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A minimal length versus the Unruh effect

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ABSTRACT

In this Letter we study the radiation measured by an accelerated detector, coupled to a scalar field, in the presence of a fundamental minimal length. The latter is implemented by means of a modified momentum space Green's function. After calibrating the detector, we find that the net flux of field quanta is negligible, and that there is no Planckian spectrum. We discuss possible interpretations of this result, and we comment on experimental implications in heavy ion collisions and atomic systems.

The emergence of a minimal length in a quantum spacetime is an old idea, dating back to the early times of Quantum Gravity [1]. In most cases, it turns out to be the crucial signature in every phenomenon that takes place on a background that departs from a purely classical description. In this general framework, the study of the Unruh effect in the presence of a minimal length can lead both to profound insights and simple phenomenological predictions. In fact, acceleration radiation has a prominent role in a variety of physical contexts: beyond the theoretical case of an accelerated detector, the Unruh radiation might affect the transverse polarization of electrons and positrons in particle storage rings (Sokolov–Ternov effect) [2,3], and the onset of the Quark Gluon Plasma (QGP) due to heavy ions collisions [4]. The Unruh effect might have non-negligible imprints in low energy physics too, such as the dynamics of electrons in Penning traps, of atoms in microwave cavities, and of ultraintense lasers (for a review see Ref. [5] and references therein). Finally, its companion effect, i.e. the Hawking radiation, is extensively investigated in analog models of gravity, such as Bose–Einstein condensates (BEC) [6–8].

The presence of a minimal length ℓ is testable only if one can perform experiments at energies around the scale $M_* = 1/\ell$. However, we recall that low energy systems are also endowed with relevant microscopic scales whose global effects, though important, cannot be described by the larger scale effective models often in use. On the other hand, fine tuning experiments in condensed matter systems and very high energy particle collisions are now in progress and could reveal key information about the interplay between the Unruh effect and the existence of a coarse-grained background in the system [9]. It is therefore imperative to have an accurate description of the acceleration radiation in the presence of a minimal length.

The energy scale associated with a minimal length is typically seen as the frontier beyond which local Lorentz symmetry is violated, and it is usually set to be of the order of the Planck mass, as in the vector–tensor theories of gravitation [10]. In other cases, such as in analog models in BEC [6,7], this energy scale is much smaller. In both contexts the violation appears as a modification of the dispersion relation. This possibility was widely studied in relation to the transplanckian problem in cosmology (see e.g. [11,12]), and to the robustness of both Hawking emission [13,14] and Unruh effect [15]. The lesson learnt from these works is that the minimal length associated with modified dispersion relations has a negligible impact on these phenomena.

The acceleration radiation was also studied in the case when the minimal length is introduced to cure the divergent ultraviolet (UV) behaviour of the field theory. For example, in [16] the propagator is modified via path integral duality, and it is finite in the UV regime. In [17,18], the same propagator is found by deforming the action of the Lorentz group. As for modified dispersion relations, the effect on both the Unruh effect [17–19] and on the Hawking radiation [20] is negligible.

Lorentz-violating models are increasingly disfavored by observations, see e.g. [21]. Therefore, it seems more sensible to implement a Lorentz invariant length ℓ in the theory. In the following, we do not assume any particular value for ℓ, which presumably depends on the details of the underlying quantum gravitational theory. A natural choice would be a value of the order of the

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Planck length (10^{-35} m), which could be larger in the presence of extra-dimensions, as discussed at the end of the Letter.

With this spirit in mind, in this Letter we assume that the Euclidean momentum space propagator is given by [22–24]

$$G_\ell(p^2) = \frac{e^{-\ell^2 p^2/2}}{p^2 + m^2}, \tag{1}$$

where $p^2 = p_0^2 + |\vec{p}|^2$. A similar propagator was already successfully employed in the context of both black hole physics [25–30] (for a review see Ref. [31] and further references therein), and of inflationary cosmology [32]. The main result is that the divergent short distance behavior of the conventional solutions to field equations (including the ones on curved spacetimes) is cured while, as expected, the quantum fluctuations of the manifold do not occur at scales larger than ℓ, where the classical description of gravity efficiently works. In particular, the divergent behavior of the black hole evaporation in the Planck phase has been regularized. In the new scenario, the terminal stage of the Hawking quantum emission is in fact characterized by a thermodynamically stable (positive heat capacity) phase of cooling of the black hole, often called the "SCRAM phase" [33,34].

We begin our discussion by briefly recalling the main features of the Unruh effect, as presented in Ref. [35]. We consider a detector, moving in a flat background spacetime along a trajectory $x^\alpha(\tau)$, where τ is the detector proper time. We assume that the detector moves through a region permeated by a quantum scalar field ϕ, and that the interaction between the two can be described in terms of the Lagrangian $L_{\text{int}} = \gamma\ \mu(\tau)\phi[x^\alpha(\tau)]$, where γ is a small coupling constant and μ is the detector monopole momentum operator. Due to the interaction with the field, the detector will undergo a transition from the ground state E_0 to an excited state $E > E_0$. As γ is small, we can derive the transition probability $\Gamma = \int dE |\Psi|^2$ by squaring the first order amplitude

$$\Psi = i\langle E; \psi| \int_{-\infty}^{\infty} L_{\text{int}}\, d\tau |0_{\text{M}}; E_0\rangle, \tag{2}$$

where $|0_{\text{M}}\rangle$ is the Minkowski vacuum, and $|\psi\rangle$ is the field excited state. At the lowest order, the monopole operator is well approximated by $\mu(\tau) = e^{iH_0\tau}\mu(0)e^{-iH_0\tau}$, hence we can separate the contributions of the detector and the field to the amplitude by writing

$$\Psi = i\gamma\langle E|\mu(0)|E_0\rangle \int_{-\infty}^{\infty} d\tau e^{i\Delta E\tau}\langle\psi|\phi(x)|0_M\rangle, \tag{3}$$

where $\Delta E = E - E_0$. From this one sees that, at first order, the state $|\psi\rangle$ can only contain a single field quantum. However, to find the transition probability, we need to take in account transitions to all possible energies, thus

$$\Gamma = \gamma^2 \sum_E \bigl|\langle E|\mu(0)|E_0\rangle\bigr|^2 \mathcal{F}(\Delta E), \tag{4}$$

where the detector response function $\mathcal{F}(\Delta E)$ is given by

$$\mathcal{F}(\Delta E) = \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' e^{-i\Delta\tau\Delta E} G^+\bigl(x(\tau), x(\tau')\bigr). \tag{5}$$

Here, $\Delta\tau = \tau - \tau'$, and G^+ is the positive frequency Wightman–Green function. We stress that the response function is fully specified in terms of the properties of the field, and it does not depend on the choice of the detector, whose sensitivity is given only by $S = \gamma^2 \sum |\langle E|\mu(0)|E_0\rangle|^2$. The double integration in Eq. (5) means that the flux of particles interacting with the detector diverges as soon as the detector-field system reaches an equilibrium configuration. Therefore, one usually considers the transition probability per unit proper time, $\dot{\Gamma} = S\dot{\mathcal{F}}$, where we define the response rate

$$\dot{\mathcal{F}} = \int_{-\infty}^{\infty} d\Delta\tau e^{-i\Delta\tau\Delta E} G^+(\Delta x). \tag{6}$$

In this expression, $\Delta x^2 = \eta_{\mu\nu}(x^\mu - x'^\mu)(x^\nu - x'^\nu)$ is the Minkowski proper time interval squared. For an inertial detector moving with constant velocity v, one has $\Delta x^2 = \Delta\tau^2/(1 - v^2)$, and $G^+(\Delta x)$ diverges when $\Delta\tau \to 0$. However, as no other singularities occur on the integration path, one can show that $\dot{\mathcal{F}}$ vanishes by means of the $i\epsilon$ prescription. On the contrary, when the trajectory is not inertial, the Minkowski interval has the form $\Delta x^2 = f(\Delta\tau)$, where f is a non-constant and finite function. Therefore, the integrand function in (6) exhibits poles corresponding to each zero of $f(\Delta\tau)$ and the rate is no longer vanishing. For example, for a uniformly accelerated detector, with acceleration $1/\alpha$, coupled to a massless scalar field, one finds a non-vanishing rate $\dot{\mathcal{F}} \sim \exp(-2\pi\alpha\Delta E)$. Thus, we learn that the detector feels an incoming radiation of quanta, as if it was coupled to a thermal bath at the temperature $T = 1/2\pi\alpha k_B$ [36].

The above calculations can also be performed in Euclidean space, upon the analytic continuation $i\tau = \tau_E$. Then, the response rate formula becomes

$$\dot{\mathcal{F}} = i \int_{i\infty}^{-i\infty} d\Delta\tau_E e^{\Delta\tau_E\Delta E} G_E^+(\Delta x), \tag{7}$$

where G_E^+ is the Euclidean Wightman function. A detector with uniform acceleration $1/\alpha$ on the Euclidean plane typically follows a circular trajectory of the form $\alpha^2\sin^2(\Delta\tau_E/2\alpha)$. Below we will find more convenient to work in Euclidean space, thus we will use Eq. (7), instead of (6) to calculate the radiation flux.

We now proceed with the implementation of a minimal length in the framework of the Unruh effect, by adopting the propagator (1). We see that the minimal length appears in the damping factor, and this is physically interpreted as a blurring, or delocalization, occurring at each point on a manifold when probed by high momenta. However, at lower momenta the presence of ℓ is actually negligible and, usually, one can work with the ordinary field theory. The Euclidean propagator in coordinate space can be found by calculating the Fourier transform of the Schwinger representation

$$\frac{e^{-\ell^2 p^2/2}}{p^2 + m^2} = e^{\ell^2 m^2/2} \int_{\ell^2/2}^{\infty} ds e^{-s(p^2+m^2)}. \tag{8}$$

In the massless case, we find that the modified Euclidean Wightman–Green function is [40]

$$G_\ell^E(\Delta x) = -\frac{1}{4\pi^2(\Delta\vec{x}^2 + \Delta t_E^2)}\bigl[1 - e^{-(\Delta t_E^2 + \Delta\vec{x}^2)/2\ell^2}\bigr]. \tag{9}$$

The theory behaves nicely, as G_ℓ^E reduces to its conventional form in the limit $\ell \to 0$. More importantly, the above function shows its regularity at coincident points: in the double limit $(\Delta t_E, \Delta\vec{x}^2) \to (0, 0)$ one has $G_\ell^E \to -1/8\pi^2\ell^2$. The same holds for the massive case, as one can show that, in the coincidence limit,

$$G_\ell^E \sim -\frac{1}{8\pi^2\ell^2} + m^2 e^{m^2\ell^2/2} E_1\bigl(m^2\ell^2/2\bigr), \tag{10}$$