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Top quark effects on the virtual photon structure function at ILC

Yoshio Kitadono

Institute of Particle and Nuclear Study, High Energy Accelerator Organization (KEK), 1-1 Oho, Tsukuba, Ibaraki, 305-0801, Japan

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ABSTRACT

We investigated top quark effects on virtual photon structure functions by pQCD. We include the top quark mass effects on the virtual photon structure function with the quark parton model and with the operator product expansion up to the next-to-leading order in QCD. We also consider the threshold effect on the running coupling constant in the calculation to the effective photon structure function with a matching condition. The numerical calculations are investigated in the kinematical region expected at the future international linear collider.

1. Introduction

The Large Hadron Collider (LHC) has restarted since last year. One of the most important tasks of the LHC is to discover the Higgs particle which will be the origin of the mass of the particles and the other beyond standard model search is going on now [1]. If the new physics beyond the standard model is discovered, the precise measurement will be done at the future International Linear Collider (ILC) [2]. In such a case we need to know the background from the standard model, especially QCD at high energies.

It is known that the two-photon exchange process ($e^+ + e^- \to \gamma^*\gamma^* \to$ hadrons) is dominated over the one-photon exchange process ($e^+ + e^- \to \gamma^* \to$ hadrons) in the electron–positron collision [3,4]. The cross section in this two-photon process is characterised by photon structure functions and see Refs. [5–8] for reviews. The photon structure functions have two types, namely real photon structure functions $F^{\gamma}_{2,L}(x, Q^2)$, $g^{\gamma}_1(x, Q^2)$ and virtual photon structure functions $F^{\gamma}_{2,L}(x, Q^2, P^2)$, $g^{\gamma}_1(x, Q^2, P^2)$, where $Q^2 = -q^2$ is a squared momentum of the probe photon, $P^2 = -p^2$ is a squared momentum of the target photon, x is Bjorken variable in the two-photon process respectively. While the real photon structure functions need to include non-perturbative effects like the vector meson dominance, the perturbative part in the virtual photon structure functions dominates at the kinematical region $\Lambda^2_{QCD} \ll P^2 \ll Q^2$. Therefore we consider the virtual photon structure functions, especially unpolarised functions, in order to avoid the non-perturbative effect in this Letter.

Much work on the photon structure functions have been carried out for both the real photon target [9–17] and the virtual photon target [18–23]. Although the heavy quark effects on the photon structure functions have been studied [24–27], their phenomenological applications were to charm quark or to bottom quark due to the kinematical constraints of experiments. We can expect easily that top quark effects on the photon structure functions will be important at the ILC. We know that top quark have $(2e/3)$ charge in the proton unit of the electro-magnetic charge and the top quark is the heaviest quark in the standard model. The large electro-magnetic charge of up-type quarks compared to down-type quarks relatively will enhance the value of the photon structure function, but the large mass of the top quark will reduce that of photon structure functions. We have to study the size of the top quark effects on photon structure functions at the ILC.

In this Letter, we consider the top quark effects on the unpolarised virtual photon structure functions with the method based on the operator product expansion (OPE) improved by the renormalisation group equation (RGE), and with the method based on the quark parton model (QPM). The top quark effects by OPE and QPM at ILC are discussed in the next section and the numerical calculation based on the framework with OPE and QPM are discussed and the results are shown in the Section 3. Final section is devoted to a conclusion.

2. Top quark effects on virtual photon structure functions

We can incorporate the top quark effects by the two methods, namely OPE supplemented by the RGE and QPM. We use the

E-mail address: kitadono@post.kek.jp.

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formalism in Ref. [28] for the calculation of photon structure functions by OPE and we use the results in Ref. [24] by QPM.

2.1. Operator product expansion

Let us define the n-th moment of photon structure functions by the equation

$$M^{\gamma}_{2(L)}\big(n, Q^2, P^2\big) = \int_0^1 dx\, x^{n-2} F^{\gamma}_{2(L)}\big(x, Q^2, P^2\big), \tag{1}$$

where $n-2$ is due to the our convention of structure functions. In the formalism of Ref. [28], it is assumed that we divide the n_f quarks system into two parts, namely $n_f - 1$ massless quarks system and one massive quark. We apply this formalism to the virtual photon structure functions at the ILC and we assume that u, d, s, c, b are massless quarks and t is the massive quark at the kinematical region expected at the ILC. The moment by OPE including heavy quark effects can be summarised as the following form,

$$\begin{aligned} &M^{\gamma}_{2(L)}\big(n, Q^2, P^2\big) \\ &\quad = M^{\gamma}_{2(L)}\big(n, Q^2, P^2\big)_{\text{massless}} + \Delta M^{\gamma}_{2(L)}\big(n, Q^2, P^2\big), \end{aligned} \tag{2}$$

where the first term means the contribution from massless quarks and is given as

$$\begin{aligned} &M^{\gamma}_{2(L)}\big(n, Q^2, P^2\big)_{\text{massless}} \Big/ \frac{\alpha}{8\pi\beta_0} \\ &\quad = \frac{4\pi}{\alpha_s(Q^2)} \sum_{i=\pm,NS} \mathcal{L}^n_{2(L),i}\big(1 - r^{d^n_i+1}\big) \\ &\qquad + \sum_{i=\pm,NS} \mathcal{A}^n_{2(L),i}\big(1 - r^{d^n_i}\big) + \sum_{i=\pm,NS} \mathcal{B}^n_{2(L),i}\big(1 - r^{d^n_i+1}\big) \\ &\qquad + \mathcal{C}^n_{2(L)}, \end{aligned} \tag{3}$$

where $r = \alpha_s(Q^2)/\alpha_s(P^2)$ is the ratio of coupling constant with different scales, d^n_i corresponds to the eigen-values of one-loop hadronic anomalous dimension matrix, the sum runs over the index to the same eigen-values. These forms of the perturbative expansion are common for M^{γ}_2 and M^{γ}_L up to the next-to-leading order (NLO) in QCD. The long expression to the coefficients $\mathcal{L}^n_{2(L),i}, \mathcal{A}^n_{2(L),i}, \mathcal{B}^n_{2(L),i}$ and $\mathcal{C}^n_{2(L)}$ are given in Ref. [18]. The above moment consists of $n_f - 1$ massless quarks (u, d, s, c, b).

On the other hand, the heavy quark effects are incorporated in the moment of the effective structure function up to the NLO in QCD with OPE supplemented by the mass-independent RGE formalism and the moment is given by the form,

$$\begin{aligned} &\Delta M^{\gamma}_2\big(n, Q^2, P^2\big) / \frac{\alpha}{8\pi\beta_0} \\ &\quad = \sum_{i=\pm,NS} \Delta\mathcal{A}^n_i\big(1 - r^{d^n_i}\big) + \sum_{i=\pm,NS} \Delta\mathcal{B}^n_i\big(1 - r^{d^n_i+1}\big) \\ &\qquad + \Delta\mathcal{C}^n, \end{aligned} \tag{4}$$

where the expression of the coefficients $\Delta\mathcal{A}^n_i, \Delta\mathcal{B}^n_i$ and $\Delta\mathcal{C}^n$ are given in Ref. [28] and all finite coefficients are related with the variation of the operator matrix element for the top quark due to mass effects. The variation by the heavy quark effects to coefficients in M^{γ}_L is zero up to this order. The above variation of the moment due to the top quark mass consists of one-massive quark (t) as we mentioned previously. We reconstruct the structure functions from the moment by Mellin inversion numerically,

$$F^{\gamma}_{2(L)}\big(x, Q^2, P^2\big) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dn\, x^{-n-1} M^{\gamma}_{2(L)}\big(n, Q^2, P^2\big), \tag{5}$$

where c is a positive constant. Although we choose $c = 1.5$, generally speaking, the result is independent of choice of constant c.

2.2. Quark parton model

The effects of the heavy quark mass are incorporated by the heavy quark propagator in related QED box diagrams. The structure functions by QPM are given by the equations,

$$F^{\gamma}_2\big|^{\text{QPM}} = \frac{x}{\tilde{\beta}^2}\left(W_{TT} + W_{LT} - \frac{1}{2}W_{TL} - \frac{1}{2}W_{LL}\right), \tag{6}$$

$$F^{\gamma}_L\big|^{\text{QPM}} = x\left(W_{TT} - \frac{1}{2}W_{LL}\right), \tag{7}$$

where $\tilde{\beta} = \sqrt{1 - p^2q^2/(p\cdot q)^2}$ and the explicit expressions of W_{TT}, W_{LT}, W_{TL}, and W_{LL} are given by the equations of Appendix B in Ref. [24]. Although the above normalisation is different from one in Ref. [24], we use $2W_{TT}, \ldots, 2W_{LL}$ in Ref. [24]. This convention is compatible with the normalisation used in Ref. [28]. In QPM results, all structure functions $W_{TT}, \ldots, W_{LL}$ are expressed by the factors β, $\tilde{\beta}$, L, Q^2, P^2 and x. The parameters β and L are given by $\beta = \sqrt{1 - \frac{(4m^2+P^2)}{Q^2}\frac{x}{(1-x)}}$, $L = \log(\frac{1+\beta}{1-\beta})$, and the threshold effect of the heavy quark mass is controlled by these factors. The factors β and L vanish at a maximum point of Bjorken variable $x_{\max} = \frac{1}{1+\frac{4m^2+P^2}{Q^2}}$, where this maximum point in Bjorken x is derived by the condition $s = (p+q)^2 \geqslant 4m^2$. Therefore structure functions $F^{\gamma}_2, F^{\gamma}_L$ by QPM are limited in the range $0 \leqslant x \leqslant x_{\max}$ and are insured to vanish at the points $x = 0, x_{\max}$, but the structure functions by OPE is not guaranteed to vanish at the points.

3. Numerical calculations

The effective photon structure function is often measured in experiments. This effective structure function is proportional to the total cross section of the two-photon process and is given by the equation,

$$F^{\gamma}_{\text{eff}}\big(x, Q^2, P^2\big) = F^{\gamma}_2\big(x, Q^2, P^2\big) + \frac{3}{2}F^{\gamma}_L\big(x, Q^2, P^2\big). \tag{8}$$

We used following masses for both QPM and OPE as inputs in this Letter,

$$\begin{aligned} &m_u = 0.003\ \text{GeV}, \qquad m_d = 0.006\ \text{GeV}, \\ &m_s = 0.12\ \text{GeV}, \qquad m_c = 1.3\ \text{GeV}, \\ &m_b = 4.2\ \text{GeV}, \qquad m_t = 170\ \text{GeV}, \end{aligned} \tag{9}$$

where we consider all quarks are massive in QPM and the top quark is the massive particle in OPE.

3.1. Q^2 and P^2 dependence in a moment of the effective photon structure function

We calculate the P^2 dependence to the moment with $n = 2$, $Q^2 = 3000\ \text{GeV}^2$, $Q^2 = 30\,000\ \text{GeV}^2$ by OPE,

$$M^{\gamma}_{\text{eff}}\big(n = 2, Q^2, P^2\big) = \int_0^1 dx\, F^{\gamma}_{\text{eff}}\big(x, Q^2, P^2\big), \tag{10}$$