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Twist three distribution $e(x)$: Sum rules and equation of motion relations

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ABSTRACT

We investigate the twist three distribution function $e(x)$ in light-front Hamiltonian perturbation theory. In light-front gauge, by eliminating the constrained field, we find a mass term, an intrinsic transverse momentum dependent term and a 'genuine twist three' quark–gluon interaction term in the operator. The equation of motion relation, moment relation and the sum rules are satisfied for a quark at one loop. We compare the results with other model calculations.

1. Introduction

Twist three parton distribution $e(x)$ has not been explored broadly in the literature so far. $e(x)$ is spin independent and chiral odd. Therefore it is difficult to measure it experimentally as it can combine only with another chiral odd object. It was shown in [1] that $e(x)$ contributes to the unpolarized Drell–Yan process at the level of twist four. $e(x)$ enters together with the chirally odd Collin's fragmentation function $H_1^{\perp}$ in the azimuthal asymmetry in semi-inclusive deep inelastic scattering (DIS) of longitudinally polarized electrons off unpolarized nucleons. This asymmetry was measured [2–4], however, apart from $e(x)$, several other distribution and fragmentation functions also appear in this asymmetry. In [5], another possibility of measuring $e(x)$ is proposed through two hadron production in polarized semi-inclusive DIS; where $e(x)$ is coupled to a two hadron fragmentation function. The first calculation of $e(x)$ was done in [1], in MIT bag model. Further it was calculated in chiral quark soliton model [6,7], spectator model [8] and perturbative one loop model [9]. The Q^2 evolution of $e(x)$ has been calculated in [10]. A nice review of the general properties of $e(x)$ and the various model calculations can be found in [11]. In chiral quark soliton model, a $\delta(x)$ singularity was observed in $e(x)$, which was later found also in a perturbative calculation [9]. However, no such $\delta(x)$ term was found in the spectator model as well as in MIT bag model. It is to be noted that as $x=0$ cannot be experimentally achieved, no direct experimental verification of the $\delta(x)$ contribution is possible. Another interesting aspect of $e(x)$ is that the first moment of the flavor singlet combination of $e(x)$ is related to the pion nucleon sigma term $\sigma_{\pi N}$ [1]. The flavor non-singlet part of $e(x)$ satisfies a sum rule connecting it to the hadronic mass difference between neutron and proton [6]. The first moment of $e(x)$ obeys the sum rule

$$\int_{-1}^{1} dx\, e(x) = \frac{1}{2M}\langle P,S|\bar{\psi}(0)\psi(0)|P,S\rangle. \tag{1}$$

The second moment of $e(x)$ obeys another sum rule [1]:

$$\int_{-1}^{1} dx\, x e(x) = \frac{m}{M} N_q; \tag{2}$$

where m is the mass of the quark and M is the mass of the proton. N_q is the number of quarks of a given flavor. These sum rules are not satisfied in the bag model as the QCD equation of motion is modified in the bag. They are not satisfied in the spectator model either. In QCD equation of motion method [11], the first sum rule is saturated by the $\delta(x)$ contribution only, whereas in chiral quark soliton model, only a part of the contribution comes from the $\delta(x)$ term. In [9], $e(x)$ has been calculated for a quark dressed with a gluon at one loop in QCD in light-front gauge. Starting from the Feynman diagram in $A^+=0$ gauge, k^- was integrated out to get to perform the calculation in light-front time-ordered method. A $\delta(x)$ term was found which was shown to be related to the $k^+=0$ modes.

The disagreement between different model calculations for $e(x)$ makes it an interesting object for further study. In this work, we

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calculate $e(x)$ for a quark at one loop in QCD using light-front Hamiltonian perturbation theory in $A^+ = 0$ gauge. Instead of using the Feynman diagrams, we expand the state in Fock space in terms of multiparton light-front wavefunctions. The advantage is that these wave functions are Lorentz boost invariant, so we can truncate the Fock space expansion to a few particle sector in a boost invariant way. The two particle light-front wave functions (LFWFs) can be calculated analytically for a quark at one loop using the light-front Hamiltonian. The partons are on-mass shell objects having non-vanishing transverse momenta. They can be called field theoretic partons. The distribution functions can be calculated at the scale Q^2 using the multiparton LFWFs. To $O(\alpha_s)$ this is an exact calculation. Another interesting aspect is the nature of ultraviolet (UV) divergence in $e(x)$. Being a twist three object, $e(x)$ contains a 'bad' component of quark field. As a result, it is expected to have a different UV divergence as compared to the logarithmic divergence in twist two distributions $f_1(x)$. In the following, we present our calculation.

2. $e(x)$ for a quark at one loop

The twist three distribution $e(x)$ is given by [1]:

$$e(x) = \frac{P^+}{M} \int \frac{dy^-}{8\pi} e^{\frac{i}{2} x P^+ y^-} \langle P, S | \bar{\psi}(0) \psi(y^-) | P, S \rangle. \tag{3}$$

We introduce the projection operators $\Lambda^\pm = \frac{1}{2}\gamma_0 \gamma_\pm$ and project out the light-front good and bad components of the quark field:

$$\psi^{(\pm)} = \Lambda^\pm \psi. \tag{4}$$

In light-front gauge, $A^+ = 0$, the 'bad' component, $\psi^{(-)}$ is constrained, and the equation of constraint is given by [12]

$$\psi^-(y^-) = \frac{1}{i\partial^+} \left(i\alpha^\perp \cdot \partial^\perp + g\alpha^\perp \cdot A^\perp + \beta m \right) \psi^{(+)}(y^-); \tag{5}$$

where the operator $\frac{1}{\partial^+}$ is defined as [12]

$$\frac{1}{\partial^+} f(x^-) = \frac{1}{4} \int_{-\infty}^{\infty} dy^- \, \epsilon(x^- - y^-) f(y^-). \tag{6}$$

The antisymmetric step function is given by

$$\epsilon(x^-) = -\frac{i}{\pi} \mathcal{P} \int \frac{d\omega}{\omega} e^{\frac{i}{2}\omega x^-}. \tag{7}$$

$\mathcal{P}$ denotes the principal value. For the dynamical field the operator in Eq. (3), can be written as

$$\begin{aligned} O_e &= \bar{\psi}(0)\psi(y^-) \\ &= \psi^{(-)\dagger}(0)\gamma_0 \psi^{(+)}(y^-) + \psi^{(+)\dagger}(0)\gamma_0 \psi^{(-)}(y^-). \end{aligned} \tag{8}$$

Using the constraint equation for $\psi^{(-)}$ this can be written as:

$$O_e = O_m + O_g + O_k; \tag{9}$$

where

$$O_m = m\psi^{(+)\dagger}(0) \left[\left(-\frac{\overleftarrow{1}}{i\partial^+} \right) + \left(\frac{\overrightarrow{1}}{i\partial^+} \right) \right] \psi^{(+)}(y^-); \tag{10}$$

$$O_k = \psi^{(+)\dagger}(0) \left[\frac{\overrightarrow{\not\partial^\perp}}{\partial^+} - \frac{\overleftarrow{\not\partial^\perp}}{\partial^+} \right] \psi^{(+)}(y^-); \tag{11}$$

$$O_g = g\psi^{(+)\dagger}(0) \left[\not{A}_T \left(\frac{\overleftarrow{1}}{i\partial^+} \right) + \left(\frac{\overrightarrow{1}}{i\partial^+} \right) \not{A}_T \right] \psi^{(+)}(y^-). \tag{12}$$

O_m is the mass term, O_k is the transverse momentum dependent term and O_g is the explicit quark–gluon interaction term, also called the 'genuine twist three' term.

For ψ^+ we use two component formalism [12]

$$\psi^+ = \begin{pmatrix} \xi \\ 0 \end{pmatrix}. \tag{13}$$

The two component field $\xi(y)$ has the Fock space expansion

$$\xi(y) = \sum_\lambda \chi_\lambda \int \frac{dk^+ d^2k^\perp}{2(2\pi)^3 \sqrt{k^+}} \left[b_\lambda^\dagger(k) e^{iky} + d_{-\lambda}(k) e^{-iky} \right]; \tag{14}$$

with

$$\chi_\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad \chi_\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{15}$$

For a dressed quark state of momentum P and helicity σ:

$$\begin{aligned} |P, \sigma\rangle &= \phi_1 b^\dagger(P, \sigma)|0\rangle \\ &\quad + \sum_{\sigma_1, \lambda_2} \int \frac{dk_1^+ d^2k_1^\perp}{\sqrt{2(2\pi)^3 k_1^+}} \int \frac{dk_2^+ d^2k_2^\perp}{\sqrt{2(2\pi)^3 k_2^+}} \sqrt{2(2\pi)^3 P^+} \\ &\quad \times \delta^3(P - k_1 - k_2) \phi_2(P, \sigma | k_1, \sigma_1; k_2, \lambda_2) b^\dagger(k_1, \sigma_1) \\ &\quad \times a^\dagger(k_2, \lambda_2)|0\rangle. \end{aligned} \tag{16}$$

Here $a^\dagger$ and $b^\dagger$ are bare gluon and quark creation operators respectively and ϕ_1 and ϕ_2 are the multiparton wave functions. They are the probability amplitudes to find one bare quark and one quark plus gluon inside the dressed quark state respectively. We introduce Jacobi momenta x_i, $q_i{}^\perp$ such that $\sum_i x_i = 1$ and $\sum_i q_i{}^\perp = 0$. They are defined as

$$x_i = \frac{k_i^+}{P^+}, \qquad q_i^\perp = k_i^\perp - x_i P^\perp. \tag{17}$$

Also, we introduce the wave functions,

$$\psi_1 = \phi_1, \qquad \psi_2(x_i, q_i^\perp) = \sqrt{P^+} \phi_2(k_i^+, k_i{}^\perp); \tag{18}$$

which are independent of the total transverse momentum $P^\perp$ of the state and are boost invariant. The state is normalized as,

$$\langle P', \lambda' | P, \lambda \rangle = 2(2\pi)^3 P^+ \delta_{\lambda,\lambda'} \delta(P^+ - P'^+) \delta^2(P^\perp - P'^\perp). \tag{19}$$

The two particle wave function depends on the helicities of the electron and photon. Using the eigenvalue equation for the light-cone Hamiltonian, this can be written as [13],

$$\begin{aligned} &\psi^\sigma_{2\sigma_1,\lambda}(x, q^\perp) \\ &\quad = \frac{x(1-x)}{(q^\perp)^2 + m^2(1-x)^2} \frac{1}{\sqrt{(1-x)}} \frac{g}{\sqrt{2(2\pi)^3}} T^a \chi^\dagger_{\sigma_1} \\ &\qquad \times \left[-2\frac{q^\perp}{1-x} - \frac{\tilde{\sigma}^\perp \cdot q^\perp}{x} \tilde{\sigma}^\perp + im\tilde{\sigma}^\perp \frac{(1-x)}{x} \right] \chi_\sigma \epsilon_\lambda^{\perp *} \psi_1. \end{aligned} \tag{20}$$

m is the bare mass of the quark, $\tilde{\sigma}_1 = \sigma_2$, $\tilde{\sigma}_2 = -\sigma_1$. ψ_1 actually gives the normalization of the state [13]:

$$|\psi_1|^2 = 1 - \frac{\alpha_s}{2\pi} C_f \int_\epsilon^{1-\epsilon} dx \frac{1+x^2}{1-x} \log\frac{Q^2}{\mu^2}, \tag{21}$$

to order α_s. Here ϵ is a small cutoff on x. We have taken the cutoff on the transverse momenta to be Q. This gives the large scale of the process. The above expression is derived using Eqs. (19), (16)