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# Galactic dynamos and slow decay of magnetic fields from torsion modes of Lorentz violation

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ABSTRACT

Campanelli et al. (2009) [9] have recently shown that electromagnetic fluctuations in Lorentz violation theories Colladay and Kostelecky (1998) [8] along differential rotation may induce galactic dynamo amplification of magnetic fields from primordial seeds. In this Letter, instead of using the Maxwell–Chern–Simmons–Lagrangian used in their paper, one adopts the Lagrangean of the type $R_{ijkl}F^{ij}F^{kl}$, where $R_{ijkl}$ represent the torsion modes, without dynamical curvature. This so-called Minkowski–Cartan spacetime $\mathbf{M}^4$ torsion modes, allows us to handle QED vacuum in any Ricci scalar electrodynamics Lagrangean. It is shown that axial-torsion modes electrodynamics allows us to obtain a slow decay of magnetic fields. Thus primordial seed fields are amplified from differential rotation and protogalaxy turbulence as galactic dynamo action. This is mainly due to the fact that the collapse of the protogalaxy induces a strong suppression of the magnetic field decay. Magnetic field anisotropies are also considered. To resume, photon-torsion axial coupling in the quantum electrodynamics (QED) framework in Riemann flat contortioned spacetime may induce galactic dynamos. Fourier space transformation are used to compute electrodynamic equations.



## 1. Introduction

Earlier Vachaspati and Enqvist and Olesen [1,2] have investigated primordial magnetic fields of electroweak origin. Their magnetic field is generated by the electroweak phase transition of the universe. In their work, either the Higgs field or the magnetic field itself, are stochastic variables. Random values of the magnetic field in the present universe, is fully consistent with what is required to seed galactic dynamo mechanism. Our approach here is based on previous work by de Sabbata and Gasperini [3,4] on a perturbative approach to QED generalized Maxwell equation with totally skew torsion. This allows one to perform photon-torsion perturbative calculation production of virtual pairs on vacuum polarization effect or QED. In this Letter we address the non-minimal extension of QED, to Riemann–Cartan geometry. Actually Drummond and Hathrell [5] have investigated its Riemannian version. Here photon-torsion coupling comes from the interaction between the Riemann–Cartan tensor and electromagnetic field tensor in the Lagrangean action term of the type $R_{ijkl}F^{ij}F^{kl}$ where $i, j = 0, 1, 2, 3$. Therefore the usual problems of the noninteraction between photons and torsion which appears in the usual Maxwell electrodynamics [6] do not appear here. More recently, Battefeld et al. [7] have investigated the amplification of primordial magnetic fields in string cosmology. In this Letter to emphasize the role played by gravitational torsion in the amplification of the magnetic fields, in the same way was done with their amplification from metric fluctuations, one considers that the torsion-free Riemannian tensor vanishes. The plan of the Letter is as follows: In Section 2 we consider the formulation of the Riemann–Cartan (RC) QED vacuum electrodynamics, where torsion modes live in Minkowski–Cartan spacetime background. In Section 3 the Riemann-flat case is presented and geometrical optics in non-Riemannian spacetime along with ray equations are presented. In Section 4 galactic magnetic dynamos are obtained by Fourier analyzing the wave equations. In this section Lorentz violation [8] originating from the presence of a homogeneity growth of torsion modes induces a slower decay of magnetic fields. The presence of the differential rotation and the collapse of protogalaxies, act together to accelerate suppression of the magnetic field decay and revert this process into a dynamo amplification in galaxies. Section 5 contains conclusions and discussions.

## 2. Flat torsion modes QED in Minkowski–Cartan spacetime

Since the torsion effects are in general too weak [3,4], throughout the Letter second order effects on torsion are neglected in the electrodynamics and curvature expressions. In this section we consider a simple cosmological application concerning the electrodynamics in vacuum QED spacetime background. The Lagrangean used in this Letter is obtained from the work of Drummond et al. [5]

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$$W = \frac{1}{m^2}\int d^4x\,(-g)^{\frac{1}{2}}\left(-\frac{1}{4}F^{ij}F_{ij} + aR^{*}F^{ij}F_{ij} + bR^{*}{}_{ik}F^{il}F^{k}{}_{l} + cR^{*}{}_{ijkl}F^{ij}F^{kl} + dD_iF^{ij}D_kF^{k}{}_{j}\right). \tag{1}$$

The physical constants $a, b, c, d$ can be obtained by means of the conventional Feynman diagram techniques [5]. The field equations obtained are [5]

$$D_iF^{ik} + \frac{1}{m_e^2}D_i\left[4aRF^{ik} + 2b\left(R^{i}{}_{l}F^{lk} - R^{k}{}_{l}F^{li}\right) + 4cR^{ik}{}_{lr}F^{lr}\right] = 0, \tag{2}$$

$$D_iF_{jk} + D_jF_{ki} + D_kF_{ij} = 0 \tag{3}$$

where $D_i$ is the Riemannian covariant derivative, while $F^{ij} = \partial^iA^j - \partial^jA^i$ is the electromagnetic field tensor non-minimally coupled to gravity. Here $A^i$ is the electromagnetic vector potential, $R$ is the Riemannian Ricci scalar, $R_{ik}$ Ricci tensor and $R_{ijkl}$ is the Riemann curvature tensor. Before we apply it to de Sitter model, let us consider several simplifications. The first one concerns the fact that the photon is treated as a test particle, and the second considers simplifications on the torsion field. The Riemann–Cartan curvature tensor reads

$$R^{*ij}{}_{kl} = R^{ij}{}_{kl} + D^iK^{j}{}_{kl} - D^jK^{i}{}_{kl} + \left[K^i, K^j\right]_{kl}. \tag{4}$$

The last term here shall be dropped since we are just considering the first order terms on the contortion tensor. Quantities with an upper asterisk represent RC geometrical quantities. We also consider only the axial part of the contortion tensor $K_{ijk}$ in the form

$$K^i = \epsilon^{ijkl}K_{jkl} \tag{5}$$

to simplify Eq. (2) we consider the expression for the Ricci tensor

$$R^{*i}{}_{k} = R^{i}{}_{k} - \epsilon^{i}{}_{klm}D^{[l}K^{m]} \tag{6}$$

where $\epsilon^{i}{}_{klm}$ is the totally skew-symmetric Levi-Civita symbol. By considering the axial torsion as a gradient of a dilaton field $\phi$ one obtains

$$K^i = D^i\phi. \tag{7}$$

Expression (7) in first approximation becomes

$$\partial^{[l}K^{m]} = 0. \tag{8}$$

Thus expression (6) reduces to

$$R^{*i}{}_{k} = R^{i}{}_{k}. \tag{9}$$

Note that in the Riemann-flat case considered in the next section, $R_{ijkl} = 0$ and $R^{*i}{}_{k} = 0$ which strongly simplifies the Maxwell type equations. The advantage of using the Riemannian Lagrangean above is that one now is allowed to work in vacuum instead in Lagrangeans of the Ricci type $R^nF^2$ used by Campanelli et al. [9]. In this section the curvature is

$$R_{ijkl} = 0. \tag{10}$$

Thus the Einstein equations in vacuum $R_{ik} = 0$ reduce to $R^{*i}{}_{k} = 0$. The constants $a$, $b$, $c$ are given as in Drummond and Hathrell [5]

$$a = \frac{-5\alpha}{720\pi}, \tag{11}$$

$$b = \frac{26\alpha}{720\pi}, \tag{12}$$

$$c = \frac{-\alpha}{360\pi}. \tag{13}$$

Substitution of these expressions into the Maxwell-like equation yields

$$\left(1 + 2\xi^2K\right)D_iF^{i}{}_{k} = \epsilon_{klmn}D^iK^nD_iF^{lm}. \tag{14}$$

Here $\xi^2 = \frac{\alpha}{90\pi {m_e}^2}$ where $m_e$ is the electron mass and $\alpha$ is the fine structure constant. This result provides interesting applications in cosmology such as in the study of optical activity in cosmologies with torsion which Kalb–Ramond cosmology is an example [10].

## 3. Riemann-flat linear electrodynamics and magnetic dynamos from inhomogeneous torsion

In this section we shall be concerned with the application of linear electrodynamics with torsion in the Riemann-flat case. Here the Riemann curvature tensor vanishes. In particular we shall investigate the non-Riemannian geometrical optics associated with that. Earlier L.L. Smalley [11] has investigated the extension of Riemannian to a non-Riemannian RC geometrical optics in the usual electrodynamics. However, from his approach was not clear if the torsion was really able to couple with photon. Since the metric considered here is the Minkowskian one $\eta_{ij}$, we note that the Riemannian Christoffel connection vanishes and the Riemannian derivative operator $D_k$ may be replaced by the partial derivative operator $\partial_k$. With these simplifications in mind, Maxwell-like Eq. (2) becomes

$$\partial_iF^{ij} + \xi^2R^{ij}{}_{kl}\partial_iF^{kl} = 0 \tag{15}$$

which reduces to

$$\partial_iF^{ik} + \xi^2\left[\epsilon^{k}{}_{jlm}\partial^iK^m - \epsilon^{i}{}_{jlm}\partial^kK^m\right]\partial_iF^{jl} = 0. \tag{16}$$

We may also note that when the contortion is parallel transported the equations reduce to the usual Maxwell equation

$$D_iF^{il} = 0. \tag{17}$$

Since we are considering non-minimal coupling of the Lorentz condition on the vector potential

$$\partial_iA^i = 0 \tag{18}$$

substitution into the Maxwell-like equation yields the wave equation for the vector electromagnetic potential as

$$\Box A^i + \xi^2\left[\epsilon^{k}{}_{jlm}\partial^iK^m - \epsilon^{i}{}_{jlm}\partial^kK^m\right]\partial_kF^{jl} = 0. \tag{19}$$

By considering the Fourier spectra of this expression, one obtains

$$\left[\partial^2\eta + k^2\right]A^a + i\xi^2\left[\epsilon^{a}{}_{de0}\left[k^dA^e - k^eA^d\right]K^0\right]k_2 = 0. \tag{20}$$

From the symmetries of Levi-Civita object, this expression can be simplified by contraction with the wave-vector $k_a$. This simply yields

$$\left[\partial^2\eta + k^2\right]k_aA^a = 0. \tag{21}$$

In this expression $\eta$ represents the conformal cosmic coordinate of the flat spacetime

$$ds^2 = d\eta^2 - dx^2 \tag{22}$$

plus torsion effects. Solution of the wave equation yields

$$A^a = \frac{a^0}{k^2}e^{ik\eta}k^a \tag{23}$$

where $a = 1, 2, 3$. This solution is similar to the one found by Bassett et al. [12] in the case of nonlinear electrodynamic magnetic