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$f(R)$ gravity and crossing the phantom divide barrier

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ABSTRACT

The $f(R)$ gravity models formulated in Einstein conformal frame are equivalent to Einstein gravity together with a minimally coupled scalar field. We shall explore phantom behavior of $f(R)$ models in this frame and compare the results with those of the usual notion of phantom scalar field.

1. Introduction

There are strong observational evidences that the expansion of the universe is accelerating. These observations are based on type Ia supernova [1], cosmic microwave background radiation [2], large scale structure surveys [3] and weak lensing [4]. There are two classes of models aim at explaining this phenomenon: in the first class, one modifies the laws of gravity whereby a late-time acceleration is produced. A family of these modified gravity models is obtained by replacing the Ricci scalar R in the usual Einstein–Hilbert Lagrangian density for some function $f(R)$ [5,6]. In the second class, one invokes a new matter component usually referred to as dark energy. This component is described by an equation of state parameter $\omega \equiv \frac{p}{\rho}$, namely the ratio of the homogeneous dark energy pressure over the energy density. For a cosmic speed up, one should have $\omega < -\frac{1}{3}$ which corresponds to an exotic pressure $p < -\rho/3$. Recent analysis of the latest and the most reliable dataset (the Gold dataset [7]) have indicated that significantly better fits are obtained by allowing a redshift dependent equation of state parameter [8]. In particular, these observations favor the models that allow the equation of state parameter crossing the line corresponding to $\omega = -1$, the phantom divide line (PDL), in the near past. It is therefore important to construct dynamical models that provide a redshift dependent equation of state parameter and allow for crossing the phantom barrier.

Most simple models of this kind employ a scalar field coupled minimally to curvature with negative kinetic energy which referred to as phantom field [9,10]. In contrast to these models, one may consider models which exhibit phantom behavior due to curvature corrections to gravitational equations rather than introducing exotic matter systems. Recently, there is a number of attempts to find phantom behavior in $f(R)$ gravity models. It is shown that one may realize crossing the PDL in this framework without recourse to any extra component relating to matter degrees of freedom with exotic behavior [11,12]. Following these attempts, we intend to explore phantom behavior in some $f(R)$ gravity models which have a viable cosmology, i.e. a matter-dominated epoch followed by a late-time acceleration. In contrast to [12], we shall consider $f(R)$ gravity models in Einstein conformal frame. It should be noted that mathematical equivalence of Jordan and Einstein conformal frames does not generally imply that they are also physically equivalent. In fact it is shown that some physical systems can be differently interpreted in different conformal frames [13,14]. The physical status of the two conformal frames is an open question which we are not going to address here. Our motivation to work in Einstein conformal frame is that in this frame, $f(R)$ models consist of Einstein gravity plus an additional dynamical degree of freedom, the scalar partner of the metric tensor. This suggests that it is this scalar degree of freedom which drives late-time acceleration in cosmologically viable $f(R)$ models. We compare this scalar degree of freedom with the usual notion of phantom scalar field. We shall show that behaviors of this scalar field attributed to $f(R)$ models which allow crossing the PDL are similar to those of a quintessence field with a negative potential rather than a phantom with a wrong kinetic term.

2. Phantom as a minimally coupled scalar field

The simplest class of models that provides a redshift dependent equation of state parameter is a scalar field minimally coupled to

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gravity whose dynamics is determined by a properly chosen potential function $V(\varphi)$. Such models are described by the Lagrangian density[1]

$$L=\frac{1}{2}\sqrt{-g}\big(R-\alpha g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi-2V(\varphi)\big) \tag{1}$$

where $\alpha=+1$ for quintessence and $\alpha=-1$ for phantom. The distinguished feature of the phantom field is that its kinetic term enters (1) with opposite sign in contrast to the quintessence or ordinary matter. The Einstein field equations which follow (1) are

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=T_{\mu\nu} \tag{2}$$

with

$$T_{\mu\nu}=\alpha\partial_\mu\varphi\partial_\nu\varphi-\frac{1}{2}\alpha g_{\mu\nu}\partial_\gamma\varphi\partial^\gamma\varphi-g_{\mu\nu}V(\varphi) \tag{3}$$

In a homogeneous and isotropic spacetime, φ is a function of time alone. In this case, one may compare (3) with the stress tensor of a perfect fluid with energy density ρ_φ and pressure p_φ. This leads to the following identifications

$$\rho_\varphi=\frac{1}{2}\alpha\dot{\varphi}^2+V(\varphi),\qquad p_\varphi=\frac{1}{2}\alpha\dot{\varphi}^2-V(\varphi) \tag{4}$$

The equation of state parameter is then given by

$$\omega_\varphi=\frac{\frac{1}{2}\alpha\dot{\varphi}^2-V(\varphi)}{\frac{1}{2}\alpha\dot{\varphi}^2+V(\varphi)} \tag{5}$$

In the case of a quintessence (phantom) field with $V(\varphi)>0$ ($V(\varphi)<0$) the equation of state parameter remains in the range $-1<\omega_\varphi<1$. In the limit of small kinetic term (slow-roll potentials [15]), it approaches $\omega_\varphi=-1$ but does not cross this line. The phantom barrier can be crossed by either a phantom field ($\alpha<0$) with $V(\varphi)>0$ or a quintessence field ($\alpha>0$) with $V(\varphi)<0$, when we have $2|V(\varphi)|>\dot{\varphi}^2$. This situation corresponds to

$$\rho_\varphi>0,\qquad p_\varphi<0,\qquad V(\varphi)>0\quad \textit{phantom} \tag{6}$$

$$\rho_\varphi<0,\qquad p_\varphi>0,\qquad V(\varphi)<0\quad \textit{quintessence} \tag{7}$$

Here it is assumed that the scalar field has a canonical kinetic term $\pm\frac{1}{2}\dot{\varphi}^2$. It is shown [16] that any minimally coupled scalar field with a generalized kinetic term (k-essence Lagrangian [17]) cannot lead to crossing the PDL through a stable trajectory. However, there are models that employ Lagrangians containing multiple fields [18] or scalar fields with non-minimal coupling [19] which in principle can achieve crossing the barrier.

There are some remarks to do with respect to $V(\varphi)<0$ appearing in (7). In fact, the role of negative potentials in cosmological dynamics has been recently investigated by some authors [20]. One of the important points about the cosmological models containing such potentials is that they predict that the universe may end in a singularity even if it is not closed. For more clarification, consider a model containing different kinds of energy densities such as matter, radiation, scalar fields and so on. The Friedmann equation in a flat universe is $H^2\propto\rho_t$ with $\rho_t=\Sigma_i\rho_i$ being the sum of all energy densities. It is clear that the universe expands forever if $\rho_t>0$. However, if the contribution of some kind of energy is negative so that $\rho_i<0$, then it is possible to have $H^2=0$ at finite time and the size of the universe starts to decrease.[2] We will return to this issue in the context of $f(R)$ gravity models in the next section.

[1] We use the unit system $8\pi G=\hbar=c=1$ and the metric signature $(-,+,+,+)$.

[2] For a more detailed discussion see, e.g., [21].

The possibility of existing a fluid with a surenegative pressure ($\omega<-1$) leads to problems such as vacuum instability and violation of energy conditions [22]. For a perfect fluid with energy density ρ and pressure p, the weak energy condition requires that $\rho\geqslant 0$ and $\rho+p\geqslant 0$. These state that the energy density is positive and the pressure is not too large compared to the energy density. The null energy condition $\rho+p\geqslant 0$ is a special case of the latter and implies that energy density can be negative if there is a compensating positive pressure. The strong energy condition as a hallmark of general relativity states that $\rho+p\geqslant 0$ and $\rho+3p\geqslant 0$. It implies the null energy condition and excludes excessively large negative pressures. The null dominant energy condition is a statement that $\rho\geqslant|p|$. The physical motivation of this condition is to prevent vacuum instability or propagation of energy outside the light cone. Applying to an equation of state $p=\omega\rho$ with a constant ω, it means that $\omega\geqslant-1$. Violation of all these reasonable constraints by phantom, gives an unusual feature to this principal energy component of the universe. There are however some remarks concerning how these unusual features may be circumvented [22,23].

3. $f(R)$ gravity

Let us consider an $f(R)$ gravity model described by the action

$$S=\frac{1}{2}\int d^4x\sqrt{-g}f(R)+S_m(g_{\mu\nu},\psi) \tag{8}$$

where g is the determinant of $g_{\mu\nu}$, $f(R)$ is an unknown function of the scalar curvature R and S_m is the matter action depending on the metric $g_{\mu\nu}$ and some matter field ψ. It is well known that these models are equivalent to a scalar field minimally coupled to gravity with an appropriate potential function. In fact, we may use a new set of variables

$$\bar{g}_{\mu\nu}=pg_{\mu\nu} \tag{9}$$

$$\phi=\frac{1}{2\beta}\ln p \tag{10}$$

where $p\equiv\frac{df}{dR}=f'(R)$ and $\beta=\sqrt{\frac{1}{6}}$. This is indeed a conformal transformation which transforms the above action in the Jordan frame to the Einstein frame [13,24,25]

$$S=\frac{1}{2}\int d^4x\sqrt{-\bar{g}}\big\{\bar{R}-\bar{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-2V(\phi)\big\}+S_m\big(\bar{g}_{\mu\nu}e^{2\beta\phi},\psi\big) \tag{11}$$

In the Einstein frame, ϕ is a minimally coupled scalar field with a self-interacting potential which is given by

$$V\big(\phi(R)\big)=\frac{Rf'(R)-f(R)}{2f'^2(R)} \tag{12}$$

Note that the conformal transformation induces the coupling of the scalar field ϕ with the matter sector. The strength of this coupling β, is fixed to be $\sqrt{\frac{1}{6}}$ and is the same for all types of matter fields.

Variation of the action (11) with respect to $\bar{g}_{\mu\nu}$, gives the gravitational field equations

$$\bar{G}_{\mu\nu}=T^{\phi}_{\mu\nu}+\bar{T}^m_{\mu\nu} \tag{13}$$

where

$$\bar{T}^m_{\mu\nu}=\frac{-2}{\sqrt{-g}}\frac{\delta S_m}{\delta\bar{g}^{\mu\nu}} \tag{14}$$

$$T^{\phi}_{\mu\nu}=\partial_\mu\phi\partial_\nu\phi-\frac{1}{2}\bar{g}_{\mu\nu}\partial_\gamma\phi\partial^\gamma\phi-V(\phi)\bar{g}_{\mu\nu} \tag{15}$$