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Cosmography: Supernovae Union2, Baryon Acoustic Oscillation, observational Hubble data and Gamma ray bursts

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ABSTRACT

In this Letter, a parametrization describing the kinematical state of the universe via cosmographic approach is considered, where the minimum input is the assumption of the cosmological principle, i.e. the Friedmann–Robertson–Walker metric. A distinguished feature is that the result does not depend on any gravity theory and dark energy models. As a result, a series of cosmographic parameters (deceleration parameter q_0, jerk parameter j_0 and snap parameter s_0) are constrained from the cosmic observations which include type Ia supernovae (SN) Union2, the Baryon Acoustic Oscillation (BAO), the observational Hubble data (OHD), the high redshift Gamma ray bursts (GRBs). By using Markov Chain Monte Carlo (MCMC) method, we find the best fit values of cosmographic parameters in 1σ regions: $H_0 = 74.299^{+4.932}_{-4.287}$, $q_0 = -0.386^{+0.655}_{-0.618}$, $j_0 = -4.925^{+6.658}_{-7.297}$ and $s_0 = -26.404^{+20.964}_{-9.097}$ which are improved remarkably. The values of q_0 and j_0 are consistent with flat ΛCDM model in 1σ region. But the value of s_0 of flat ΛCDM model will go beyond the 1σ region.

1. Introduction

The kinematical approach to describe the status of universe is interesting for its distinguished feature that it does not rely on any dynamical gravity theory and dark energy models. Then it becomes crucial for its potential ability to distinguish cosmological models when a flood of dark energy models and modified gravity theories are proposed to explain the current accelerated expansion of our universe. This late time accelerated expansion of our universe was firstly revealed by two teams' observation of type Ia supernovae [1,2]. In general, via the Taylor expansion of the scale factor $a(t)$ in terms of cosmic time t, the dimensionless coefficients q_0, j_0 and s_0 named deceleration, jerk and snap parameters are defined respectively, for the detailed forms please see Eqs. (8), (9), (10) in the following. For convenience, they are dubbed as *cosmographic parameters*. These cosmographic parameters, which current values can be determined by cosmic observations, describe the kinematical status of our universe. For example, the present value of Hubble parameter H_0 describes the present expansion rate of our universe, and a negative value of q_0 means that our universe is undergoing an accelerated expansion. This kind of approach is also called cosmography [3,4], cosmokinetics [5,6], or Friedmann-less cosmology [7,8]. Recently, this approach was considered by using SN in Ref. [9], SN + GRBs in Ref. [10] and SN + OHD + BAO in [11], where the current status of our universe can be read. On the other hand, for a concrete dark energy model or gravity theory, when the Friedmann equation is arrived the corresponding cosmographic parameters can be derived by simple calculation. As a consequence, the corresponding parameter spaces can be fixed from cosmographic parameters space without implementing annoying data fitting procedure. However, the reliability of the cosmographic approach depends crucially on how the cosmographic parameter space is shrunk, in other words, the improvement of the figure of merit (FoM). That is the main motivation of this Letter. In general, when more cosmic observational data sets are added to constrain model parameter space, the more degeneracies between model parameters will be broken. Also the FoM will be improved. So, to investigate the current status of our universe and to improve the FoM, the cosmographic parameters will be determined by more cosmic observations. When the SN and GRBs are used as distance indicators, the Hubble parameter H_0 and the absolute magnitudes of SN and GRBs are treated as notorious parameters and marginalized. That is to say, SN and GRBs cannot fix the current value of Hubble parameter H_0. That is what the authors have done in Refs. [9,10] where the cosmographic parameters q_0, j_0 and s_0 were investigated. However, the cosmographic parameters permeate in a

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relative larger space. Of course, to describe the kinematical status of our universe well, one has to shrink the parameter space efficiently. Fortunately, when the Hubble parameter H_0 is fixed as done in Ref. [11], the parameter space is pinned down effectively. When the snap parameter s_0 is included, high redshift observations should be added. So, in this Letter we are going to use SN, BAO, GRBs, OHD to investigate the cosmographic approach. When SN data sets are used, the systematic errors are included. The BAO are detected in the clustering of the combined 2dFGRS and SDSS main galaxy samples, so it is helpful to break the degeneracies between parameters. The OHD data sets are used to fix the Hubble parameter H_0. Higher redshift data points are from GRBs where the correlation parameters are calibrated via cosmographic approach synchronously. For the detailed description of these data sets, please see Appendix A.

This Letter is structured as follows. In Section 2, the definition of cosmographic parameters and basic expansions with respect to redshift z are presented, where to consider the convergence issue, the map from $z \in (0,\infty)$ to $y = z/(1+z) \in (0,1)$ is adopted. To the expansion truncation problem, we compare the expansions with ΛCDM model in the range of redshift involved in this Letter. The relative departure of Hubble parameter from that of ΛCDM model is up to 20% at the redshift $z \sim 1.75$. The difference of distance modulus between the expansion of luminosity distance and that of ΛCDM model is less than 1.6. Section 3 are the main results of this Letter. To obtain these results, the cosmic observational data sets from SN Ia, BAO, OHD and GRBs and MCMC method are used. The detailed descriptions are shown in Appendix A. The main points of this Letter are listed as follows: (1) BAO and OHD are used to shrink the model parameter space.[1] (2) The calibration of GRBs and constraint to cosmographic parameters are carried out synchronously. In this way the so-called circular problem is removed. We summarize the results in Table 1 and Figs. 2 and 3. Section 4 is a brief conclusion.

2. Cosmographic parameters

The minimum input of the cosmographic approach is the assumption of the cosmological principle, i.e. the Friedmann–Robertson–Walker (FRW) metric

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right], \tag{1}$$

where the parameter $k = 1, 0, -1$ denotes spatial curvature for closed, flat and open geometries respectively. In this Letter, we only consider the spatially flat case $k = 0$.

The Hubble parameter $H(z)$ can be expanded as

$$H(z) = H_0 + \frac{dH}{dz}\bigg|_0 z + \frac{1}{2}\frac{d^2H}{dz^2}\bigg|_0 z^2 + \frac{1}{3!}\frac{d^3H}{dz^3}\bigg|_0 z^3 + \cdots, \tag{2}$$

where the subscript '0' denotes the value at the present epoch and $z = 1/a(t) - 1$. Via the relation

$$\frac{dt}{dz} = -\frac{1}{(1+z)H(z)}, \tag{3}$$

one has

$$\frac{dH}{dz}\bigg|_0 = -\frac{\dot{H}}{(1+z)H}\bigg|_0 = (1+q_0)H_0, \tag{4}$$

$$\begin{aligned}\frac{d^2H}{dz^2}\bigg|_0 &= \frac{\ddot{H}}{(1+z)^2H^2}\bigg|_0 + \dot{H}\left(\frac{1}{(1+z)^2H} - \frac{\dot{H}}{(1+z)^2H^3}\right)\bigg|_0\\ &= (j_0 + 3q_0 + 2)H_0 - \left(q_0^2 + 3q_0 + 2\right)H_0\\ &= \left(j_0 - q_0^2\right)H_0,\end{aligned} \tag{5}$$

$$\begin{aligned}\frac{d^3H}{dz^3}\bigg|_0 &= -\frac{H^{(3)}}{(1+z)^3H^3}\bigg|_0 - 3\frac{\ddot{H}}{(1+z)^2H^2}\left(\frac{1}{1+z} + \frac{1}{H}\frac{dH}{dz}\right)\bigg|_0\\ &\quad + \frac{\dot{H}}{(1+z)H}\left[-\frac{2}{(1+z)^2} - \frac{2}{(1+z)H}\frac{dH}{dz} - \frac{2}{H^2}\left(\frac{dH}{dz}\right)^2 + \frac{1}{H}\frac{d^2H}{dz^2}\right]\bigg|_0\\ &= \left(6 + 12q_0 + 3q_0^2 + 4j_0 - s_0\right)H_0\\ &\quad - 3(2+3q_0+j_0)(2+q_0)H_0\\ &\quad + (1+q_0)\left[2 + 2(1+q_0) + 2(1+q_0)^2 + q_0^2 - j_0\right]H_0\\ &= \left[3q_0^3 + 3q_0^2 - j_0(3+4q_0) - s_0\right]H_0,\end{aligned} \tag{6}$$

where the cosmographic parameters are defined as follows

$$H_0 \equiv \frac{da(t)}{dt}\frac{1}{a(t)}\bigg|_0 \equiv \frac{\dot{a}(t)}{a(t)}\bigg|_0, \tag{7}$$

$$q_0 \equiv -\frac{1}{H^2}\frac{d^2a(t)}{dt^2}\frac{1}{a(t)}\bigg|_0 \equiv -\frac{1}{H^2}\frac{\ddot{a}(t)}{a(t)}\bigg|_0, \tag{8}$$

$$j_0 \equiv \frac{1}{H^3}\frac{d^3a(t)}{dt^3}\frac{1}{a(t)}\bigg|_0 \equiv \frac{1}{H^3}\frac{a^{(3)}(t)}{a(t)}\bigg|_0, \tag{9}$$

$$s_0 \equiv \frac{1}{H^4}\frac{d^4a(t)}{dt^4}\frac{1}{a(t)}\bigg|_0 \equiv \frac{1}{H^4}\frac{a^{(4)}(t)}{a(t)}\bigg|_0. \tag{10}$$

Then the Hubble parameter can be rewritten in terms of the cosmographic parameters as

$$\begin{aligned}H(z) = H_0\big\{&1 + (1+q_0)z + \left(j_0 - q_0^2\right)z^2/2\\ &+ \left[3q_0^3 + 3q_0^2 - j_0(3+4q_0) - s_0\right]z^3/6 + \cdots\big\}.\end{aligned} \tag{11}$$

For a spatially flat FRW universe, the luminosity distance can also be expanded in terms of redshift z with the cosmographic parameters

$$\begin{aligned}d_L(z) = cH_0^{-1}\big\{&z + (1-q_0)z^2/2 - \left(1 - q_0 - 3q_0^2 + j_0\right)z^3/6\\ &+ \left[2 - 2q_0 - 15q_0^2 - 15q_0^3 + 5j_0\right.\\ &\left.+ 10q_0j_0 + s_0\right]z^4/24 + \cdots\big\}.\end{aligned} \tag{12}$$

Via the relation $d_A(z) = d_L(z)/(1+z)^2$, one has the expansion of $d_A(z)$

$$\begin{aligned}d_A(z) = cH_0^{-1}\big\{&z - (3+q_0)z^2/2 + \left(11 - j_0 + 7q_0 + 3q_0^2\right)z^3/6\\ &+ \left(-50 + 13j_0 - 46q_0 + 10j_0q_0\right.\\ &\left.- 39q_0^2 - 15q_0^3 + s_0\right)z^4/24 + \cdots\big\}.\end{aligned} \tag{13}$$

To avoid problems with the convergence of the series for the highest redshift objects, these relations are recast in terms of the new variable $y = z/(1+z)$ [12,13]

$$\begin{aligned}H(y) = H_0\big\{&1 + (1+q_0)y + \left(1 + q_0 + j_0/2 - q_0^2/2\right)y^2\\ &+ \left(6 + 3j_0 + 6q_0 - 4q_0j_0 - 3q_0^2 + 3q_0^3 - s_0\right)y^3/6\\ &+ \left(1 + q_0 - 2j_0q_0 + 3q_0^3/2 - s_0/2\right)y^4 + \mathcal{O}\left(y^5\right)\big\},\end{aligned} \tag{14}$$

$$\begin{aligned}d_L(y) = cH_0^{-1}\big\{&y + (3-q_0)y^2/2 + \left(11 - j_0 - 5q_0 + 3q_0^2\right)y^3/6\\ &+ \left(50 - 7j_0 - 26q_0 + 10q_0j_0 + 21q_0^2\right.\\ &\left.- 15q_0^3 + s_0\right)y^4/24 + \mathcal{O}\left(y^5\right)\big\},\end{aligned} \tag{15}$$

[1] After our work, the papers used BAO and OHD appeared in arXiv: J.Q. Xia et al., arXiv:1103.0378 and S. Capozziello et al., arXiv:1104.3096.