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Investigation of meson masses for real and imaginary chemical potential using the three-flavor PNJL model

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ABSTRACT

We investigate chemical-potential (μ) and temperature (T) dependence of scalar and pseudo-scalar meson masses for both real and imaginary μ, using the Polyakov-loop extended Nambu–Jona-Lasinio (PNJL) model with three-flavor quarks. A three-flavor phase diagram is drawn in μ^2–T plane where positive (negative) μ^2 corresponds to positive (imaginary) μ. A critical surface is plotted as a function of light- and strange-quark current mass and μ^2. We show that μ-dependence of the six-quark Kobayashi–Maskawa–'t Hooft (KMT) determinant interaction originated in $U_A(1)$ anomaly can be determined from lattice QCD data on η' meson mass around $\mu = 0$ and $\mu = i\pi T/3$ with T slightly above the critical temperature at $\mu = 0$ where the chiral symmetry is restored at $\mu = 0$ but broken at $\mu = i\pi T/3$, if it is measured in future.

1. Introduction

In recent theoretical studies, novel scenarios for QCD phase structure at finite real chemical potential (μ_R) are suggested; for example, the quarkyonic phase [1–4], the multi critical-endpoint generation [5–8] and the Lifshitz-point induced by the inhomogeneous phase [9]. Thus, *qualitative or speculative* investigation of QCD phase diagram is progressing well.

Nevertheless, *quantitative or more conclusive* understanding of QCD phase diagram is quite poor. The principal reason is the sign problem in the first-principle lattice QCD (LQCD) simulation at finite μ_R. Several methods such as the reweighting method [10], the Taylor expansion method [11], the analytic continuation from imaginary chemical potential μ_I to μ_R [12–14] and so on were proposed so far to circumvent the sign problem. However, they do not reach the $\mu_R/T \gtrsim 1$ region yet, where T is temperature. For this reason, effective models such as the Nambu–Jona-Lasinio (NJL) model were used so far to investigate qualitative properties of the phase structure at finite μ_R.

The effective-model approach, however, is ambiguous particularly in determining the interaction part. For example, the six-quark Kobayashi–Maskawa–'t Hooft (KMT) determinant interaction [15,16] is necessary to introduce $U_A(1)$ anomaly into the effective model, but the strength G_D of the KMT interaction is not well determined. Very recently, it has been pointed out in Ref. [17] that the strength G_D may have μ_R dependence through the suppression of instanton density due to Debye screening. If G_D has such a μ_R dependence, the phase diagram will be changed largely [17].

Thus, a new approach should be proposed for quantitative or more reliable investigation of QCD phase diagram at finite μ_R. As a possible answer, recently, we proposed the *imaginary chemical potential matching approach* (the μ_I-matching approach) [18,19]. In this approach, interactions of the effective model are determined from LQCD data at finite μ_I where no sign problem comes out. After the determination, a phase structure at finite μ_R is predicted with the effective model. The most important point in this approach is whether the model taken can reproduce the Roberge–Weiss (RW) periodicity and the RW transition at finite μ_I [20]. In our previous works [18], we showed that the Polyakov-loop extended Nambu–Jona-Lasinio (PNJL) model [21] can do it, because the thermodynamical potential of the PNJL model is invariant under the extended $\mathbb{Z}_3$ transformation,

$$e^{\pm i\theta} \to e^{\pm i\theta} e^{\pm i\frac{2\pi k}{3}},$$
$$\Phi(\theta) \to \Phi(\theta) e^{-i\frac{2\pi k}{3}}, \qquad \bar{\Phi}(\theta) \to \bar{\Phi}(\theta) e^{i\frac{2\pi k}{3}}, \tag{1}$$

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where the chemical potential μ is given by $\mu = i\mu_{\rm I} = i\theta T$. Here, Φ and $\bar{\Phi}$ denote the Polyakov-loop and its conjugate, respectively. This symmetry ensures the RW periodicity. Since, the PNJL model is designed to treat the confinement mechanism approximately in addition to the chiral symmetry breaking, we can investigate not only the chiral transition but also the deconfinement transition with the PNJL model. We also showed by using the PNJL model that the crossover deconfinement transition that takes place at finite θ becomes stronger as θ increases and eventually at $\theta = \pi/3$ it changes into the RW phase transition [18].

The $U_{\rm A}(1)$ anomaly is related to instantons. The anomaly can be taken into account in the NJL and PNJL models. In the three-flavor case, it is described by the effective six-quark KMT determinant interaction [15,16], as mentioned above. The $U_{\rm A}(1)$ anomaly restoration at finite T is investigated in the case of $\mu = 0$ by the NJL model [22] that can reproduce LQCD data [23]. For the case of finite $\mu_{\rm R}$, the μ dependence of the anomaly restoration strongly depends on that of the coupling constant G_D of the KMT determinant interaction [17], if G_D has the μ dependence. However, the μ dependence of G_D is unclear because LQCD data is not feasible at finite $\mu_{\rm R}$ and also theoretical understanding on the μ dependence of the instanton density is not sufficient. Therefore, the phase structure in the three-flavor system is more ambiguous than in the two-flavor system.

Recently, scalar and pseudo-scalar meson masses are investigated in the $\mu_{\rm R}$ region by the PNJL model with three-flavor quarks; for example, see [24,25]. In this Letter, we investigate scalar and pseudo-scalar meson masses in both the $\mu_{\rm R}$ and the $\mu_{\rm I}$ region, using the three-flavor PNJL model. We show η' meson mass is sensitive to G_D particularly near $\theta = \pi/3$. This means that the θ dependence of η' meson mass is a good quantity to determine μ dependence of G_D. At the present stage, there is no reliable LQCD data particular on meson masses for the case of finite $\mu_{\rm I}$. Therefore, our investigation is limited to only a qualitative level.

2. Three-flavor PNJL model

Lagrangian density of the three-flavor PNJL model is

$$\begin{aligned}\mathcal{L}_{\rm PNJL} &= \bar{q}(i\gamma_\nu D^\nu - \hat{m}_0)q + G_S\sum_{a=0}^{8}\left[(\bar{q}\lambda_a q)^2 + (\bar{q}i\gamma_5\lambda_a q)^2\right] \\ &\quad - G_D\left[\det_{ij}\bar{q}_i(1+\gamma_5)q_j + \det_{ij}\bar{q}_i(1-\gamma_5)q_j\right] \\ &\quad - \mathcal{U}\left(\Phi[A], \bar{\Phi}[A], T\right),\end{aligned} \tag{2}$$

where $D^\nu = \partial^\nu + iA^\nu = \partial^\nu + i\delta^\nu_0 g A^0_a \lambda_a/2$ with the gauge coupling g and the Gell-Mann matrices λ_a. Three-flavor quark fields $q = (q_u, q_d, q_s)$ have current quark masses $\hat{m}_0 = {\rm diag}(m_u, m_d, m_s)$. The Polyakov-loop potential $\mathcal{U}$ is defined later in (7) and (8). In the interaction part, G_S and G_D denote coupling constants of the scalar-type four-quark and the KMT determinant interaction, respectively. The determinant $\det_{ij}$ runs in the flavor space and then the KMT determinant interaction breaks the $U_{\rm A}(1)$ symmetry explicitly.

In the PNJL model, the gauge field A_μ is treated as a homogeneous and static background field. The Polyakov-loop Φ and its conjugate $\bar{\Phi}$ are given by

$$\Phi = \frac{1}{3}{\rm tr}_c(L), \qquad \bar{\Phi} = \frac{1}{3}{\rm tr}_c(\bar{L}) \tag{3}$$

where $L = \exp(iA_4/T)$ with $A_4 = iA_0$ in Euclidean space. In the Polyakov-gauge, A_4 is diagonal in the color space.

We make the mean field approximation (MFA) to the quark–antiquark interactions in (2) in the following way. In (2), the operator product $\bar{q}_i q_j$ is first divided into $\bar{q}_i q_j = \sigma_{ij} + (\bar{q}_i q_j)'$ with the mean field (the chiral condensate) $\sigma_{ij} \equiv \langle \bar{q}_i q_j\rangle$ and the fluctuation $(\bar{q}_i q_j)'$ where $i, j = u, d, s$. Ignoring higher-order terms of $(\bar{q}_i q_j)'$ in the rewritten Lagrangian and re-substituting $(\bar{q}_i q_j)' = \bar{q}_i q_j - \sigma_{ij}$ into the approximated Lagrangian, one can obtain a linearized Lagrangian based on MFA:

Table 1
Summary of the parameter set in the NJL sector [26].

m_l (MeV)	m_s (MeV)	Λ (MeV)	$G_S\Lambda^2$	$G_D(0)\Lambda^5$
5.5	140.7	602.3	1.835	−12.36

$$\begin{aligned}\mathcal{L}^{\rm MFA}_{\rm PNJL} &= \bar{q}_i\left(i\gamma_\nu\partial^\nu + i\gamma_0 A_4 - M_{ii}\right)q_i \\ &\quad - \left(\sum_{i=u,d,s} 2G_S\sigma_{ii}^2 - 4G_D\sigma_{uu}\sigma_{dd}\sigma_{ss}\right) \\ &\quad - \mathcal{U}\left(\Phi[A], \bar{\Phi}[A], T\right),\end{aligned} \tag{4}$$

where the dynamical quark mass M_{ii} is defined by $M_{ii} = m_i - 4G_S\sigma_{ii} + 2G_D\sigma_{jj}\sigma_{kk}$ with $i \neq j \neq k$.

In this study, we impose the isospin symmetry for u–d sector and then we use $m_l = m_u = m_d$. The thermodynamical potential becomes

$$\begin{aligned}\Omega_{\rm PNJL} &= -2\sum_{f=u,d,s}\int\frac{d^3p}{(2\pi)^3}\Big[N_c E_{p,f} \\ &\quad + \frac{1}{\beta}\ln\left[1 + 3\left(\Phi + \bar{\Phi}e^{-\beta(E_{p,f}-\mu_f)}\right)e^{-\beta(E_{p,f}-\mu_f)} + e^{-3\beta(E_{p,f}-\mu_f)}\right] \\ &\quad + \frac{1}{\beta}\ln\left[1 + 3\left(\bar{\Phi} + \Phi e^{-\beta(E_{p,f}+\mu_f)}\right)e^{-\beta(E_{p,f}+\mu_f)} + e^{-3\beta(E_{p,f}+\mu_f)}\right]\Big] \\ &\quad + \left(\sum_{i=u,d,s} 2G_S\sigma_{ii}^2 - 4G_D\sigma_{uu}\sigma_{dd}\sigma_{ss}\right) \\ &\quad + \mathcal{U}\left(\Phi[A], \bar{\Phi}[A], T\right).\end{aligned} \tag{5}$$

We take the three-dimensional momentum cutoff,

$$\int\frac{d^3p}{(2\pi)^3} \to \frac{1}{2\pi^2}\int_0^\Lambda dp\, p^2, \tag{6}$$

because this model is non-renormalizable. Hence, the present model has five parameters G_S, G_D, m_l, m_s and Λ.

We use the parameter set of Ref. [26]; the parameters are determined to fit empirical values on meson masses of π, K and η' and the pion decay constant, while m_l is fixed at 5.5 MeV. The parameter set thus determined is shown in Table 1.

We also use $\mathcal{U}$ of Ref. [27] that is fitted to LQCD data in the pure gauge limit at finite T [28,29]:

$$\frac{\mathcal{U}}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\bar{\Phi}^3 + \Phi^3\right) + \frac{b_4}{4}(\bar{\Phi}\Phi)^2, \tag{7}$$

$$b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3. \tag{8}$$

We take the original value 270 MeV for T_0; see Table 2 for other parameters.