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Lorentz and CPT violation in QED revisited: A missing analysis

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ABSTRACT

We investigate the breakdown of Lorentz symmetry in QED by a CPT violating interaction term consisting of the coupling of an axial fermion current with a constant vector field *b*, in the framework of algebraic renormalization — a regularization-independent method. We show, to all orders in perturbation theory, that a CPT-odd and Lorentz violating Chern–Simons-like term, definitively, is not radiatively induced by the axial coupling of the fermions with the constant vector *b*.

1. Introduction

The quantum electrodynamics (QED) with violation of Lorentz and CPT have been studied intensively in recent years. Among several issues, the possible generation of a Chern–Simons-like term induced by radiative corrections arising from a CPT and Lorentz violating term in the fermionic sector has been a recurrent theme in the literature. We particularly mention the following works [1–18] (and references cited therein), where many controversies have emerged from the discussion whether this Chern–Simons-like term could be generated by means of radiative corrections arising from the axial coupling of charged fermions to a constant vector b_μ responsible for the breakdown of Lorentz symmetry.

In this Letter, we reassess the discussion on the radiative generation of a Chern–Simons-like term induced from quantum corrections in the extended QED. Concerning to extended QED with a term which violates the Lorentz and CPT symmetries, most of the papers were devoted to discuss the gauge invariance of the model *only*, putting aside a more specific way how Lorentz invariance is broken. Here, we will discuss the latter point, giving attention to the requirement that the breakdown of Lorentz symmetry arising from the axial coupling of charged fermions to a constant vector b_μ be *soft* in the sense of Symanzik [19–22], i.e., has power-counting dimension less than four or, equivalently, is negligible in the deep Euclidean region of energy–momentum space. To the best of our knowledge this has not been investigated in details. In switching on the radiative corrections, it is a nontrivial task to study the effects of such a symmetry breaking. In particular, one has to ask how the corresponding Ward identity that characterizes the breaking behaves at the quantum level. Our aim is to show that, to the contrary of the claims found in the literature, radiative corrections arising from the axial coupling of charged fermions to a constant vector b_μ do not induce a Lorentz- and CPT-violating Chern–Simons-like term in the QED action.

2. Extended QED in the classical approximation

2.1. The classical theory

We start by considering an action for extended QED with a term which violates the Lorentz and CPT symmetries in the matter sector only. In the tree approximation, the classical action of extended QED with one Dirac spinor that we are considering here is given by:

$$\Sigma = \Sigma_{\rm S} + \Sigma_{\rm SB} + \Sigma_{\rm IR} + \Sigma_{\rm gf}, \tag{2.1}$$

where

$$\Sigma_{\rm S} = \int d^4x \left\{ i\bar{\psi}\gamma^\mu(\partial_\mu + ieA_\mu)\psi - m\bar{\psi}\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \right\}, \tag{2.2}$$

is the symmetric part of Σ under gauge and Lorentz transformations. The term

$$\Sigma_{\rm SB} = -\int d^4x\, b_\mu \bar{\psi}\gamma_5\gamma^\mu\psi, \tag{2.3}$$

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is the symmetry-breaking part of Σ that breaks the manifest Lorentz covariance by the presence of a constant vector b_μ which selects a preferential direction in Minkowski space–time, breaking its isotropy, as well as it breaks CPT.[1]

$$\Sigma_{\rm IR} = \int d^4x \frac{1}{2}\lambda^2 A_\mu A^\mu, \tag{2.4}$$

is a mass term for the photon field,[2] introduced in order to avoid infrared singularities and

$$\Sigma_{\rm gf} = -\int d^4x \frac{1}{2\xi}\left(\partial_\mu A^\mu\right)^2, \tag{2.5}$$

is a gauge-fixing action.

2.2. The symmetries

2.2.1. Discrete symmetries

The discrete symmetries of the theory are the following ones.

Charge Conjugation C. Assuming the Dirac representation of the γ-matrices [26], the charge conjugation transformations read:

$$\begin{aligned}
&\psi \xrightarrow{C} \psi^c = C\bar{\psi}^t, \\
&\bar{\psi} \xrightarrow{C} \bar{\psi}^c = -\psi^t C^{-1}, \\
&A_\mu \xrightarrow{C} A_\mu^c = -A_\mu, \\
&C\gamma_\mu C = \gamma_\mu^t, \\
&C\gamma_5 C = -\gamma_5^t = -\gamma_5,
\end{aligned} \tag{2.6}$$

where C is the charge conjugation matrix, with $C^2 = -1$. All terms in the action Σ (2.1) are invariant under charge conjugation.

Parity P.

$$\begin{aligned}
&x \xrightarrow{P} \left(x^0, -\vec{x}\right), \\
&\psi \xrightarrow{P} \gamma^0\psi, \\
&\bar{\psi} \xrightarrow{P} \bar{\psi}\gamma^0, \\
&A_\mu \xrightarrow{P} A^\mu.
\end{aligned} \tag{2.7}$$

All terms of the action are invariant, excepted the Lorentz breaking term $\Sigma_{\rm SB}$ (2.3), which transforms under parity as

$$\bar{\psi} b_\mu \gamma_5 \gamma^\mu \psi \xrightarrow{P} \begin{cases} -\bar{\psi} b_0 \gamma_5 \gamma^0 \psi, \\ \bar{\psi} b_i \gamma_5 \gamma^i \psi \quad (i = 1, 2, 3). \end{cases} \tag{2.8}$$

Time Reversal T.

$$\begin{aligned}
&\psi \xrightarrow{T} T\psi, \\
&\bar{\psi} \xrightarrow{T} \bar{\psi} T, \\
&A_\mu \xrightarrow{T} A_\mu, \\
&T\gamma^\mu T = \gamma_\mu^T = \gamma^{\mu *}, \\
&T\gamma_5 T = \gamma_5.
\end{aligned} \tag{2.9}$$

Under time reversal transformation, the broken Lorentz term, $\Sigma_{\rm SB}$ (2.3), transforms as below:

$$\bar{\psi} b_\mu \gamma_5 \gamma^\mu \psi \xrightarrow{T} \begin{cases} \bar{\psi} b_0 \gamma_5 \gamma^0 \psi, \\ -\bar{\psi} b_i \gamma_5 \gamma^i \psi \quad (i = 1, 2, 3), \end{cases} \tag{2.10}$$

which implies time reversal violation, whereas the other terms in the action Σ (2.1) remain invariant.

Therefore, the action for extended QED, Σ (2.1), has CPT symmetry broken by the Lorentz breaking term, $\Sigma_{\rm SB}$ (2.3):

$$\bar{\psi} b_\mu \gamma_5 \gamma^\mu \psi \xrightarrow{\rm CPT} -\bar{\psi} b_\mu \gamma_5 \gamma^\mu \psi. \tag{2.11}$$

2.2.2. Continuous symmetries: the functional identities

The $U(1)$ gauge transformations are given by:

$$\begin{aligned}
&\delta_{\rm g} A_\mu(x) = \frac{1}{e}\partial_\mu \omega(x), \\
&\delta_{\rm g} \psi(x) = -i\omega(x)\psi(x), \\
&\delta_{\rm g} \bar{\psi}(x) = i\omega(x)\bar{\psi}(x),
\end{aligned} \tag{2.12}$$

which are broken by the gauge-fixing and infrared regulator terms.

Subjected to the $U(1)$ gauge transformations (2.12), the action Σ (2.1) transforms as given by the following Ward identity:

$$\mathcal{W}_{\rm g}\Sigma = -\frac{1}{e\xi}\left(\Box + e\xi\lambda^2\right)\partial_\mu A^\mu, \tag{2.13}$$

with the Ward operator associated to the gauge transformations

$$\mathcal{W}_{\rm g}(x) = -\frac{1}{e}\partial_\mu \frac{\delta}{\delta A_\mu(x)} + i\bar{\psi}(x)\frac{\overrightarrow{\delta}}{\delta\bar{\psi}(x)} - i\frac{\overleftarrow{\delta}}{\delta\psi(x)}\psi(x). \tag{2.14}$$

Note that the right-hand side of (2.13) being linear in the quantum field A_μ, will not be submitted to renormalization, i.e., it will remain a classical breaking [20,22].

On the other hand, the Lorentz symmetry is broken by the presence of the constant vector b_μ. The fields A_μ and ψ transform under infinitesimal Lorentz transformations $\delta x^\mu = \epsilon^\mu{}_\nu x^\nu$, with $\epsilon_{\mu\nu} = -\epsilon_{\nu\mu}$, as

$$\begin{aligned}
&\delta_{\rm L} A_\mu = -\epsilon^\lambda{}_\nu x^\nu \partial_\lambda A_\mu + \epsilon_\mu{}^\nu A_\nu \equiv \frac{1}{2}\epsilon^{\alpha\beta}\delta_{{\rm L}\alpha\beta} A_\mu, \\
&\delta_{\rm L}\psi = -\epsilon^\lambda{}_\nu x^\nu \partial_\lambda \psi - \frac{i}{4}\epsilon^{\mu\nu}\sigma_{\mu\nu}\psi \equiv \frac{1}{2}\epsilon^{\alpha\beta}\delta_{{\rm L}\alpha\beta}\psi,
\end{aligned} \tag{2.15}$$

where $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$.

It should be noticed that the Lorentz breaking (2.3) is not linear in the dynamical fields, therefore will be renormalized. It is however a "*soft breaking*", since its UV power-counting dimension[3] is less than 4, namely 3. According to Symanzik [19,20], a theory with soft symmetry breaking is renormalizable if the radiative corrections do not induce a breakdown of the symmetry by terms of UV power-counting dimension equal to 4 – called hard breaking terms. Concretely, according to the Weinberg theorem [27,28,24], this means that the symmetry of the theory in the asymptotic deep Euclidean region of momentum space is preserved by the radiative corrections. In order to control the Lorentz breaking and, in particular, its power-counting properties, following Symanzik [19,20], and [29] for the specific case of Lorentz breaking, we introduce an external field $\beta_\mu(x)$, of dimension 1 and transforming under Lorentz transformations according to

$$\begin{aligned}
\delta_{\rm L}\beta_\mu(x) &= -\epsilon^\lambda{}_\nu x^\nu \partial_\lambda \beta_\mu(x) + \epsilon_\mu{}^\nu\left(\beta_\nu(x) + b_\nu\right) \\
&\equiv \frac{1}{2}\epsilon^{\alpha\beta}\delta_{{\rm L}\alpha\beta}\beta_\mu(x).
\end{aligned} \tag{2.16}$$

[1] Greenberg proved that CPT invariance is necessary, but not sufficient, for Lorentz invariance [23].

[2] As we shall see, the gauge invariance properties are not spoiled by the photon mass: this is a peculiarity of the Abelian case [24]. This was studied in details for the QED in Ref. [25] using the BPHZ scheme.

[3] The UV power-counting dimensions of A_μ and ψ are $d_A = 1$ and $d_\psi = \frac{3}{2}$.