Physics Letters B 696 (2011) 74–78

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Electromagnetic properties of dark matter: Dipole moments and charge form factor

Vernon Barger [a], Wai-Yee Keung [b], Danny Marfatia [c,*]

[a] Department of Physics, University of Wisconsin, Madison, WI 53706, USA
[b] Department of Physics, University of Illinois, Chicago, IL 60607, USA
[c] Department of Physics and Astronomy, University of Kansas, Lawrence, KS 66045, USA

ARTICLE INFO

Article history:
Received 3 September 2010
Received in revised form 1 December 2010
Accepted 2 December 2010
Available online 8 December 2010
Editor: S. Dodelson

Keywords:
Dark matter
Electric dipole moment
Magnetic dipole moment
Electric charge form factor

ABSTRACT

A neutral dark matter particle may possess an electric dipole moment (EDM) or a magnetic dipole moment (MDM), so that its scattering with nuclei is governed by electromagnetic interactions. If the moments are associated with relevant operators of dimension-5, they may be detectable in direct search experiments. We calculate complete expressions of the scattering cross sections and the recoil energy spectra for dark matter with these attributes. We also provide useful formulae pertinent to dark matter that interacts via an electric charge form factor (CFF) which is related to the charge radius defined by an effective dimension-6 operator. We show that a 7 GeV dark matter particle with an EDM, MDM or CFF easily reproduces the CoGeNT excess while remaining consistent with null searches.

1. Introduction

The nature of the dark matter (DM) particle is unknown. There are many well studied scenarios guided by theoretical niceties such as the Minimal Supersymmetric Standard Model, models with extra dimensions, and the Little Higgs Models with T-parity, all of which posit that the DM particle interacts primarily via weak interactions. On the experimental side, direct DM searches are looking for signals of the recoiling nuclei from DM–nucleus scattering. We take the uncommon view that the scattering process may be electromagnetic in nature. The interaction occurs through the electric dipole moment (EDM) or the magnetic dipole moment (MDM) of dark matter. The interactions are described by dimension-5 operators for non-self-conjugate particles, such as Dirac DM, but not for Majorana DM. The EDM and MDM of DM can be induced by underlying short distance physics at the one-loop order or higher. As there are no strong reasons against large CP violation in the DM sector, we cannot neglect the EDM possibility. In fact, for comparable short distance cutoffs, we find that an EDM may give the dominant contribution to DM–nucleus scattering because it directly couples to the nuclear charge. In the EDM case, the recoil energy E_R distribution is highly enhanced in the low E_R region as $1/(v_r^2 E_R)$ (where v_r is the speed of the DM particle in the rest frame of the nucleus) in contrast with that from MDM which goes as $1/E_R$.

While DM–nucleus scattering due to the MDM of DM has been studied extensively in the literature, the relevant formulae with the correct dependence on the nuclear charge Z and nuclear moment $\mu_{Z,A}$ are not available. In this work, we provide analytic expressions for the scattering cross sections for the MDM and EDM cases with careful expansions in the relative velocity and recoil energy. To complete the treatment of electromagnetic properties of DM, we extend our analysis to the dimension-6 operator, which is the electric charge form factor (CFF) slope or the charge radius of the neutral DM. The operator has a structure that is similar to that of the spin-independent (SI) interaction.

2. Electric dipole moment of dark matter

The effective non-relativistic Hamiltonian of the EDM of a particle with spin $\boldsymbol{S}$ is

$$H_{\text{eff}} = -\mathrm{d}\boldsymbol{E}\cdot\boldsymbol{S}/S,$$

with the normalization chosen to agree with the standard form for a spin $\frac{1}{2}$ particle, *i.e.*, $-\mathrm{d}\boldsymbol{E}\cdot\boldsymbol{\sigma}$, where d is dimensionful, and the electromagnetic energy density is $(E^2+B^2)/2$. As the electric field of the nucleus $\boldsymbol{E} = -\operatorname{grad}\phi$, we identify the gradient with the momentum transfer $\boldsymbol{q}$. Therefore, the direct scattering of the DM particle χ and the nucleus N, $\chi N \to \chi N$, via the interaction

* Corresponding author.
E-mail address: marfatia@ku.edu (D. Marfatia).

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2010.12.008

between the DM electric dipole (with moment d_χ) and the nuclear charge Ze (with $e^2 = 4\pi\alpha$) is

$$\mathcal{M} = \boldsymbol{S} \cdot i\boldsymbol{q}\left(\frac{\mathrm{d}_\chi}{S}\frac{1}{\boldsymbol{q}^2}Ze\right).$$

It is important to note that the momentum transfer $\boldsymbol{q}$ is Galilean invariant and is conveniently related to the center-of-mass momentum $\boldsymbol{p}$. Hence $d\boldsymbol{q}^2 = 2|\boldsymbol{p}|^2 d\cos\theta = 4|\boldsymbol{p}|^2\frac{d\Omega}{4\pi}$, and $|\boldsymbol{p}| = m_r v_r$, where $m_r \equiv \frac{m_A m_\chi}{m_A + m_\chi}$ is the reduced mass; m_A and m_χ are the masses of the nucleus and DM particle, respectively. $\boldsymbol{q}^2$ ranges from 0 to $(2m_r v_r)^2$, as is easily checked in the center-of-mass frame in which the momenta have equal magnitudes $m_r v_r$.[1] We use the spin relation,

$$\mathrm{Tr}\left(S^i S^j\right) = (2S+1)\delta^{ij} S(S+1)/3,$$

to obtain the spin-averaged differential cross section,

$$d\sigma_{EDM}(\chi N) = \frac{1}{4\pi}\mathrm{d}_\chi^2 Z^2 e^2 \frac{(S+1)}{3S}\frac{1}{v_r^2}\frac{d\boldsymbol{q}^2}{\boldsymbol{q}^2}\left|G_E\left(\boldsymbol{q}^2\right)\right|^2. \quad (1)$$

We have included the nuclear charge form factor $|G_E(\boldsymbol{q}^2)|^2$ to incorporate elastic scattering effects off a heavy nucleus.[2] Accounting for a difference in convention in the definition of e, our formula agrees with that of Ref. [6].

We relate $\boldsymbol{q}^2$ to the nuclear recoil energy in the lab frame, $\boldsymbol{q}^2 = 2m_A E_R$, and find

$$\frac{d\sigma_{EDM}}{dE_R} = \frac{1}{4\pi}\mathrm{d}_\chi^2 Z^2 e^2 \frac{(S+1)}{3S}\frac{1}{v_r^2}\frac{1}{E_R}\left|G_E\left(\boldsymbol{q}^2\right)\right|^2. \quad (2)$$

The $1/(v_r^2 E_R)$ dependence is characteristic of the EDM of the DM particle.[3]

To have an EDM, the DM particle cannot be self-conjugate. Consequently, for $S = \frac{1}{2}$, the particle has to be Dirac. Note that the spin factor $\frac{S+1}{3S}$ becomes 1 for $S = \frac{1}{2}$ in our numerical illustrations. Our result also applies to the anti-dark matter particle under the assumption that CPT is conserved.

3. Magnetic dipole moment of dark matter

In the static limit, the DM magnetic moment can only couple to the nuclear magnetic moment via the induced magnetic field. The non-relativistic effective Hamiltonian of a magnetic moment of a particle with spin $\boldsymbol{S}$ subject to a magnetic field $\boldsymbol{B}$ is, in our convention

$$H_{\text{eff}} = -\mu \boldsymbol{B} \cdot \boldsymbol{S}/S,$$

which agrees with the standard form for a spin $\frac{1}{2}$ particle, *i.e.*, $-\mu\boldsymbol{B}\cdot\boldsymbol{\sigma}$, where μ is dimensionful. Since $\boldsymbol{B} = \nabla \times \boldsymbol{A}$, we identify the curl $\boldsymbol{\nabla}\times$ with the momentum transfer $\boldsymbol{q}\times$. The direct scattering off the nucleus of spin I via the interaction between the nuclear magnetic moments $\mu_{Z,A}$ and the magnetic moment μ_χ of the DM particle is described by

$$\mathcal{M} = \boldsymbol{S} \times \boldsymbol{q} \cdot \left(\frac{\mu_\chi}{S}\frac{1}{\boldsymbol{q}^2}\frac{\mu_{Z,A}}{I}\right)\boldsymbol{I} \times \boldsymbol{q}.$$

---

[1] The DM particle velocity in the frame of the galactic halo is usually described [1] by a Maxwellian distribution,

$$f^G(\boldsymbol{v})\,d^3\boldsymbol{v} = \frac{N}{(v_0\sqrt{\pi})^3}e^{-v^2/v_0^2}\,d^3\boldsymbol{v},$$

where the most probable velocity is $v_0 = 230$ km/s, and the distribution is cut off at the escape velocity $v_{\text{esc}} = 600$ km/s. The normalization N is close to 1, or more precisely,

$$N = \frac{1}{\mathrm{erf}(v_{\text{esc}}/v_0) - \frac{2}{\sqrt{\pi}}(v_{\text{esc}}/v_0)\exp(-v_{\text{esc}}^2/v_0^2)}.$$

The one-variable velocity distribution is

$$f_1^G(v)\,dv = \frac{4v^2 N}{v_0^3\sqrt{\pi}}e^{-v^2/v_0^2}\,dv.$$

However, since the solar system is moving at a speed $v_E = 244$ km/s with respect to the halo [2] and $v_E \sim v_0$, we need to use a more relevant distribution of the relative velocity, $\boldsymbol{v}_r = \boldsymbol{v} - \boldsymbol{v}_E$. As an approximation we ignore the seasonal motion of the Earth around the Sun with a relative speed of about 30 km/s. Then,

$$f(\boldsymbol{v}_r)\,d^3\boldsymbol{v}_r = f^G(\boldsymbol{v})\,d^3\boldsymbol{v} = f^G(\boldsymbol{v}_r + \boldsymbol{v}_E)\,d^3\boldsymbol{v}_r \quad \text{because } d^3\boldsymbol{v} = d^3\boldsymbol{v}_r.$$

For a fixed v_r, we integrate the polar angle between $\boldsymbol{v}_r$ and $\boldsymbol{v}_E$ to obtain

$$f_1(v_r)\,dv_r = N\frac{v_r\,dv_r}{v_E v_0\sqrt{\pi}}\left(e^{-(\min(v_r - v_E, v_{\text{esc}}))^2/v_0^2} - e^{-(\min(v_r + v_E, v_{\text{esc}}))^2/v_0^2}\right).$$

If $v_r > v_{\text{esc}} + v_E$, $f_1(v_r) = 0$; see Refs. [1,3].

[2] A good nuclear form factor can be found in Ref. [4]. The spatial charge distribution is parameterized by the Fermi distribution $\rho(\mathbf{r}) = \rho_0/(1 + e^{(r-c)/a_0})$, where the radius at which the density is $\rho_0/2$ is $c = (1.18A^{\frac{1}{3}} - 0.48)$ fm and the edge thickness parameter is $a_0 = 0.57$ fm [5]. The form factor that is valid for nuclei with a well-developed core (*i.e.*, with atomic masses above 20) is obtained by the Fourier transform in the limit $c \gg a_0$,

$$G_E(q) = \left[\frac{\pi a_0}{c}\frac{\sin(qc)\cosh(\pi a_0 q)}{\sinh^2(\pi a_0 q)} - \frac{\cos(qc)}{\sinh(\pi a_0 q)}\right]\frac{4\pi^2\rho a_0 c}{q},$$

$$\rho_0 = \frac{3}{4\pi c^3}\frac{1}{1 + (a_0\pi/c)^2}.$$

Note that $G_E(0) = 1$.

[3] The differential reaction rate (per unit detector mass) is

$$\frac{dR}{dE_R} = \frac{\rho_0}{m_\chi}\frac{1}{m_A}\int_{v_{\min}}^{\infty} v_r f_1(v_r)\frac{d\sigma}{dE_R}\,dv_r,$$

where the local DM density $\rho_0 = 0.3$ GeV/cm^3 and $v_{\min} = \sqrt{\frac{m_A E_R}{2m_r^2}}$. dR/dE_R includes contributions from both χ and its conjugate $\bar{\chi}$ for they have the same cross sections.

In the non-relativistic limit, the differential cross section can be Maclaurin expanded in powers of v_r. The two most important contributions are

$$d\sigma \sim \frac{1}{v_r^2}d\{\sigma_-\} + d\{\sigma_+\},$$

with v_r independent coefficients denoted by brackets. (For example, in Eq. (1), $d\{\sigma_-\}$ is the coefficient of v_r^{-2}, and $d\{\sigma_+\} = 0$.) Usually, the first term is the relevant one (as in the EDM, CFF, or SI cases). However, in certain cases like MDM, the second term may compete due to the $1/E_R$ enhancement from the low energy virtual photon propagator. On integrating, we find

$$\frac{dR}{dE_R} = \frac{\rho_0}{m_\chi}\frac{1}{m_A}\left[\frac{d\{\sigma_-\}}{dE_R}\frac{1}{v_0}I_- + \frac{d\{\sigma_+\}}{dE_R}v_0 I_+\right],$$

where the dimensionless integrals are defined by

$$\frac{I_-}{N} = \frac{v_0}{2v_E}\left[\mathrm{erf}\left(\frac{v_u}{v_0}\right) - \mathrm{erf}\left(\frac{v_d}{v_0}\right) - \frac{2}{\sqrt{\pi}}\left(\frac{v_u}{v_0} - \frac{v_d}{v_0}\right)e^{-v_{\text{esc}}^2/v_0^2}\right],$$

and

$$\frac{I_+}{N} = \left(\frac{v_d}{2v_E\sqrt{\pi}} + \frac{1}{\sqrt{\pi}}\right)e^{-v_d^2/v_0^2} - \left(\frac{v_u}{2v_E\sqrt{\pi}} - \frac{1}{\sqrt{\pi}}\right)e^{-v_u^2/v_0^2}$$
$$+ \frac{v_0}{4v_E}\left(1 + \frac{2v_E^2}{v_0^2}\right)\left(\mathrm{erf}\left(\frac{v_u}{v_0}\right) - \mathrm{erf}\left(\frac{v_d}{v_0}\right)\right)$$
$$- \frac{1}{\sqrt{\pi}}\left[2 + \frac{1}{3v_E v_0^2}\left((v_{\min} + v_{\text{esc}} - v_d)^3 - (v_{\min} + v_{\text{esc}} - v_u)^3\right)\right]e^{-v_{\text{esc}}^2/v_0^2},$$

with the shorthand $v_u = \min(v_{\min} + v_E, v_{\text{esc}})$, $v_d = \min(v_{\min} - v_E, v_{\text{esc}})$. Note that $I_- = 0$ for $v_{\min} > v_{\text{esc}} + v_E$.