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$O(\alpha_s)$ heavy flavor corrections to charged current deep inelastic scattering in Mellin space

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ABSTRACT

We provide a fast and precise Mellin space implementation of the $O(\alpha_s)$ heavy flavor Wilson coefficients for charged current deep inelastic scattering processes. They are of importance for the extraction of the strange quark distribution in neutrino–nucleon scattering and the QCD analyses of the HERA charged current data. Errors in the literature are corrected. We also discuss a series of more general parton parameterizations in Mellin space.

1. Introduction

The scale evolution in Quantum Chromodynamics (QCD) can be performed very efficiently in Mellin space. There the evolution equations of the twist-2 parton distributions and the corresponding observables become ordinary differential equations, unlike in momentum fraction space, where integro-differential equations have to be solved numerically. In Mellin space, moreover, the evolution equations can be solved analytically, provided that the anomalous dimensions and Wilson coefficients can be represented at complex values of the Mellin variable N, cf. [1]. The transformation mediating between both representations is

$$\mathrm{M}\big[f(x)\big](N) = \int_0^1 dx\, x^{N-1} f(x), \tag{1}$$

where $f(x)$ may be distribution valued. The anomalous dimensions and massless Wilson coefficients to 3-loop order are functions of the nested harmonic sums [2,3], see Refs. [4]. The nested harmonic sums $S_{\vec{a}}(N)$ are meromorphic functions in the complex plane with poles at the non-positive integers and in case that the first k indices a_i are equal to one, they diverge $\propto \ln^k(N)$ for large values of N, [5]. They obey recursion relations for arguments $N \to (N+1)$ in terms of harmonic sums of lower weight, which allow for shifts parallel to the real axis. Moreover, analytic asymptotic representations have been derived, [5,6], which are valid in the region of large values of N, $|\arg(N)| < \pi$. One may derive effective numerical representations using the MINIMAX method [7], cf. [8,9]. In this way the QCD observables at the level of twist-2 are represented analytically at general scales Q^2, parameterizing the parton density functions by appropriate non-perturbative distributions at a starting scale Q_0^2. This also applies for a wide class of other processes, cf. [10].

In many precision analyses heavy quark contributions have to be considered, which are more difficult to represent in Mellin space. For deep inelastic scattering the contributions up to the $O(\alpha_s^2)$ neutral current heavy flavor Wilson coefficients [11], which are available in semi-analytic form in momentum fraction space, were given in Mellin space in Ref. [12]. The running mass effects have been described in [13].

In the present Letter a fast and precise implementation of the $O(\alpha_s)$ charged current heavy flavor Wilson coefficients for deep inelastic scattering is presented. In Section 2 we derive the corresponding Mellin space representations for the charged current structure functions up to $O(\alpha_s)$. As a significant part of the data emerges for large values of Q^2, we also derive the representation in Section 3 which is

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valid for $Q^2 \gg m_c^2$, with $Q^2 = -q^2$, q being the four-momentum transfer, and m_c the charm quark mass. We illustrate the precision of the Mellin space implementation by comparing to the structure functions in momentum fraction space in Section 4. Finally we remark on the Mellin space implementations of various x-space distributions used in different analyses in the literature in Section 5, which may be important to account for a higher flexibility in the choice of the non-perturbative input distributions. This will allow to perform analogous analyses in Mellin space in the future.

2. The scattering cross section

The charged current deep inelastic scattering cross section for $\nu(\bar{\nu})p \to l(\bar{l}) + X$, $l(\bar{l})p \to \nu(\bar{\nu}) + X$ in case of charm quark production is given by

$$\frac{d^2\sigma}{dxdy} = \frac{G_F^2 S}{4\pi} \frac{M_W^4}{(M_W^2 + Q^2)^2} \left\{ y^2 2x\mathbb{F}_1^c(x, Q^2, m_c^2) + 2\left(1 - y - y^2\frac{M^2x^2}{Q^2}\right)\mathbb{F}_2^c(x, Q^2, m_c^2) \pm \left[1 - (1-y)^2\right] x\mathbb{F}_3^c(x, Q^2, m_c^2)\right\}. \quad (2)$$

Here $x = Q^2/yS$, $y = q.P/l.P$ are the Bjorken variables, l and P are the lepton and nucleon momentum, $S = 2l.P$, G_F is the Fermi constant, M_W the mass of the W-boson, M the nucleon mass, and $\mathbb{F}_i(x, Q^2)$ are the structure functions. The $\pm$ signs in (2) refer to incoming neutrinos (anti-neutrinos) or charged leptons (anti-leptons), respectively. Note that the scattering cross section does depend on the masses of the initial state particles, cf. [14]. Due to the inclusive kinematics the dependence on m_c is only implicit.

In the twist-2 approximation, invoking the parton model, one may decompose the nucleon wave function into individual partons and consider the excitation of single charm quarks in the transitions

$$s' = s|V_{cs}|^2 + d|V_{cd}|^2 \to c, \qquad \bar{s}' = \bar{s}|V_{cs}|^2 + \bar{d}|V_{cd}|^2 \to \bar{c}, \quad (3)$$

with V_{ij} the CKM matrix elements [15], and s and d the strange and down quark parton distributions. Here $\bar{d}$ and $\bar{s}$ denote the corresponding anti-quark distributions. For this transition the Bjorken variable x and the momentum fraction of the struck parton ξ are related by

$$x = \xi\lambda \leqslant \lambda \quad (4)$$

with $\lambda = Q^2/(Q^2 + m_c^2)$. This phenomenon is called slow rescaling [16]. The heavy quark structure functions and parton densities can be expressed as a function of $\xi \in [0, 1]$, preserving Mellin symmetry.

The $O(\alpha_s)$ charged current heavy flavor Wilson coefficients were calculated in [17] and corrected in [18] later. We will follow Ref. [18] and work in the fixed flavor number scheme (FFNS).[1] We define

$$\mathcal{F}_1^c = \mathbb{F}_1^c, \qquad \mathcal{F}_2^c = \mathbb{F}_2^c/2\xi, \qquad \mathcal{F}_3^c = \mathbb{F}_3^c/2. \quad (5)$$

To $O(\alpha_s)$ one obtains, after a Mellin transformation (1) over ξ,

$$\mathcal{F}_i^c(N, Q^2) = s'(N, \mu^2) + a_s\left[H_i^{(1),q}\left(N, \frac{m^2}{\mu^2}, \frac{Q^2}{\mu^2}\right)s'(N, \mu^2) + H_i^{(1),g}\left(N, \frac{m^2}{\mu^2}, \frac{Q^2}{\mu^2}\right)g(N, \mu^2)\right], \quad (6)$$

for W^+ exchange. In case of W^- exchange $\bar{s}'$ replaces s'. Here $a_s = \alpha_s/(4\pi)$ denotes the strong coupling constant and g the gluon distribution. The massive Wilson coefficients for charged current deep inelastic scattering in Mellin space are given by[2]

$$H_i^q\left(N, \frac{m^2}{\mu^2}, \frac{Q^2}{\mu^2}\right) = 1 + \sum_{k=1}^{\infty} a_s^k H_i^{(k),q}\left(N, \frac{m^2}{\mu^2}, \frac{Q^2}{\mu^2}\right), \quad (7)$$

$$H_i^g\left(N, \frac{m^2}{\mu^2}, \frac{Q^2}{\mu^2}\right) = \sum_{k=1}^{\infty} a_s^k H_i^{(k),g}\left(N, \frac{m^2}{\mu^2}, \frac{Q^2}{\mu^2}\right) \quad (8)$$

with

$$H_i^{(1),q}\left(N, \frac{m^2}{\mu^2}, \frac{Q^2}{\mu^2}\right) = \frac{1}{2}P_{qq}^{(0)}(N)\ln\left(\frac{Q^2 + m_c^2}{\mu^2}\right) + h_i^q(\lambda, N), \quad i = 1, 2, 3, \quad (9)$$

$$H_{1,2}^{(1),g}\left(N, \frac{m^2}{\mu^2}, \frac{Q^2}{\mu^2}\right) = \frac{1}{4}P_{qg}^{(0)}(N)\ln\left(\frac{Q^2 + m_c^2}{\mu^2}\right) + \frac{1}{4}\tilde{P}(\lambda, N) + h_{1,2}^g(\lambda, N), \quad (10)$$

$$H_3^{(1),g}\left(N, \frac{m^2}{\mu^2}, \frac{Q^2}{\mu^2}\right) = \frac{1}{4}P_{qg}^{(0)}(N)\ln\left(\frac{Q^2 + m_c^2}{\mu^2}\right) - \frac{1}{4}\tilde{P}(\lambda, N) + h_3^g(\lambda, N). \quad (11)$$

Here the leading order splitting functions are

[1] The use of a variable flavor number scheme needs care, since the scales at which one massive flavor can be dealt with as massless are process dependent. The corresponding scales are usually not $\mu^2 \sim m^2$, cf. Ref. [19].

[2] Throughout the present Letter we consider the scale derivative in the renormalization group operator as $\partial/\partial\ln(\mu^2)$.