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The nucleon electric dipole form factor from dimension-six time-reversal violation

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ABSTRACT

We calculate the electric dipole form factor of the nucleon that arises as a low-energy manifestation of time-reversal violation in quark–gluon interactions of effective dimension 6: the quark electric and chromoelectric dipole moments, and the gluon chromoelectric dipole moment. We use the framework of two-flavor chiral perturbation theory to one loop.

Electric dipole moments (EDMs) [1,2] provide stringent bounds on sources of time-reversal (T) violation beyond the phase of the quark-mixing matrix [3]. Experiments are in preparation [4] which aim to improve the current bound on the neutron EDM, $|d_n| < 2.9 \times 10^{-26}\ e\,\mathrm{cm}$ [5], by nearly two orders of magnitude. Novel ideas exist [6] also for the measurement of EDMs of charged particles in storage rings, including the proton—for which an indirect bound, $|d_p| < 7.9 \times 10^{-25}\ e\,\mathrm{cm}$, has been extracted from the atomic Hg EDM [7]—and the deuteron. Since the Standard Model prediction [8,9] is orders of magnitude away from current experimental limits, a signal in this new crop of experiments would be a clear sign of new physics.

The momentum dependence of an EDM is the electric dipole form factor (EDFF). Together with the well-known parity (P) and T-preserving electric and magnetic form factors and the P-violating, T-preserving anapole form factor, the P- and T-violating EDFF completely specifies the Lorentz-invariant electromagnetic current of a particle with spin 1/2. Although the full momentum dependence of a nuclear EDFF will not be measured anytime soon, the radius of the form factor provides a contribution to the Schiff moment (SM) of the corresponding atom, because it produces a short-range electron–nucleus interaction.

There has been some recent interest [10–12] on the nucleon EDFF stemming from the lowest-dimension T violation in strong interactions, the QCD $\bar{\theta}$ term. As other low-energy observables, both the EDM and the SM of hadrons and nuclei are difficult to calculate directly in QCD. Attempts have been made to extract the nucleon EDM from lattice simulations [13], but a signal with dynamical quarks remains elusive. One possible way to extract the EDM in this case relies on a extrapolation of the EDFF to zero momentum, which provides another motivation to look at the EDFF. QCD-inspired models have also been brought to bear on the nucleon EDFF [11].

We would like to use a framework flexible enough to formulate the nucleon EDFF in the wider context of other low-energy T-violating observables such as the EDMs of nuclei. Such framework exists in the form of an effective field theory, chiral perturbation theory (ChPT) [14–16]. (For introductions, see for example Refs. [17,18].) Since it correctly incorporates the approximate chiral symmetry of QCD, ChPT provides not only a model-independent description of low-energy physics but also the quark-mass dependence of observables, which is useful in the extrapolation of lattice results to realistic values of the pion mass. The nucleon EDFF from the $\bar{\theta}$ term has in fact been calculated in this framework [10,12], and some implications of the particular way the $\bar{\theta}$ term breaks chiral symmetry were discussed in Ref. [19]. (For earlier work on the neutron EDM in ChPT, see for example Refs. [20,21].) The momentum dependence of the EDFF is given by the pion cloud [10,22]: the scale for momentum variation is the pion mass and the SM is determined by a T-violating pion–nucleon coupling. Assuming naturalness of ChPT's low-energy constants (LECs), one can use an estimate of this coupling based on $SU(3)$ symmetry to derive [20]

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a bound on $\bar{\theta}$, $\bar{\theta} \lesssim 2.5 \times 10^{-10}$ [12] from the current limit on the neutron EDM. ChPT extrapolation formulas for the nucleon EDM in lattice QCD can be found in Ref. [23].

The smallness of $\bar{\theta}$ leaves room for other sources of T violation in the strong interactions. Here we calculate in ChPT the nucleon EDFF arising from the effectively dimension-6 interactions involving quark and gluon fields that violate T [24,25]: the quark electric dipole moment (qEDM), which couples quarks and photons; the quark chromoelectric dipole moment (qCEDM), which couples quarks and gluons; and the Weinberg operator, which couples three gluons and can be identified as the gluon chromoelectric dipole moment (gCEDM). These higher-dimension operators can have their origin in an ultraviolet-complete theory at a high-energy scale, such as, for example, supersymmetric extensions of the Standard Model. We construct the interactions among nucleons, pions and photons that stem from the underlying quark–gluon operators and use them to calculate the EDFF to the order where the momentum dependence first appears. As we will see, the sizes of the proton and neutron EDMs and SMs partially reflect the underlying sources of T violation. While much effort has already been put into estimating the EDMs from these sources [1,2], the full EDFF apparently has been previously considered only within a particular chiral quark model [26]. Other implications of the different chiral transformation properties [27] of the dimension-6 operators will be considered in a subsequent paper [28].

Well below the scale $M_{\not{T}}$ characteristic of T violation, we expect T-violating effects to be captured by the lowest-dimension interactions among Standard Model fields that respect the theory's $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge symmetry. Just above the characteristic QCD scale $M_{\mathrm{QCD}} \sim 1$ GeV, strong interactions are described by the most general Lagrangian with Lorentz, and color and electromagnetic gauge invariance among the lightest quarks ($q = (ud)^T$), gluons (G^a_μ), and photons (A_μ). The effectively dimension-6 T-violating terms at this scale can be written as

$$\begin{aligned}\mathcal{L}_{\not{T}} = &-\frac{i}{2}\bar{q}(d_0 + d_3\tau_3)\sigma^{\mu\nu}\gamma^5 q F_{\mu\nu} \\ &-\frac{i}{2}\bar{q}(\tilde{d}_0 + \tilde{d}_3\tau_3)\sigma^{\mu\nu}\gamma^5\lambda^a q G^a_{\mu\nu} \\ &+\frac{d_W}{6}\epsilon^{\mu\nu\lambda\sigma} f^{abc} G^a_{\mu\rho} G^{b,\rho}_{\nu} G^c_{\lambda\sigma},\end{aligned} \tag{1}$$

in terms of the photon and gluon field strengths $F_{\mu\nu}$ and $G^a_{\mu\nu}$, respectively, the standard products of gamma matrices γ^5 and $\sigma^{\mu\nu}$ in spin space, the totally antisymmetric symbol $\epsilon^{\mu\nu\lambda\sigma}$, the Pauli matrix τ_3 in isospin space, the Gell-Mann matrices λ^a in color space, and the associated Gell-Mann coefficients f^{abc}. In Eq. (1) the first (second) term represents the isoscalar d_0 ($\tilde{d}_0$) and isovector d_3 ($\tilde{d}_3$) components of the qEDM (qCEDM). Although these interactions have canonical dimension 5, they originate just above the Standard Model scale M_W from dimension-6 operators [24] involving in addition the carrier of electroweak symmetry breaking (the Higgs field). They are thus proportional to the vacuum expectation value of the Higgs field, which we can trade for the ratio of the quark mass to Yukawa coupling, m_q/f_q. Writing the proportionality constant as $e\delta_q f_q/M^2_{\not{T}}$ ($4\pi\tilde{\delta}_q f_q/M^2_{\not{T}}$),

$$d_i \sim \mathcal{O}\left(e\delta\frac{\bar{m}}{M^2_{\not{T}}}\right), \qquad \tilde{d}_i \sim \mathcal{O}\left(4\pi\tilde{\delta}\frac{\bar{m}}{M^2_{\not{T}}}\right), \tag{2}$$

in terms of the average light-quark mass $\bar{m}$ and the dimensionless factors δ and $\tilde{\delta}$ representing typical values of δ_q and $\tilde{\delta}_q$. The third term in Eq. (1) [25] is the gCEDM, with

$$d_W \sim \mathcal{O}\left(\frac{4\pi w}{M^2_{\not{T}}}\right) \tag{3}$$

in terms of a dimensionless parameter w. The sizes of δ, $\tilde{\delta}$ and w depend on the exact mechanisms of electroweak and T breaking and on the running to the low energies where non-perturbative QCD effects take over. The minimal assumption is that they are $\mathcal{O}(1)$, $\mathcal{O}(g_s/4\pi)$ and $\mathcal{O}((g_s/4\pi)^3)$, respectively, with g_s the strong-coupling constant. However they can be much smaller (when parameters encoding T-violating beyond the Standard Model are small) or much larger (since f_q is unnaturally small). In the Standard Model itself, where $M_{\not{T}} = M_W$, they are suppressed [9] by the Jarlskog parameter [29] $J_{CP} \simeq 3 \times 10^{-5}$. In supersymmetric models with various simplifying, universality assumptions of a soft-breaking sector with a common scale M_{SUSY}, one has $M_{\not{T}} = M_{SUSY}$ and the size of the dimensionless parameters is given by the minimal assumption times a factor which is [2,30,31], roughly (neglecting electroweak parameters), $A_{CP} = (g_s/4\pi)^2 \sin\phi$, with ϕ a phase encoding T violation. Allowing for non-diagonal terms in the soft-breaking sfermion mass matrices, enhancements of the type $m_b/m_d \sim 10^3$ or even $m_t/m_u \sim 10^5$ are possible (although they might be associated with other, smaller phases) [2].

Since we are interested in light systems, we are integrating out all degrees of freedom associated with quarks heavier than up and down. The effects of qEDMs and qCEDMs of such quarks are discussed briefly at the end. T-violating four-quark operators are effectively dimension-8 because again electroweak gauge invariance requires insertions of the Higgs field. Since higher-dimension operators are suppressed by more inverse powers of the large scale $M_{\not{T}}$, we expect them to be generically less important at low energies and we concentrate here on the dimension-6 operators in Eq. (1). It is of course possible that in particular models the coefficients of the effectively dimension-6 operators are suppressed enough to make higher-dimension operators numerically important; low-energy implications of four-quark operators, which also contain representations of chiral symmetry we consider, have recently been studied in Ref. [32].

At momenta Q comparable to the pion mass, $Q \sim m_\pi \ll M_{\mathrm{QCD}}$, interactions among nucleons, pions and photons are described by the most general Lagrangian involving these degrees that transforms properly under the symmetries of the QCD. Ignoring quark masses and charges and the $\bar{\theta}$ term, the dimension-4 QCD terms are invariant under a chiral $SU(2)_L \times SU(2)_R \sim SO(4)$ symmetry, which is spontaneously broken down to its diagonal, isospin subgroup $SU(2)_V \sim SO(3)$. The corresponding Goldstone bosons are identified as the pions, which provide a non-linear realization of chiral symmetry. Pion interactions proceed through a covariant derivative, which in stereographic coordinates [17] $\boldsymbol{\pi}$ for the pions is written as

$$D_\mu\boldsymbol{\pi} = D^{-1}\partial_\mu\boldsymbol{\pi}, \tag{4}$$

with $D = 1 + \boldsymbol{\pi}^2/F^2_\pi$ and $F_\pi \simeq 186$ MeV the pion decay constant. Nucleons are described by an isospin-1/2 field N, and the nucleon covariant derivative is

$$\mathcal{D}_\mu N = \left(\partial_\mu + \frac{i}{F^2_\pi}\boldsymbol{\tau}\cdot\boldsymbol{\pi}\times D_\mu\boldsymbol{\pi}\right)N. \tag{5}$$

We define $\mathcal{D}^\dagger$ through $\bar{N}\mathcal{D}^\dagger \equiv \overline{\mathcal{D}N}$, and use the shorthand notation

$$\begin{aligned}&\mathcal{D}^\mu_\pm \equiv \mathcal{D}^\mu \pm \mathcal{D}^{\dagger\mu}, \\ &\mathcal{D}^\mu_\pm\mathcal{D}^\nu_\pm \equiv \mathcal{D}^\mu\mathcal{D}^\nu + \mathcal{D}^{\dagger\mu}\mathcal{D}^{\dagger\nu} \pm \mathcal{D}^{\dagger\mu}\mathcal{D}^\nu \pm \mathcal{D}^{\dagger\nu}\mathcal{D}^\mu,\end{aligned} \tag{6}$$

and

$$\begin{aligned}&\tau_i\mathcal{D}^\mu_\pm \equiv \tau_i\mathcal{D}^\mu \pm \mathcal{D}^{\dagger\mu}\tau_i, \\ &\tau_i\mathcal{D}^\mu_\pm\mathcal{D}^\nu_\pm \equiv \tau_i\mathcal{D}^\mu\mathcal{D}^\nu + \mathcal{D}^{\dagger\mu}\mathcal{D}^{\dagger\nu}\tau_i \pm \mathcal{D}^{\dagger\mu}\tau_i\mathcal{D}^\nu \pm \mathcal{D}^{\dagger\nu}\tau_i\mathcal{D}^\mu.\end{aligned} \tag{7}$$