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Higher-dimensional conformal field theories in the Coulomb branch

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A B S T R A C T

We use the AdS/CFT correspondence to study flows of $\mathcal{N}=4$ SYM to non-conformal theories. The dual geometries can be seen as sourced by a Wigner's semicircle distribution of D3 branes. We consider two cases, the first case corresponds to a point in the Coulomb branch and the theory flows to a six-dimensional conformal field theory. In the second case a mass is introduced for a hypermultiplet and the theory flows to a five-dimensional conformal field theory. We argue from the gravity and the field theory side that the low energy theories correspond to the $(2,0)$ theory in six dimensions and to a theory with exceptional global symmetry E_1 in five dimensions.

1. Introduction

It has been known for some time how to describe generic points in the Coulomb moduli space of $\mathcal{N}=4$ super-Yang–Mills (SYM) theory using the AdS/CFT correspondence [1–3]. The expectation values of the six real scalar fields ϕ_i of $\mathcal{N}=4$ SYM translate into a distribution of D3 branes in the transverse six-dimensional space [4]. In the near horizon limit the ten-dimensional geometry takes the form

$$ds^2_{9,1}=H^{-1/2}\eta_{\mu\nu}\,dx^\mu\,dx^\nu+H^{1/2}\sum_{i=1}^{6}\left(dy^i\right)^2, \tag{1}$$

where the harmonic function $H(\vec{y})$ is determined by a distribution function $\sigma(\vec{w})$

$$H(\vec{y})=\int d^6w\,\sigma(\vec{w})|\vec{y}-\vec{w}|^{-4}. \tag{2}$$

The geometry is asymptotically $AdS_5\times S^5$.

A special family of solutions was studied in Ref. [5] starting from $d=5$ $\mathcal{N}=8$ supergravity (SUGRA) and uplifting to ten dimensions. The associated distribution functions σ_n have support on an n-dimensional ball and preserve a $SO(n)\times SO(6-n)$ symmetry in the internal space. In the field theory, they correspond to expectation values of operators in the $\mathbf{20}'$ representation of the $SO(6)_R$ R-symmetry group of $\mathcal{N}=4$ SYM, given by the symmetric traceless combinations $\mathrm{tr}\,\phi_{(i}\phi_{j)}$. The most symmetric configuration, with $SO(5)$ symmetry, is a Wigner's semicircle distribution on an interval[1]

$$\sigma_1(\vec{\phi})=\frac{2}{\pi\Lambda^2}\sqrt{\Lambda^2-\phi_1^2}\,\Theta\left(\Lambda^2-\phi_1^2\right)\prod_{i=2}^{6}\delta(\phi_i), \tag{3}$$

associated to operators of the form $\sim\xi^{ij}\,\mathrm{tr}(\phi_i\phi_j)$, with $\xi^{ij}=\mathrm{diag}(5,-1,-1,-1,-1,-1)$. This class of solutions has a gapless continuum spectrum, and at low temperatures the entropy density scales as $s\sim T^5$ [6].

In the $\mathcal{N}=2^*$ SYM theory the field content of $\mathcal{N}=4$ SYM is divided in an $\mathcal{N}=2$ vector multiplet and a hypermultiplet, and a mass is introduced for the last. The holographic duals for some configurations in the moduli space were constructed using a truncation of $d=5$ $\mathcal{N}=8$ SUGRA [7,8]. One class of solutions also correspond to a Wigner's semicircle distribution [9–11]. In this case the low temperature entropy density scales as $s\sim T^4$ [6]. In addition, the speed of sound approaches the value $c_s^2=1/4$ and the bulk over shear viscosity ratio saturates the bound $\zeta/\eta\geqslant 2(1/3-c_s^2)=1/6$ [12]. This is consistent with having a five-dimensional conformal field theory (CFT) compactified on a circle [13].

At finite temperature the Coulomb moduli space is lifted in general, but the low temperature regime is dominated by an infrared effective theory close to special points where the free energy is minimized (see e.g. [14]). In the cases we study, the local minima correspond to the semicircle distributions. We will show that the near horizon geometry of the zero temperature holographic duals

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[1] The $n=5$ distribution is thought to be unphysical.

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can be seen as an AdS_7 compactified on a torus in the $\mathcal{N}=4$ SYM case and an AdS_6 geometry compactified on a circle in the $\mathcal{N}=2^*$ SYM case. Therefore, the dual effective theory is a $d=6$ CFT in the $\mathcal{N}=4$ theory and a $d=5$ CFT in the $\mathcal{N}=2^*$ theory, explaining the behavior of thermodynamic quantities. We will also argue that in the large-N limit a sector of the $\mathcal{N}=4$ low energy theory maps to the $(2,0)$ theory on the M5 brane [15] while in the $\mathcal{N}=2^*$ theory a similar sector maps to a D4/D8/O8 intersection with $N_f=0$ flavors [16].

Finally, we will argue that the mechanism giving rise to the effective higher-dimensional theories is dimensional (de)construction [17,18]. Given the distribution of eigenvalues in the Coulomb branch, the gauge group in these examples will be broken to a number of group factors of order N, and the spectrum of charged massive states will include masses scaling as $\sim 1/N$, that can be grouped to fill a Kaluza–Klein tower in the large-N limit, effectively producing the additional dimensions.

2. The $\mathcal{N}=4$ flow to a $d=6$ CFT

The holographic dual to the $SO(5)$ symmetric flow of $\mathcal{N}=4$ SYM belongs to a larger family of solutions of maximally supersymmetric SUGRAs with non-trivial profiles for scalars belonging to the $SL(N,\mathbb{R})/SO(N)$ coset ($N=8,6,5$ for $d=4,5,7$ dimensions). The relevant action is [5,19]

$$e^{-1}\mathcal{L}_d = R - \frac{1}{2}\sum_{i=1}^{N-1}(\partial\varphi_i)^2 - V, \tag{4}$$

where the scalar potential is

$$V = -\frac{g^2}{2}\left[(\operatorname{tr} M)^2 - 2\operatorname{tr} M^2\right], \tag{5}$$

and M is a diagonal $N\times N$ matrix with eigenvalues $X_i = e^{\beta_i}$, $i=1,\dots,N$, satisfying $\det M = 1$. The exponents β_i are linear combinations of the scalar fields φ_i, but we will not need the explicit expressions.

When $d=5$, the $SO(5)$ symmetric solution is

$$ds^2_{4,1} = (gr)^2 H^{1/6}\eta_{\mu\nu}\,dx^\mu\,dx^\nu + \frac{dr^2}{(gr)^2 H^{1/3}}. \tag{6}$$

Where $H = 1+\ell^2/r^2$. The profile for the scalars $\beta_i = (1-6\delta_{i6})\beta/\sqrt{15}$ is determined by the flow equations for a single function β

$$H^{1/6} r\frac{d\beta}{dr} = -\frac{1}{2}\frac{\partial}{\partial\beta}\operatorname{tr} M. \tag{7}$$

In the near horizon limit $u^2 = 1/(g^2\ell r)\to\infty$, the metric becomes

$$ds^2_{4,1} \simeq \frac{(g\ell u)^{-4/3}}{u^2}\left[\eta_{\mu\nu}\,dx^\mu\,dx^\nu + \frac{4}{g^2}\,du^2\right]. \tag{8}$$

The scalars asymptote to

$$X_{i<6} = X \simeq 2(g\ell u)^{2/3}, \qquad X_6 = X^{-5} \sim u^{-10/3}. \tag{9}$$

In this limit the leading term in the scalar potential (5) is

$$V_\infty = -\frac{15g^2}{2}X^2. \tag{10}$$

The metric (8) and scalar profiles (9) are solutions of the action (4) with the potential (10).

2.1. Lift to eleven dimensions

The maximally symmetric solution to (4) in $d=7$ dimensions is an AdS_7 space

$$ds^2_{6,1} = \frac{1}{\tilde u^2}\left[\eta_{\mu\nu}\,d\tilde x^\mu\,d\tilde x^\nu + \delta_{ab}\,dy^a\,dy^b + \frac{4}{g^2}\,d\tilde u^2\right]. \tag{11}$$

Here we set y^a, $a=1,2$ to be coordinates along a torus. The scalars have values $\tilde X_{i=1,\dots,5}=1$, so the value of the potential is

$$V_{d=7} = -\frac{15g^2}{2}. \tag{12}$$

Writing the metric (11) as

$$ds^2_{6,1} = e^{-2\phi}\,ds^2_{4,1} + e^{3\phi}\left(dy_1^2+dy_2^2\right), \tag{13}$$

and reducing along the torus gives (8), (9) and (10) if we identify

$$X = e^{-\phi}, \qquad \tilde u = 2^{3/2}g\ell u, \qquad \tilde x^\mu = \sqrt{2}g\ell x^\mu. \tag{14}$$

We have shown that the $\mathcal{N}=4$ flow can be lifted to an AdS_7 solution of $d=7$ $\mathcal{N}=2$ SUGRA. This geometry, in turn, can be lifted to an $AdS_7\times S^4$ solution of $d=11$ SUGRA, that is the near horizon geometry of a stack of M5 branes [19]. Therefore, in the large-N and strong coupling approximation we are using, the infrared dynamics of the $SO(5)$ symmetric flow coincides with the $d=6$ $(2,0)$ CFT of the M5 brane at least in the subsector we have studied. In the field theory side this involves the components of the $d=6$ energy–momentum tensor $T_{\mu\nu}$, T_{ab} that reduce to the $d=4$ energy–momentum tensor $T_{\mu\nu}$ plus a scalar field $T_{11}=T_{22}$, $T_{12}=0$. This is enough to explain the scaling of the entropy density $s\sim T^5$ and to make the prediction that, in the low temperature regime, the speed of sound will approach the value $c_s^2=1/5$, while the bulk over shear viscosity ratio will saturate the bound $\zeta/\eta \geqslant 2(1/3-c_s^2)=4/15$.

3. A large-N equivalence for $\mathcal{N}=2^*$ SYM

An $\mathcal{N}=2$ CFT that is equivalent to $\mathcal{N}=4$ SYM in the large-N limit can be constructed by doing a simple orientifold projection. The holographic dual is an $AdS_5\times S^5/\mathbf{Z}_2$ orbifold geometry, with an orientifold O7 plane and $N_f=4$ D7 branes sitting at the orbifold point [20]. The orientifold breaks the isometry group of the five-sphere $SO(6)\simeq SU(4)\to SU(2)\times SU(2)_R\times U(1)_R$ and does a $\mathbf{Z}_2\subset U(1)_R$ projection of supergravity fields. In the field theory this is interpreted in terms of the breaking of and the projection with respect to the R-symmetry group of the $\mathcal{N}=4$ theory. The field content is an $\mathcal{N}=2$ theory with an $USp(2N)$ vector multiplet, a hypermultiplet in the antisymmetric representation and $SO(8)$ flavor group. In terms of $\mathcal{N}=1$ superfields the matter content is a vector multiplet W_α, a chiral multiplet in the adjoint representation X, two chiral multiplets in the antisymmetric representation A, $\tilde A$ and 8 chiral multiplets in the fundamental representation Q^i, $\tilde Q_i$. There is a $USp(2)\simeq SU(2)$ symmetry that rotates the antisymmetric multiplets.

The $\mathbf{20}'$ and $\mathbf{10}$ Kaluza–Klein modes of the dilaton and two-form potential are projected as

$$\begin{aligned}&\mathbf{20}'\to(\mathbf{3},\mathbf{3})_0\oplus(\mathbf{1},\mathbf{1})_4\oplus(\mathbf{1},\mathbf{1})_{-4}\oplus(\mathbf{1},\mathbf{1})_0,\\ &\mathbf{10}\to(\mathbf{3},\mathbf{1})_{-2}\oplus(\mathbf{1},\mathbf{3})_2.\end{aligned} \tag{15}$$

The first row correspond to modes dual to operators of conformal dimension $\Delta=2$, while the second row is dual to $\Delta=3$ operators.

This can be compared with the $\mathcal{N}=2^*$ theory. The breaking of supersymmetry implies that the maximal R-symmetry group is