Dark energy and QCD ghost

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It has been suggested that the dark energy that explains the observed accelerating expansion of the universe may arise due to the contribution to the vacuum energy of the QCD ghost in a time-dependent background. The argument uses a four-dimensional simplified model. In this Letter, we put the discussion in more realistic model keeping all components of the QCD vector ghost and show that indeed QCD ghost produces dark energy proportional to the Hubble parameter $H\Lambda_{QCD}^3$ (Λ_{QCD} is the QCD mass scale) which has the right magnitude $\sim (3 \times 10^{-3} \text{ eV})^4$.

1. Introduction

The recent cosmological observations have confirmed the existence of the early inflationary epoch and the accelerated expansion of the present universe [1–3]. Observational result is consistent with the picture that the universe has an unknown form of energy density, named the dark energy, about 75% of the total energy density. The simplest possibility is the existence of the vacuum energy or cosmological constant whose origin is yet to be identified.

Such vacuum energy is easily incorporated in the quantum field theory. In the standard model of particle physics, we have Higgs field which produces electroweak phase transition, which changes the value of the vacuum energy. It has also been known that the vacuum fluctuations in quantum field theory naturally induce such a vacuum energy, but the problem is how to control the size of it. The contribution of quantum fluctuations in known fields up to 300 GeV, which is about the highest energy at which the current theories have been verified, gives a vacuum energy density of order $(300 \text{ GeV})^4$. This is vastly larger than the observed dark energy density $(3 \times 10^{-3} \text{ eV})^4$ by a factor of order 10^{56}. Assuming the tree-level contribution is zero, it is a great challenge how to understand the origin of this tiny energy density.

Recently a very interesting suggestion on the origin of a cosmological constant is made, without introducing new degrees of freedom beyond what are already known, with the cosmological constant of just the right magnitude to give the observed expansion [4]. In this proposal, it is claimed that the cosmological constant arises from the contribution of the ghost fields which

are supposed to be present in the low-energy effective theory of QCD [5–9]. The ghosts are required to exist for the resolution of the $U(1)$ problem, but are completely decoupled from the physical sector [9]. The above claim is that the ghosts are decoupled from the physical states and make no contribution in the flat Minkowski space, but once they are in the curved space or time-dependent background, the cancellation of their contribution to the vacuum energy is off-set, leaving a small energy density $\rho \sim H\Lambda_{QCD}^3$, where H is the Hubble parameter and Λ_{QCD} is the QCD mass scale of order a hundred MeV. With $H \sim 10^{-33}$ eV, this gives the right magnitude. This coincidence is remarkable and suggests that we are on the right track.

However in this proposal, the authors discuss a four-dimensional model similar to the one based on the Schwinger model (proposed by Kogut and Susskind [10]), keeping only the longitudinal and scalar components of the QCD ghost. These scalar fields have positive and negative norms and cancel with each other, leaving no trace in the physical subspace, but it is argued that they have small contribution to the vacuum energy in the curved space or time-dependent background. (Similar system is used in a mechanism of supersymmetry breaking in Ref. [11].) However it is known that the QCD ghost must be intrinsically vector field in order for the $U(1)$ problem to be consistently resolved within the framework of QCD [12]. It is thus an interesting and important question to examine if the mechanism works even if we formulate the proposal keeping all the modes of the vector ghost.

In the next section, we briefly recapitulate how the $U(1)$ problem is resolved by the vector ghost following [9], and also show how the ghost decouples from the physical sector. In Section 3, following the discussions in Ref. [13], we discuss the mechanism of generating a tiny contribution to the vacuum energy in the Rindler

space as a typical example of the time-dependent background [14, 15]. We argue that due to the change of the definition of the vacuum, we indeed obtain small contribution to the vacuum energy proportional to the Hubble parameter. We find that our result is factor 2 larger than the previous estimate. Section 4 is devoted to the discussions and conclusions.

2. Decoupling of the vector ghost in the Minkowski space

We consider the low-energy effective Lagrangian [7–9]

$$\mathcal{L} = \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}m_{NS}^2 S^2 + \frac{1}{2F_S^2(m_S^2 - m_{NS}^2)}\big(\partial_\mu K^\mu\big)^2$$
$$- \frac{1}{F_S}S\partial_\mu K^\mu, \tag{1}$$

where S is a flavor-singlet pseudoscalar field with the decay constant F_S, K^μ is an axial vector "field", which in QCD corresponds to the gluonic current

$$K^\mu = 2N_f \frac{g^2}{16\pi^2}\epsilon^{\mu\nu\lambda\sigma}A_\nu^a\left(\partial_\lambda A_\sigma^a + \frac{1}{3}gf^{abc}A_\lambda^b A_\sigma^c\right), \tag{2}$$

where N_f is the number of flavors.

The Lagrangian (1) is invariant under the gauge transformation

$$K^\mu \to K^\mu + \epsilon^{\mu\nu\lambda\sigma}\partial_\nu \Lambda_{\lambda\sigma}, \tag{3}$$

where $\Lambda_{\lambda\sigma}$ denotes an arbitrary antisymmetric tensor. In fact this transformation reflects the color gauge invariance of the underlying QCD. Under the QCD gauge transformation, the gluonic current transforms as [9,12]

$$\big[Q_B, K^\mu\big] = 2iN_f \frac{g^2}{16\pi^2}\epsilon^{\mu\nu\lambda\sigma}\partial_\nu\big(C^a\partial_\lambda A_\sigma^a\big), \tag{4}$$

where Q_B is the BRST charge and C^a is the Faddeev–Popov ghost.

To quantize this system, we have to break the gauge invariance under (3). This can be done by adding the term

$$\frac{1}{4F_S^2(m_S^2 - m_{NS}^2)\alpha}(\partial_\mu K_\nu - \partial_\nu K_\mu)^2, \tag{5}$$

where α is a gauge parameter. The simplest case is to choose $\alpha = 1$. One can then derive the Feynman rules as follows:

$$K_\mu\text{-propagator:} \quad \frac{i\eta_{\mu\nu}}{k^2}F_S^2(m_S^2 - m_{NS}^2),$$
$$S\text{-propagator:} \quad \frac{i}{k^2 - m_{NS}^2}, \qquad S - K_\mu\text{-mixing:} \quad \frac{1}{F_S}k_\mu. \tag{6}$$

It appears that the system (1) describes a scalar field S with mass m_{NS} and a massless vector. However it is not difficult to show that the mass of the scalar S gets shifted to m_S due to the mixing of the scalar and vector modes. This is the approach that Veneziano took in [6]. Alternatively, in the general gauge, one can derive the two-point functions

$$T\langle K_\mu(x)K_\nu(y)\rangle_M = \int \frac{d^4k}{(2\pi)^4}e^{-ik\cdot(x-y)}i\left\{\frac{k^2 - m_{NS}^2}{k^2 - m_S^2}\frac{k_\mu k_\nu}{(k^2)^2}\right.$$
$$\left. + \frac{\alpha}{k^2}\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\right\}F_S^2(m_S^2 - m_{NS}^2),$$

$$T\langle S(x)S(y)\rangle_M = \int \frac{d^4k}{(2\pi)^4}e^{-ik\cdot(x-y)}\frac{i}{k^2 - m_S^2},$$

$$T\langle S(x)K_\mu(y)\rangle_M = \int \frac{d^4k}{(2\pi)^4}e^{-ik\cdot(x-y)}\frac{F_S(m_S^2 - m_{NS}^2)}{k^2(k^2 - m_S^2)}, \tag{7}$$

where T denotes the time-order and the subscript M stands for the expectation value by the Minkowski vacuum. We see that the two-point function of S clearly shows that it has the shifted mass m_S instead of the original m_{NS}, confirming the above claim. This is one of the consequences of the massless mode K_μ and gives a resolution of the $U(1)$ problem. What happens to the massless mode in the system? Because we observe no massless field in the low-energy world, it must decouple from the physical sector. The precise mechanism of this was not clear in the approach of Ref. [6] but it was simply assumed that it decouples because it is gauge-variant, as indicated above. We now show how this can be achieved in the Minkowski space.

One can derive the field equations from the Lagrangian (1):

$$\Box K_\mu - \partial_\mu\big\{(1-\alpha)\partial^\nu K_\nu + F_S(m_S^2 - m_{NS}^2)S\big\} = 0,$$
$$\Box S + m_{NS}^2 S + \frac{1}{F_S}\partial^\mu K_\mu = 0. \tag{8}$$

The first equation in (8) tells us that $\Box K_\mu$ is expressed as a gradient of a field, so we find

$$\Box(\partial_\mu K_\nu - \partial_\nu K_\mu) = 0. \tag{9}$$

Hence we can consistently impose the subsidiary condition on the physical states:

$$(\partial_\mu K_\nu - \partial_\nu K_\mu)^{(+)}|\text{phys}\rangle = 0. \tag{10}$$

In the gauge $\alpha = 1$, we can write the mode expansion of the vector field. In terms of the canonically normalized field $K'_\mu(x) \equiv F_S\sqrt{m_S^2 - m_{NS}^2}K_\mu(x)$, it takes the form

$$K'_\mu(x) = \int \frac{d^3\boldsymbol{k}}{(2\pi)^{3/2}\sqrt{2k_0}}e_\mu^{(\lambda)}\big[e^{-ik\cdot(x-y)}a(\boldsymbol{k}, \lambda)$$
$$+ e^{ik\cdot(x-y)}a^\dagger(\boldsymbol{k}, \lambda)\big], \tag{11}$$

where $e_\mu^{(1)} = (0, \boldsymbol{e}^{(1)})$ and $e_\mu^{(2)} = (0, \boldsymbol{e}^{(2)})$ represent the transverse modes, $e_\mu^{(3)} = (0, \boldsymbol{e}^{(3)})$ the longitudinal mode, and $e_\mu^{(0)} = (1, \boldsymbol{0})$ the time component with

$$\boldsymbol{e}^{(3)} = \frac{\boldsymbol{k}}{|\boldsymbol{k}|}, \qquad \boldsymbol{e}^{(1)}\cdot\boldsymbol{e}^{(2)} = \boldsymbol{e}^{(1)}\cdot\boldsymbol{e}^{(3)} = \boldsymbol{e}^{(2)}\cdot\boldsymbol{e}^{(3)} = 0. \tag{12}$$

Canonical quantization of the system then gives

$$\big[a(\boldsymbol{k}, \lambda), a^\dagger(\boldsymbol{k}', \lambda')\big] = \eta_{\lambda\lambda'}\delta^3(\boldsymbol{k} - \boldsymbol{k}'). \tag{13}$$

We see that the transverse modes have the opposite sign to the usual gauge fields. It is then easy to see that the condition (10) means that the two transverse components $a(\boldsymbol{k}, 1)$, $a(\boldsymbol{k}, 2)$ and the combination $\frac{1}{\sqrt{2}}[a(\boldsymbol{k}, 3) - a(\boldsymbol{k}, 0)]$ should annihilate the physical state. This forbids the states generated by the two transverse components and by $\frac{1}{\sqrt{2}}[a^\dagger(\boldsymbol{k}, 3) + a^\dagger(\boldsymbol{k}, 0)]$. The remaining component $\frac{1}{\sqrt{2}}[a^\dagger(\boldsymbol{k}, 3) - a^\dagger(\boldsymbol{k}, 0)]$ can only produce zero norm states, so that all the components of K_μ are completely decoupled from the physical state. Nevertheless it produces the physical effect of shifting the mass of the singlet pseudoscalar S and resolves the problem associated with the η' meson decay [9]. In the Kogut–Susskind model, similar subsidiary condition can be imposed [4,11].

We are now going to see what effects this massless mode may produce if our space is not just the Minkowski but curved space or time-dependent.