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# Quasinormal modes of a black hole in the deformed Hořava–Lifshitz gravity

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ABSTRACT

We study the quasinormal modes of the massless scalar perturbation in the background of a deformed black hole in the Hořava–Lifshitz gravity with coupling constant $\lambda = 1$. Our results show that the quasinormal frequencies depend on the parameter in the Hořava–Lifshitz gravity and the behavior of the quasinormal modes is different from those in the Reissner–Norström and Einstein–Born–Infeld black hole spacetimes. The absolute value of imaginary parts is smaller and the scalar perturbations decay more slowly in the deformed Hořava–Lifshitz black hole spacetime. This information can help us understand more about the Hořava–Lifshitz gravity.



Inspired by the Lifshitz model, Hořava [1] proposed recently a field theory model for a UV complete theory of gravity, which is a non-Lorentz invariant theory of gravity in $3+1$ dimensions. Unlike Einstein gravity, it is renormalizable by power-counting arguments. Thus, it is believed widely that it could be a candidate for Einstein's general relativity. Very recently, the Hořava–Lifshitz gravity theory has been intensively investigated in [2–11] and its cosmological applications have been studied in [12–18]. Some static spherically symmetric black hole solutions have been found in Hořava–Lifshitz theory [19–24] and the associated thermodynamic properties with those black hole solutions have been investigated in [25–28].

The four-dimensional metric in the ADM formalism can be expressed as [29]

$$ds^2_{ADM} = -N^2\,dt^2 + g_{ij}\big(dx^i - N^i\,dt\big)\big(dx^j - N^j\,dt\big) \tag{1}$$

and the Einstein–Hilbert action is given by

$$S_{EH} = \frac{1}{16\pi G}\int d^4x\,\sqrt{g}N\big(K_{ij}K^{ij} - K^2 + R - 2\Lambda\big), \tag{2}$$

where $G$ is Newton's constant and $K_{ij}$ is extrinsic curvature which takes the form

$$K_{ij} = \frac{1}{2N}(\partial_t g_{ij} - \nabla_i N_j - \nabla_j N_i). \tag{3}$$

In general, the IR vacuum of this theory is anti de Sitter (AdS) spacetime. To obtain a Minkowski vacuum in the IR sector, one can modify the theory by introducing "$\mu^4 R$" and then take the $\Lambda_W \to 0$ limit. This does not change the UV properties of the theory, but it alters the IR properties. The deformed action of the nonrelativistic renormalizable gravitational theory is given by [20]

$$S_{HL} = \int dt\,d^3x(\mathcal{L}_0 + \tilde{\mathcal{L}}_1), \tag{4}$$

$$\mathcal{L}_0 = \sqrt{g}N\left\{\frac{2}{\kappa^2}\big(K_{ij}K^{ij} - \lambda K^2\big) + \frac{\kappa^2\mu^2(\Lambda_W R - 3\Lambda_W^2)}{8(1-3\lambda)}\right\}, \tag{5}$$

$$\tilde{\mathcal{L}}_1 = \sqrt{g}N\left\{\frac{\kappa^2\mu^2(1-4\lambda)}{32(1-3\lambda)}R^2 - \frac{\kappa^2}{2w^4}\left(C_{ij} - \frac{\mu w^2}{2}R_{ij}\right)\left(C^{ij} - \frac{\mu w^2}{2}R^{ij}\right) + \mu^4 R\right\}. \tag{6}$$

Here $C_{ij}$ is the Cotton tensor, defined by

$$C^{ij} = \epsilon^{ik\ell}\nabla_k\left(R^j{}_\ell - \frac{1}{4}R\delta^j_\ell\right) = \epsilon^{ik\ell}\nabla_k R^j{}_\ell - \frac{1}{4}\epsilon^{ikj}\partial_k R. \tag{7}$$

Comparing the action to that of general relativity in the ADM formalism, one can find that the speed of light, Newton's constant and the cosmological constant are given by

$$c = \frac{\kappa^2\mu}{4}\sqrt{\frac{\Lambda_W}{1-3\lambda}}, \qquad G = \frac{\kappa^2}{32\pi c}, \qquad \Lambda = \frac{3}{2}\Lambda_W. \tag{8}$$

Taking $N^i = 0$, the spherically symmetric solutions could be obtained with the metric ansatz [19,21–27]

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$$ds^2 = -N^2(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2\big(d\theta^2 + \sin^2\theta\,d\phi^2\big). \tag{9}$$

Substituting the metric ansatz (9) into the action, and then varying the functions $N$ and $f$, one can find that the reduced Lagrangian reads

$$\tilde{\mathcal{L}}_1 = \frac{\kappa^2\mu^2 N}{8(1-3\lambda)\sqrt{f}}\bigg(\frac{\lambda-1}{2}f'^2 - \frac{2\lambda(f-1)}{r}f' + \frac{(2\lambda-1)(f-1)^2}{r^2} - 2w\big(1-f-rf'\big)\bigg) \tag{10}$$

with $w = 8\mu^2(3\lambda-1)/\kappa^2$. For $\lambda = 1$ ($w = 16\mu^2/\kappa^2$), we have a solution where $f$ and $N$ are determined to be

$$N^2 = f = \frac{2(r^2-2Mr+\alpha)}{r^2+2\alpha+\sqrt{r^4+8\alpha Mr}}, \tag{11}$$

where $\alpha = 1/(2w)$ and $M$ is an integration constant related to the mass. The metric of this black hole looks like that of Gauss–Bonnet black hole. The event horizons are given by

$$r_\pm = M \pm \sqrt{M^2-\alpha}, \tag{12}$$

and the Hawking temperature is

$$T_H = \frac{f'}{4\pi}\bigg|_{r=r_+} = \frac{\sqrt{M^2-\alpha}}{\pi\big(r_+^2+2\alpha+\sqrt{r_+^4+8\alpha Mr_+}\big)}. \tag{13}$$

The thermodynamics of this black hole has been studied in [28,30,31]. Myung [30] has calculated the ADM mass of this black hole and find that it is similar to that of four-dimensional Einstein–Born–Infeld black hole [32]. It is well known that the Born–Infeld electrodynamics is one of the important nonlinear electromagnetic theories. As the Born–Infeld scale parameter $b$ tends to zero, the Einstein–Maxwell theory is recovered and the Einstein–Born–Infeld black hole is reduced to Reissner–Norström black hole. Applying such kind of ADM mass proposed in [30], Wang et al. [31] find that the integral and differential forms of the first law of thermodynamics are still valid for the deformed Hořava–Lifshitz black hole. The potentially observable properties of this black hole were considered in [33–35]. In this Letter, our main purpose is to study the quasinormal modes of massless scalar field in this spacetime and to see what there exists some new feature in the dynamical evolution of the perturbation in the black hole in Hořava–Lifshitz theory.

The Klein–Gordon equation for a massless scalar field in this spacetime is

$$\frac{1}{\sqrt{-g}}\partial_\mu\big(\sqrt{-g}g^{\mu\nu}\partial_\nu\big)\psi = 0. \tag{14}$$

Separating $\psi = e^{-i\omega t}R(r)Y_{lm}(\theta,\phi)/r$, we can obtain the radial equation for the scalar perturbation in the deformed Hořava–Lifshitz black hole spacetime

$$\frac{d^2R(r)}{dr_*^2} + \big[\omega^2 - V(r)\big]R(r) = 0, \tag{15}$$

where $r_*$ is the tortoise coordinate (which is defined by $dr_* = \frac{1}{f}dr$) and the effective potential $V(r)$ reads

$$V(r) = \frac{2(r^2-2Mr+\alpha)}{r^2+2\alpha+\sqrt{r^4+8\alpha Mr}} \times\bigg[\frac{l(l+1)}{r^2} + \frac{1}{\alpha}\bigg(1-\frac{r^3+2\alpha M}{r\sqrt{r^4+8\alpha Mr}}\bigg)\bigg]. \tag{16}$$

Obviously, as $\alpha = 0$ the effective potential $V(r)$ can be reduced to that of the Schwarzschild black hole. When $\alpha$ increases we find that the peak value of the potential barrier gets lower for $l = 0$ and higher for $l = 1$, which is shown in Fig. 1. In the Reissner–Norström black hole background, for all $l$ the peak value of the effective potential increases with the charge $q$ of the black hole. This means that although the formulas of the outer and inner horizons are very similar, the behaviors of the effective potential are different in these black hole background. In the Einstein–Born–Infeld black hole, the variety of the effective potential with the charge $q$ is similar to that in Reissner–Norström black hole spacetime. However, the Born–Infeld scale parameter $b$ decreases the peak value of $V(r)$ for all $l$. These results imply the quasinormal modes in the deformed Hořava–Lifshitz black hole possess some different properties from those of the black holes in the Einstein–Maxwell and Einstein–Born–Infeld gravities.

We are now in a position to apply the third-order WKB approximation method approximation to evaluate the fundamental quasinormal modes ($n = 0$) of massless scalar perturbation in the deformed Hořava–Lifshitz black hole. We expect to see what effects of Hořava–Lifshitz parameter $\alpha$ can be reflected in the quasinormal modes' behavior. The formula for the complex quasinormal frequencies $\omega$ in this approximation is given by [36–38]

$$\omega^2 = \big[V_0 + \big(-2V_0''\big)^{1/2}\Lambda\big] - i\bigg(n+\frac{1}{2}\bigg)\big(-2V_0''\big)^{1/2}(1+\Omega), \tag{17}$$

where

$$\Lambda = \frac{1}{(-2V_0'')^{1/2}}\bigg\{\frac{1}{8}\bigg(\frac{V_0^{(4)}}{V_0''}\bigg)\bigg(\frac{1}{4}+\alpha^2\bigg) - \frac{1}{288}\bigg(\frac{V_0'''}{V_0''}\bigg)^2\big(7+60\alpha^2\big)\bigg\},$$

$$\Omega = \frac{1}{(-2V_0'')}\bigg\{\frac{5}{6912}\bigg(\frac{V_0'''}{V_0''}\bigg)^4\big(77+188\alpha^2\big) - \frac{1}{384}\bigg(\frac{V_0'''^2V_0^{(4)}}{V_0''^3}\bigg)\big(51+100\alpha^2\big) + \frac{1}{2304}\bigg(\frac{V_0^{(4)}}{V_0''}\bigg)^2\big(67+68\alpha^2\big) + \frac{1}{288}\bigg(\frac{V_0'''V_0^{(5)}}{V_0''^2}\bigg)\big(19+28\alpha^2\big) - \frac{1}{288}\bigg(\frac{V_0^{(6)}}{V_0''}\bigg)\big(5+4\alpha^2\big)\bigg\}, \tag{18}$$

and

$$\alpha = n+\frac{1}{2}, \qquad V_0^{(s)} = \frac{d^sV}{dr_*^s}\bigg|_{r_*=r_*(r_p)},$$

$n$ is overtone number and $r_p$ is the value of polar coordinate $r$ corresponding to the peak of the effective potential (16). Setting $M = 1$ and substituting the effective potential (16) into the formula above, we can obtain the quasinormal frequencies of scalar perturbation in the deformed Hořava–Lifshitz black hole.

In Tables 1–4, we list the fundamental quasinormal frequencies of the massless scalar perturbation field for fixed $l = 0$, 1 and 2 in the deformed Hořava–Lifshitz, the Reissner–Norström and Einstein–Born–Infeld black hole spacetimes, respectively. From Table 1 and Figs. 2 and 3, we find that with the increase of the parameter $\alpha$ the real parts decrease for $l = 0$ and increase for $l = 1$ and $l = 2$. The absolute value of imaginary parts for all $l$ decrease.