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Interacting HDE and NADE in Brans–Dicke chameleon cosmology

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ABSTRACT

Motivated by the recent work of one of us Setare and Jamil (2010) [1], we generalize this work to the case where the pressureless dark matter and the holographic dark energy do not conserve separately but interact with each other. We investigate the cosmological applications of interacting holographic dark energy in Brans–Dicke theory with chameleon scalar field which is non-minimally coupled to the matter field. We find out that in this model the phantom crossing can be constructed if the model parameters are chosen suitably. We also perform the study for the new agegraphic dark energy model and calculate some relevant cosmological parameters and their evolution.

1. Introduction

Among various scenarios to explain the acceleration of the universe expansion, the holographic dark energy (HDE) and agegraphic dark energy (ADE) models have got a lot of enthusiasm recently. These models are originated from some considerations of the features of the quantum theory of gravity. That is to say, the HDE and ADE models possess some significant features of quantum gravity. Although a complete theory of quantum gravity has not established yet today, we still can make some attempts to investigate the nature of dark energy according to some principles of quantum gravity. The former is motivated from the holographic principle [2,3]. It was shown in [4] that in quantum field theory, the UV cutoff Λ should be related to the IR cutoff L due to limit set by forming a black hole. If $\rho_D = \Lambda^4$ is the vacuum energy density caused by UV cutoff, the total energy of size L should not exceed the mass of the system-size black hole:

$$E_D \leqslant E_{BH} \rightarrow L^3 \rho_D \leqslant m_p^2 L. \tag{1}$$

If the largest cutoff L is taken for saturating this inequality, we get the energy density of HDE as

$$\rho_D = \frac{3c^2 m_p^2}{L^2} = \frac{3c^2}{8\pi G L^2}. \tag{2}$$

The HDE is thoroughly investigated in the literature in various ways (see e.g. [5] and references therein). The later (ADE) model assumes that the observed dark energy comes from the spacetime and matter field fluctuations in the universe. Following the line of quantum fluctuations of spacetime, Karolyhazy et al. [6] discussed that the distance t in Minkowski spacetime cannot be known to a better accuracy than $\delta t = \beta t_p^{2/3} t^{1/3}$ where β is a dimensionless constant of order unity. Based on Karolyhazy relation, Sasakura [7] discussed that the energy density of metric fluctuations of the Minkowski spacetime is given by (see also [8])

$$\rho_D \sim \frac{1}{t_p^2 t^2} \sim \frac{m_p^2}{t^2}, \tag{3}$$

where t_p is the reduced Planck time and t is a proper time scale. On these basis, Cai [9] proposed the energy density of the original ADE in the form

$$\rho_D = \frac{3n^2 m_p^2}{T^2}, \tag{4}$$

where T is the age of the universe. Since the original ADE model suffers from the difficulty to describe the matter-dominated epoch, the new ADE (NADE) model was proposed by Wei and Cai [10], while the time scale was chosen to be the conformal time instead of the age of the universe. The ADE models have arisen a lot of enthusiasm recently and have examined and studied in ample detail [11–14].

It is also of great interest to analyze these models in the framework of Brans–Dicke (BD) gravity. In recent years the BD theory

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of gravity got a new impetus as it arises naturally as the low energy limit of many theories of quantum gravity such as superstring theory or Kaluza–Klein theory. The motivation for studying these models in the BD theory comes from the fact that both HDE and ADE models belong to a dynamical cosmological constant, therefore we need a dynamical frame to accommodate them instead of Einstein gravity. The investigation on the HDE and ADE models in the framework of BD cosmology, have been carried out in [15–18]. In the present work, we consider a BD theory in which there is a non-minimal coupling between the scalar field and the matter field. Thus the action and the field equations are modified due to the coupling of the scalar field with the matter. This kind of scalar field usually called "chameleon" field in the literature [19]. This is due to the fact that the physical properties of the field, such as its mass, depend sensitively on the environment. Moreover, in regions of high density, the chameleon blends with its environment and becomes essentially invisible to searches for Equivalence Principle violation and fifth force [19]. Further more, it was shown [19,20] that all existing constraints from planetary orbits, such as those from lunar laser ranging, are easily satisfied in the presence of chameleon field. The reason is that the chameleon-mediated force between two large objects, such as the Earth and the Sun, is much weaker than one would naively expect. In particular, it was shown [20] that the deviations from Newtonian gravity due to the chameleon field of the Earth are suppressed by nine orders of magnitude by the thin-shell effect. Other studies on the chameleon gravity have been carried out in [21]. Our work differs from that of Ref. [17] in that we assume a non-minimal coupling between the scalar field and the matter field. It also differs from that of Ref. [1], in that we assume the pressureless dark matter and dark energy do not conserve separately but interact with each other, while the author of [1] assumes that the dark components do not interact with each other.

2. HDE in BD theory with Chameleon scalar field

We begin with the BD chameleon theory in which the scalar field is coupled non-minimally to the matter field via the action [22]

$$S=\int d^4x\sqrt{-g}\left(\phi R-\frac{\omega}{\phi}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-V(\phi)+f(\phi)L_m\right), \quad (5)$$

where R is the Ricci scalar curvature, ϕ is the BD scalar field with a potential $V(\phi)$. The chameleon field ϕ is non-minimally coupled to gravity, ω is the dimensionless BD parameter. The last term in the action indicates the interaction between the matter Lagrangian L_m and some arbitrary function $f(\phi)$ of the BD scalar field. In the limiting case $f(\phi)=1$, we obtain the standard BD theory.

The gravitational field equations derived from the action (5) with respect to the metric is

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=\frac{f(\phi)}{\phi}T_{\mu\nu}+\frac{\omega}{\phi^2}\left(\phi_\mu\phi_\nu-\frac{1}{2}g_{\mu\nu}\phi^\alpha\phi_\alpha\right)+\frac{1}{\phi}[\phi_{\mu;\nu}-g_{\mu\nu}\Box\phi]-g_{\mu\nu}\frac{V(\phi)}{2\phi}, \quad (6)$$

where $T_{\mu\nu}$ represents the stress-energy tensor for the fluid filling the spacetime which is represented by the perfect fluid

$$T_{\mu\nu}=(\rho+p)u_\mu u_\nu+pg_{\mu\nu}, \quad (7)$$

where ρ and p are the energy density and pressure of the perfect fluid which we assume to be a mixture of matter and dark energy. Also u^μ is the four-vector velocity of the fluid satisfying $u^\mu u_\mu=-1$. The Klein–Gordon equation (or the wave equation) for the scalar field is

$$\Box\phi=\frac{T}{2\omega+3}\left(f-\frac{1}{2}\phi f_{,\phi}\right)+\frac{1}{2\omega+3}(\phi V_{,\phi}-2V), \quad (8)$$

where T is the trace of (7). The homogeneous and isotropic Friedmann–Robertson–Walker (FRW) universe is described by the metric

$$ds^2=-dt^2+a^2(t)\left(\frac{dr^2}{1-kr^2}+r^2d\Omega^2\right), \quad (9)$$

where $a(t)$ is the scale factor, and $k=-1,0,+1$ corresponds to open, flat, and closed universes, respectively. Variation of action (5) with respect to metric (9) for a universe filled with dust and HDE yields the following field equations

$$H^2+\frac{k}{a^2}-\frac{\omega}{6}\frac{\dot{\phi}^2}{\phi^2}+H\frac{\dot{\phi}}{\phi}=\frac{f(\phi)}{3\phi}(\rho_M+\rho_D)+\frac{V(\phi)}{6\phi}, \quad (10)$$

$$2\frac{\ddot{a}}{a}+H^2+\frac{k}{a^2}+\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2}+2H\frac{\dot{\phi}}{\phi}+\frac{\ddot{\phi}}{\phi}=-\frac{p_D}{\phi}+\frac{V(\phi)}{2\phi}, \quad (11)$$

where $H=\dot{a}/a$ is the Hubble parameter, ρ_D, p_D and ρ_M are, respectively, the dark energy density, dark energy pressure and energy density of dust (dark matter). Here, a dot indicates differentiation with respect to the cosmic time t. The dynamical equation for the scalar field is

$$\ddot{\phi}+3H\dot{\phi}-\frac{\rho-3p}{2\omega+3}\left(f-\frac{1}{2}\phi f_{,\phi}\right)+\frac{2}{2\omega+3}\left(V-\frac{1}{2}\phi V_{,\phi}\right)=0. \quad (12)$$

We assume the HDE in the chameleon BD theory has the following form

$$\rho_D=\frac{3c^2\phi}{L^2}. \quad (13)$$

The motivation idea for taking the energy density of HDE in BD theory in the form (13) comes from the fact that in BD theory we have $\phi\propto G^{-1}$. Here the constant $3c^2$ is introduced for later convenience and the radius L is defined as

$$L=ar(t), \quad (14)$$

where the function $r(t)$ can be obtained from the following relation

$$\int_0^{r(t)}\frac{dr}{\sqrt{1-kr^2}}=\int_0^\infty\frac{dt}{a}=\frac{R_h}{a}. \quad (15)$$

It is important to note that in the non-flat universe the characteristic length which plays the role of the IR-cutoff is the radius L of the event horizon measured on the sphere of the horizon and not the radial size R_h of the horizon. Solving Eq. (15) for the general case of the non-flat FRW universe, we get

$$r(t)=\frac{1}{\sqrt{k}}\sin y, \quad (16)$$

where $y=\sqrt{k}R_h/a$. Now we define the critical energy density, ρ_{cr}, and the energy density of the curvature, ρ_k, as

$$\rho_{cr}=3\phi H^2, \qquad \rho_k=\frac{3k\phi}{a^2}. \quad (17)$$

As usual, the fractional energy densities are defined as