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Ruling out the Modified Chaplygin Gas cosmologies

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ABSTRACT

The Modified Chaplygin Gas (MCG) model belongs to the class of a unified models of dark energy (DE) and dark matter (DM). It is characterized by an equation of state (EoS) $p_c = B\rho - A/\rho^\alpha$, where the case $B = 0$ corresponds to the Generalized Chaplygin Gas (GCG) model. Using a perturbative analysis and power spectrum observational data we show that the MCG model is not a successful candidate for the cosmic medium unless $B = 0$. In this case, it reduces to the usual GCG model.

1. Introduction

The cross of different observational results at cosmological level indicates that besides the usual expected contents of the cosmic budget, like baryons and radiation, there is a dark sector, with two components, dark matter and dark energy. In principle, dark matter is present in local structures like galaxies and cluster of galaxies, suffering consequently the process of gravitational collapse. In this sense, dark matter behaves much as like ordinary matter. However, it does emit any kind of electromagnetic radiation. Dark energy, on the other hand, seems to remain a smooth, not clustered, component, driving the accelerated expansion of the universe. This property requires a negative pressure. Many different models have been evoked to describe this dark sector of the energy content of the universe, going from the inclusion of exotic components in the context of general relativity theory to modifications of the gravitational theory itself, passing by other possibilities as the breakdown of the homogeneity condition. For a recent review, see Ref. [1].

A very appealing proposal to describe the dark sector are the so-called unified models. The prototype of such model is the Chaplygin gas [2–4]. In the unified model dark matter and dark energy are described by a single fluid, which behaves as ordinary matter in the past, and as a cosmological constant term in the future. In this sense, it interpolates the different periods of evolution of the universe, including the present stage of accelerated expansion. The Chaplygin gas model leads to very good results when confronted with the observational data of supernova type Ia [5]. Concerning the matter power spectrum data, the statistic analysis leads to results competitive with the ΛCDM model, but the unified (called quartessence) scenario must be imposed from the beginning [6,7]. This means that the only pressureless component admitted is the usual baryonic one, otherwise there is a conflict between the constraints obtained from the matter power spectrum and the supernova tests.

Many variations of the Chaplygin gas model have been proposed in the literature. One of them is the Modified Chaplygin Gas (MCG) Model. The equation of state of the MCGM is

$$p_c = B\rho - A\rho^{-\alpha}, \tag{1}$$

where B, A and α are constants. When $B = 0$ we recover the Generalized Chaplygin Gas (GCG) Model, and if in addition $\alpha = 1$ we have the original Chaplygin gas model. The dynamics of the MCG model has been studied in Ref. [8], while a dynamical system analysis has been made in Ref. [9]. The evolution of the temperature function has been considered in Ref. [10]. Some background constraints were established in Refs. [11] and [12]. The analysis of the spherical collapse was made in Ref. [13], while a perturbative study, looking for some general features of the model, was carried out in Ref. [14]. In all these studies the viability of the model was concluded, but no one of them has exploited the observational data concerning the perturbative behavior of the model.

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Our intention here is to test the MCG model against the power spectrum observational data. For the background tests, as those analyzed in Refs. [11,12], the MCG reveals to lead to competitive scenarios compared with the ΛCDM and the GCG models (to give just some examples). However, the constraints coming from the power spectrum data are, in general, much more crucial since it tests not only the background framework, but also the perturbative behavior of the model. Many general constraints can be established on the parameters of the EoS (1) even not considering perturbations. For example, in the past the equation of state (1) implies, when α is positive (a requirement necessary in a perturbative analysis in order to preserve a positive sound speed) that

$$\rho_c(a \approx 0) = \frac{cte}{a^{3(1+B)}}. \tag{2}$$

In order not to spoil the usual primordial scenario of the standard model (in special nucleosynthesis), B must be smaller than 1/3, which may include negative values. On the other hand, as we will show below, the requirement that the sound speed of the MCG must be positive implies essentially that $B > 0$. Hence, the admissible values of the parameter B seems to be around $0 < B < 1/3$. These considerations will be strengthened through the power spectrum analysis to be made later in this Letter: the matter power spectrum data can be fitted only if $|B| < 10^{-6}$. Hence, essentially the only configuration possible is that corresponding to the generalized Chaplygin gas, with perhaps some possible very small deviations from it. In this sense, we can consider that the MCG model is ruled out when confronted with the power spectrum observational data.

In next section we set out the general equations of the MCG model at background and perturbative levels. In Section 3 we perform a numerical analysis comparing the theoretical results with the matter power spectrum observational data. In Section 4 we present our conclusions.

2. Basic set of equations

Our starting point are Einstein's equations coupled to a pressureless fluid, radiation and to the MCG fluid. They read,

$$R_{\mu\nu} = 8\pi G\left\{T^m_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T^m\right\} + 8\pi G\left\{T^r_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T^r\right\} + 8\pi G\left\{T^c_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T^c\right\},$$

$$T^{\mu\nu}_{m;\mu} = 0, \quad T^{\mu\nu}_{c;\mu} = 0, \quad T^{\mu\nu}_{r;\mu} = 0.$$

The superscripts (subscripts) m, r and c stand for "matter", "radiation" and "Chaplygin". We assume a perfect fluid structure for the cosmic medium as a whole and also for each of the components,

$$T^{\mu\nu}_A = (\rho_A + p_A)u^\mu_A u^\nu_A - p_A g^{\mu\nu}, \qquad A = m, c, r. \tag{3}$$

Note that for "matter" component we understand a pressureless fluid that, in principle, may include baryons and dark matter. This questions will be discussed later. Using now the flat Friedman–Robertson–Walker metric (as suggested by the Seven-year WMAP data [15]),

$$ds^2 = dt^2 - a(t)^2\left[dx^2 + dy^2 + dz^2\right],$$

and identifying all the background 4-velocities, Einstein's equations reduce to

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho_m + \frac{8\pi G}{3}\rho_r + \frac{8\pi G}{3}\rho_c, \tag{4}$$

$$2\frac{\ddot a}{a} + \left(\frac{\dot a}{a}\right)^2 = -8\pi G(p_c + p_r), \tag{5}$$

$$\dot\rho_m + 3\frac{\dot a}{a}\rho_m = 0 \quad\Rightarrow\quad \rho_m = \rho_{m0}/a^3, \tag{6}$$

$$\dot\rho_r + 4\frac{\dot a}{a}\rho_r = 0 \quad\Rightarrow\quad \rho_r = \rho_{r0}/a^4, \tag{7}$$

$$\dot\rho_c + 3\frac{\dot a}{a}(\rho_c + p_c) = 0 \quad \left(p_c = B\rho_c - A/\rho_c^\alpha\right)$$
$$\Rightarrow\quad \rho_c = \left\{A_s + \frac{1 - A_s}{a^{3(1+\alpha)(1+B)}}\right\}^{1/(1+\alpha)}. \tag{8}$$

In the above set of equations we have defined $A_s = \frac{A}{(1+B)\rho_{c0}^{1+\alpha}}$.

The perturbed equations in the synchronous coordinate condition can be established following closely the computation shown in Ref. [7]. We introduce fluctuations around the background quantities, $g_{\mu\nu} = \bar g_{\mu\nu} + h_{\mu\nu}$, $\rho = \bar\rho + \delta\rho$, $p = \bar p + \delta p$, $u^\mu = \bar u^\mu + \delta u^\mu$. The bars indicate the background quantities. The synchronous coordinate condition implies $h_{\mu 0} = 0$ and $\delta u^0 = 0$. The final perturbed equations read (see also [16]),

$$\frac{\ddot h}{2} + \frac{\dot a}{a}\dot h - 4\pi G(\delta\rho + 3\delta p) = 0, \tag{9}$$

$$\delta\dot\rho + \frac{3\dot a}{a}(\delta\rho + \delta p) + (\rho + p)\left(\theta - \frac{\dot h}{2}\right) = 0, \tag{10}$$

$$(p + \rho)\dot\theta + \left[(\dot\rho + \dot p) + \frac{5\dot a}{a}(\rho + p)\right]\theta + \frac{\nabla^2\delta p}{a^2} = 0, \tag{11}$$

where ρ and p stand for the total matter and pressure, respectively, $\theta = \delta u^i_{,i}$ and $h = h_{kk}/a^2$.

In terms of the components, we end up with the following equations:

$$\frac{\ddot h}{2} + \frac{\dot a}{a}\dot h - 4\pi G\left[\delta\rho_m + \delta\rho_c + \delta\rho_r + 3(\delta p_m + \delta p_c + \delta p_r)\right] = 0, \tag{12}$$

$$\delta\dot\rho_m + \frac{3\dot a}{a}(\delta\rho_m + \delta p_m) + (\rho_m + p_m)\left(\theta_m - \frac{\dot h}{2}\right) = 0, \tag{13}$$

$$(\rho_m + p_m)\dot\theta_m + \left[(\dot\rho_m + \dot p_m) + \frac{5\dot a}{a}(\rho_m + p_m)\right]\theta_m + \frac{\nabla^2\delta p_m}{a^2} = 0, \tag{14}$$

$$\delta\dot\rho_c + \frac{3\dot a}{a}(\delta\rho_c + \delta p_c) + (\rho_c + p_c)\left(\theta_c - \frac{\dot h}{2}\right) = 0, \tag{15}$$

$$(\rho_c + p_c)\dot\theta_c + \left[(\dot\rho_c + \dot p_c) + \frac{5\dot a}{a}(\rho_c + p_c)\right]\theta_c + \frac{\nabla^2\delta p_c}{a^2} = 0, \tag{16}$$

$$\delta\dot\rho_r + \frac{3\dot a}{a}(\delta\rho_r + \delta p_r) + (\rho_r + p_r)\left(\theta_r - \frac{\dot h}{2}\right) = 0, \tag{17}$$

$$(\rho_r + p_r)\dot\theta_r + \left[(\dot\rho_r + \dot p_r) + \frac{5\dot a}{a}(\rho_r + p_r)\right]\theta_r + \frac{\nabla^2\delta p_r}{a^2} = 0, \tag{18}$$

with $\theta_m = \delta u^i_{m,i}$, $\theta_c = \delta u^i_{c,i}$ and $\theta_r = \delta u^i_{r,i}$.

With the definitions

$$\Omega_c(a) = \Omega_{c0}\left(A_s + \frac{1 - A_s}{a^{3(1+\alpha)(1+B)}}\right)^{\frac{1}{1+\alpha}}, \tag{19}$$

$$w(a) = \frac{p_c}{\rho_c} = B - \frac{A_s(1+B)}{A_s + (1 - A_s)a^{-3(1+\alpha)(1+B)}}, \tag{20}$$

$$v_s^2(a) = B + \frac{\alpha A_s(1+B)}{A_s + (1 - A_s)a^{-3(1+\alpha)(1+B)}}, \tag{21}$$