelock theory mainly corresponds to Einstein gravity coupled to the dimensional extension of four-dimensional Euler density, that's to say, the theory referred as Einstein–Gauss–Bonnet theory. The spherically symmetric solution in five dimensions [17,24,25] takes the follow form:

$$ds^2=-f(r)\,dt^2+f^{-1}(r)\,dr^2+r^2\,d\Omega_3^2, \tag{8}$$

where $d\Omega_3^2$ is the metric of a unitary 3-sphere, and

$$f(r)=\frac{4\alpha-4M+2r^2-\Lambda r^4/3}{4\alpha+r^2+\sqrt{r^4+\frac{4}{3}\alpha\Lambda r^4+16M\alpha}}, \tag{9}$$

where M,Λ are ADM mass, cosmological constant, respectively, and α is the coupling constant of additional term that presents the ultraviolet correction to Einstein theory.

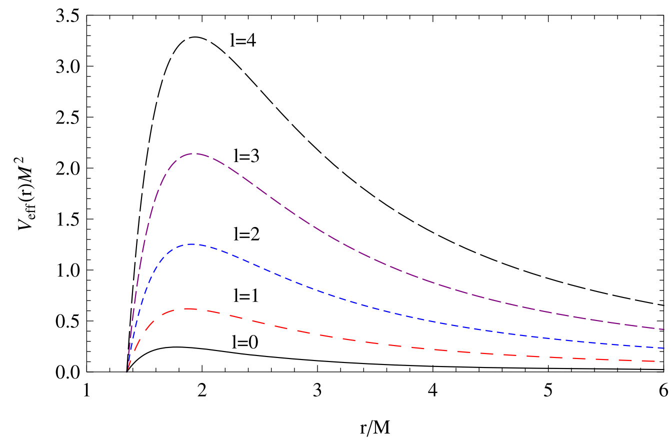

Fig. 1. The scattering potential $V_{eff}(r)$ of the Lovelock black hole for different values of the angular momentum l by fixed parameters $\alpha=0.1$, $\Lambda=0.1$. From this figure, we can see that it vanishes at the black hole horizon r_+ and it will tend to zero as $r\to\infty$.

The general perturbation equation for the massless scalar field Ψ in the curve spacetime is given by

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\right)\Psi=0. \tag{10}$$

The Klein–Gordon equation can be written in the spacetime Eq. (8) as

$$\frac{1}{f}\frac{\partial^2\Psi}{\partial t^2}-\frac{1}{r^2}\frac{\partial}{\partial r}\left(fr^2\frac{\partial\Psi}{\partial r}\right)+\frac{1}{r^2}\nabla^2\Psi=0. \tag{11}$$

The positive-frequency solutions of Eq. (11) take as follows

$$\Psi_{\omega lm}=\left(\psi_{\omega l}(r)/r\right)Y_{lm}e^{-i\omega t}, \tag{12}$$

where Y_{lm} are scalar spherical harmonic functions and l and m are the corresponding angular momentum quantum numbers. In this case, the functions $\psi_{lm}(r)$ satisfy the follow differential equation

$$f\frac{d}{dr}\left(f\frac{d\psi_{\omega l}}{dr}\right)+\left[\omega^2-V^l_{eff}(r)\right]\psi_{\omega l}=0, \tag{13}$$

where

$$\begin{aligned}V^l_{eff}(r)&=\frac{4\alpha-4M+2r^2-\Lambda r^4/3}{4\alpha+r^2+\sqrt{r^4+\frac{4}{3}\alpha\Lambda r^4+16M\alpha}}\\&\quad\times\left[\frac{l(l+2)}{r^2}+\frac{3}{4r^2}\frac{4\alpha-4M+2r^2-\Lambda r^4/3}{4\alpha+r^2+\sqrt{r^4+\frac{4}{3}\alpha\Lambda r^4+16M\alpha}}\right.\\&\quad\left.+\frac{1}{4\alpha}\left(3-\frac{3+4\alpha M}{\sqrt{9r^4+12\alpha\Lambda r^4+144M\alpha}}r^2\right)\right].\end{aligned} \tag{14}$$

The effective potential $V_{eff}(r)$ is plotted in Fig. 1. From this figure, we can see that the effective potential V depends only on the value of r, angular quantum number l, ADM mass M, cosmological constant Λ and coupling constant α, respectively, and that the peak value of potential barrier gets upper and the location of the peak point $(r=r_p)$ moves along the right when the angular momentum l increases. We also can find that it goes to zero as $r\to r_+$ and $r\to\infty$. At the same time, we can find that the height of the effective scattering potential increases as the angular momentum l increases. If we introduce the tortoise coordinate

$$x=\int\left(\frac{4\alpha-4M+2r^2-\Lambda r^4/3}{4\alpha+r^2+\sqrt{r^4+\frac{4}{3}\alpha\Lambda r^4+16M\alpha}}\right)^{-1}dr, \tag{15}$$