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Tunneling across dilaton coupled black holes in anti de Sitter spacetime

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ABSTRACT

Considering generalised action for dilaton coupled Maxwell–Einstein theory in four dimensions, Gao and Zhang obtained black holes solutions for asymptotically anti de Sitter (Ads) and de Sitter (ds) spacetimes. We study the Hawking radiation in Parikh–Wilczek's tunneling formalism as well as using Bogoliubov transformations. We compare the expression of the Hawking temperature obtained from these two different approaches. Stability and the extremality conditions for such black holes are discussed. The exact dependences of the Hawking temperature and flux on the dilaton coupling parameter are determined. It is shown that the Hawking flux increases with the dilaton coupling parameter. Finally we show that the expression for the Hawking flux obtained using Bogoliubov transformation matches exactly with flux calculated via chiral gauge and gravitational anomalies. This establishes a correspondence among all these different approaches of estimating Hawking radiation from these classes of black holes.

1. Introduction

Hawking radiation from black holes [1,2] is known to be a quantum phenomena. Parikh and Wilczek viewed this as a quantum tunneling of particles through black hole horizon [3–5]. Energy conservation as well as particle's self-gravitation effects are taken into account in this approach. Introducing Painlevé coordinate transformation, which is well behaved across the horizon, Parikh and Wilczek found a generic relation between the entropy change and the tunneling rate which is valid for a large class of black holes. In an alternative approach, Bogoliubov transformation is used in two different coordinates (such as Kruskal coordinates and asymptotic coordinates) to compute the coefficients of the energy modes. In this approach, neglecting the back scattering effect and normalising Bogoliubov coefficients, the leading thermal characteristic of Hawking radiation has been derived. In yet another approach the Hawking flux has been evaluated via chiral gauge and gravitational anomalies from a covariant boundary condition. Equivalence of these approaches in respect to thermodynamic behaviour of black holes has been an important area of study. There have been several works in recent times to explore Hawking radiation for different kinds of black holes using these approaches [6–44].

Meanwhile Gao and Zhang found the dilaton coupled de Sitter and anti de Sitter black hole solutions using appropriate Liouville type potential for the dilaton field. Dilaton coupled black hole has it's origin rooted into string theory which in the low energy effective field theory appears naturally with a dilaton-electromagnetic coupling. Anti de Sitter spacetime has received interest in the context of Ads/Cft correspondence [45–47] and also in brane world scenarios of Randall–Sundram model [48]. Our objective here is to study Hawking radiation for the class of black holes using quantum tunneling procedure, computing Bogoliubov coefficients as well as estimating gauge and gravitational anomalies to compare the leading thermal behaviour of radiation spectrum. Our results establish the equivalence of these descriptions of Hawking radiation. The dilaton coupling parameter is shown to enhance the Hawking flux significantly.

2. The model

The action describing dilaton black hole in both de Sitter and anti de Sitter spacetime can be expressed as follows [49]:

$$S=\int d^4x\sqrt{-g}\left[R-2\partial_\mu\varphi\partial^\mu\varphi-e^{-2\alpha\varphi}F^2-V(\varphi)\right] \tag{1}$$

where $V(\varphi)=\frac{-2\lambda}{3(1+\alpha^2)^2}[\alpha^2(3\alpha^2-1)e^{\frac{-2\varphi}{\alpha}}+(3-\alpha^2)e^{2\alpha\varphi}+8\alpha^2e^{\varphi\alpha-\frac{\varphi}{\alpha}}]$ is the dilaton potential for anti de Sitter black holes, φ is the dilaton field. α represents coupling parameter of the scalar field with Maxwell field $F_{\mu\nu}$ and λ is the cosmological constant.

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The interpretation of the parameter λ as the cosmological constant has been discussed in [49] where starting from a dilaton de Sitter metric in cosmic coordinate system a scale factor $a = e^{Ht}$ was obtained with H as the Hubble constant. Replacing $H^2 = \frac{\lambda}{3}$, dilaton anti de Sitter/de Sitter black hole solution has been produced.

3. Hawking Radiation by uncharged particle tunneling from dilaton anti de Sitter black hole

Considering the general action describing dilaton field coupled to electromagnetic field in presence of three Liouville type potentials, Gao and Zhang obtained dilaton black hole solutions in both de Sitter and anti de Sitter spacetime. Here we study the phenomena of particle tunneling across the dilaton anti de Sitter black hole which is described by the metric [49],

$$ds^2 = -f(r)\,dt_s^2 + g(r)\,dr^2 + h(r)\left[d\theta^2 + \sin^2\theta\,d\phi^2\right] \tag{2}$$

$$\begin{aligned} &= -\left[\left(1-\frac{r_+}{r}\right)\left(1-\frac{r_-}{r}\right)^{\frac{(1-\alpha^2)}{(1+\alpha^2)}} - \frac{\lambda}{3}r^2\left(1-\frac{r_-}{r}\right)^{\frac{2\alpha^2}{(1+\alpha^2)}}\right]dt_s^2 \\ &\quad + \left[\left(1-\frac{r_+}{r}\right)\left(1-\frac{r_-}{r}\right)^{\frac{(1-\alpha^2)}{(1+\alpha^2)}} \right. \\ &\quad \left. - \frac{\lambda}{3}r^2\left(1-\frac{r_-}{r}\right)^{\frac{2\alpha^2}{(1+\alpha^2)}}\right]^{-1} dr^2 \\ &\quad + r^2\left(1-\frac{r_-}{r}\right)^{\frac{2\alpha^2}{(1+\alpha^2)}} d\Omega \end{aligned} \tag{3}$$

The event horizons of the black hole r_+, r_-, electromagnetic charge Q and asymptotic value of dilaton field φ_0 are related through $e^{2\alpha\varphi_0} = \frac{r_+ r_-}{(1+\alpha^2)Q^2}$ for arbitrary α. For $\alpha = 1$, the expressions of $f(r)$ and $h(r)$ for both dilaton anti de Sitter and de Sitter black hole having mass M and dilaton charge $D = \frac{Q^2 e^{2\varphi_0}}{2M}$ respectively take the following form

$$f(r) = \frac{1}{g(r)} = 1 - \frac{2M}{r} - \frac{\lambda}{3}r(r-2D) \tag{4}$$

$$f(r) = \frac{1}{g(r)} = 1 - \frac{2M}{r} - H^2 r(r-2D) \tag{5}$$

and

$$h(r) = r(r-2D) \tag{6}$$

Consider a tunneling particle with energy ω and charge e across such a dilaton anti de Sitter black hole such that the total energy of the system comprising of the black hole and the particle is conserved.

As the anti de Sitter Painlevé coordinates have the same geometry as global anti de Sitter metric of constant time slices, we choose the following Painlevé-type coordinate transformation [40].

$$dt_s = dt - \sqrt{\frac{g(r)-1}{f(r)}}\,dr \tag{7}$$

After this transformation line element (2) can be written as

$$\begin{aligned} ds^2 &= -f(r)\,dt^2 + 2\sqrt{f(r)}\sqrt{g(r)-1}\,dt\,dr + dr^2 \\ &\quad + h(r)\left(d\theta^2 + \sin^2\theta\,d\phi^2\right) \end{aligned} \tag{8}$$

The advantage of Painlevé coordinate transformation is that none of the components of the metric or the inverse metric diverge. The radial velocity of the outgoing particle in such a coordinate is obtained as

$$\dot{r} = \sqrt{f(r).g(r)}\left(1 - \sqrt{1 - \frac{1}{g(r)}}\right) \tag{9}$$

During particle tunneling black hole mass M should be replaced by $M - \omega$. Following Parikh–Wilczek tunneling formalism [3–5,40], the imaginary part of the action for an outgoing particle crossing the horizon of the black holes can be written as

$$\operatorname{Im} I = \operatorname{Im} \int_{r_h(M)}^{r_h(M-\omega)} \int_0^{p_r} dp'_r\,dr \tag{10}$$

where within the integration we have used p'_r for canonical momentum conjugate to radial variable r. Considering particle's self-gravitational effect [50,51] and using Hamilton's equations we calculate the radial velocity in the dragged frame. In this frame, by an appropriate transformation, the black hole event horizon can be made to coincide with infinite redshift surface so that WKB approximation is valid. Thus radial velocity and time derivative of vector potential $\dot{A}_t$ in the dragged frame become,

$$\dot{r} = \frac{dH}{dp_r} = \frac{d(M-\omega)}{dp'_r} \tag{11}$$

and

$$\dot{A}_t = \frac{dH}{dp'_{A_t}} \tag{12}$$

where p_r and H are the canonical momentum conjugate to radial variable r and energy of the black hole respectively. Substituting Eqs. (11) and (12) in (10), we get

$$\begin{aligned} \operatorname{Im} I &= \operatorname{Im} \int_{r_h(M),Q}^{r_h(M-\omega),Q-e} \int_0^{\omega} \frac{d(M-\omega', Q-e')\,dr}{\dot{r}} \\ &= \operatorname{Im} \int_{r_h(M),Q}^{r_h(M-\omega),Q-e} \int_0^{\omega} \frac{(dM' - \frac{Q'\,dQ' e^{2\alpha\varphi_0}}{r})\,dr}{\dot{r}} \end{aligned} \tag{13}$$

where $M' = M - \omega'$, $Q' = Q - e$, $r_h(M)$ and $r_h(M-\omega)$ are the locations of the horizon before and after the emission of particle. Using the expression for $\dot{r}$ from Eq. (9), $\operatorname{Im} I$ takes the form,

$$\operatorname{Im} I = \operatorname{Im} \int_0^{\omega,Q} \int_{r_h(M-\omega,Q-e)}^{r_h(M),Q} \frac{dr\,d\omega'}{\sqrt{f(r)g(r)}(1 - \sqrt{1 - \frac{1}{g(r)}})} \tag{14}$$

Now using WKB approximation the tunneling rate of the particle can be expressed as

$$\Gamma = \Gamma_0 e^{-2\operatorname{Im} I} \tag{15}$$

where Γ_0 is the normalisation constant. From Eq. (2), it follows that for four-dimensional dilaton anti de Sitter black hole the function $g(r)$ has a singularity at $r = r_h$. Thus $g(r)$ can be written as

$$g(r) = \frac{C(r)}{(r - r_h)} \tag{16}$$

where the function $C(r)$ is regular on the horizon. Now substituting (16) in (13) and simplifying, we have the following equation,