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Sommerfeld enhancement from unparticle exchange for dark matter annihilation

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ABSTRACT

We investigate the implication of unparticle exchange for the possible Sommerfeld enhancement in dark matter annihilation process. Assuming the unparticle exchange during WIMP collision, we solve the Schrödinger equation for the effective potential, and find that the Sommerfeld enhancement factor is dictated by the scale dimension of unparticle as $1/v^{3-2d_{\mathcal{U}}}$. Numerically the Sommerfeld enhancement could be $\mathcal{O}(10\text{–}10^3)$.

1. Introduction

It has been known that our universe is made of not only the stuff of the standard model (SM) with the occupancy of mere 4%, but also dark matter and dark energy with the abundance of 22% and 74%, respectively [1]. Therefore, it must be one of the most important issue to understand what the dark matter is and how to explore it by various observations. Hopefully, through high energy colliders such as the Large Hadron Collider (LHC) at CERN, we may directly observe dark matter soon. In the mean time we may also have the chance to probe dark matter indirectly by the study of the high energy cosmic-ray.

Recently, the collaborations of PAMELA [2], ATIC [3], FERMI-LAT [4], HESS [5], *etc.* have published quite astonished events in cosmic-ray measurements, in which PAMELA observes the excess in the positron flux ratio over 10 GeV till PAMELA's observational limit of about 100 GeV, whereas others measure consistent anomalies in the electron + positron flux in the 300–1000 GeV range. Inspired by the new founds at the satellite, balloon and ground-based experiments, the excess could be readily ascribed to dark matter annihilation, even though there still are the possibilities of existing new young pulsars [6]. Although dark matter *decays* could be the origin of such anomaly, however, for making the lifetime as long as $\mathcal{O}(10^{25})$ seconds, the extreme fine-tuning on the coupling of interaction [7,8] cannot be avoided. For escaping the fine-tuning problem, hereafter, we will focus on the mechanism of dark matter *annihilation* only. In addition, the candidate of dark matter in our following analysis is regarded as weakly interacting massive particle (WIMP).

Although the WIMP annihilation could be the source for the excess of cosmic-ray, however, due to low reacting rate in the annihilating process, an enhanced boost factor of a few orders of magnitude, *e.g.* Sommerfeld enhancement [9–16], has to appear during the annihilation of WIMP. Therefore, a new force carrier in the dark matter annihilation to dictate the enhancement is required. As an example, an interesting mechanism for the Sommerfeld enhancement is arisen from the light boson exchange between dark matter [13,17], where the resulted interaction is Yukawa potential and the force carrier has the significant influence in the range of Compton wavelength, denoted by αM_χ with α being the fine structure constant of the interaction and of order 10^{-2}.

In this Letter, we study another kind of new force that may be alive in an invisible sector and dictated by scale invariant. As known that an exact scale invariant stuff cannot have a definite mass unless it is zero, therefore for distinguishing from the conventional particles, Georgi named the stuff as unparticle [18,19]. Interestingly, it is found that the unparticle with the scaling dimension $d_{\mathcal{U}}$ behaves like a non-integral number $d_{\mathcal{U}}$ of invisible

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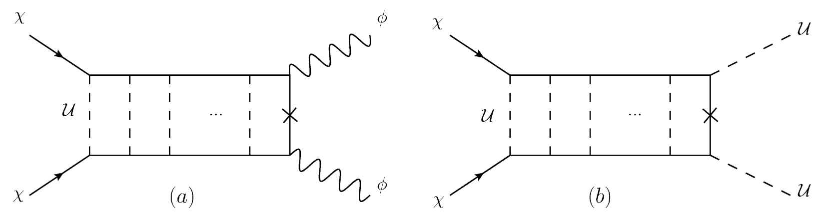

Fig. 1. Dark matter annihilation with unparticle-mediated Sommerfeld enhancement.

particles [18]. Further implications of the unparticle to colliders and low energy physics could be referred to Refs. [20–22]. In order to concentrate on the Sommerfeld effect, here we don't study the general effective interactions with unparticle, *e.g.* in our analysis, we have suppressed the interactions between unparticle and Higgs [23]. Although WIMP and unparticle both weakly couple to the SM particles, however, there is no any reason to limit that the interactions between them should be weak. Therefore, when WIMPs collide each other with small speed, we speculate that the Sommerfeld enhancement could be arisen from the unparticle exchange during the collision, sketched in Fig. 1, where the χ and $\mathcal{U}$ denotes the WIMP and unparticle, respectively. And ϕ represents the (generic light) particle that might weakly decay to SM particles and the constraints on the couplings will be controlled by current observed fluxes of cosmic rays such as electrons, positrons, antiproton, *etc*. Here, its appearance is responsible for the possible connection between dark and visible sectors, but not for the Sommerfeld enhancement. Hence, we don't further discuss the detailed couplings to the SM stuff and the related issue for this decay. Our motivation is to understand that if there exists unparticle in the invisible sector, what are its unique character on the Sommerfeld factor and the differences from Coulomb and Yukawa interactions?

The Letter is organized as follows: First, we derive the static unparticle potential, solve the associated radial Schrödinger equation with suitable boundary condition, and find the resultant formula for s-wave Sommerfeld enhancement. Then, we do the numerical analysis on the resulted Sommerfeld factor and present it as a function of involved parameters in two-dimensional contour plots. Finally we give the conclusion.

2. Unparticle potential and its Sommerfeld factor

Although the nonperturbative Sommerfeld effect can be calculated by the combined contributions of a set of ladder diagrams shown as in Fig. 1, however in the non-relativistic limit, the effect could be equivalent to solving the Schrödinger equation with an effective potential, which is arisen from the single particle exchange. In the considered mechanism, here the exchanged particle is the unparticle. Following the scheme proposed in Ref. [18], the interaction of WIMP to unparticle can be written as

$$\frac{\lambda}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \bar{\chi}^{(c)} \chi \mathcal{O}_{\mathcal{U}} \tag{1}$$

where we have assumed that the WIMP is a Dirac (or Majorana) fermion, λ is dimensionless parameter and $\Lambda_{\mathcal{U}}$ denotes the living scale of unparticle. For displaying the character of scale invariant stuff, we concentrate only on the scalar unparticle. Please note that when unparticle is realized in the framework of conformal field theories, the propagators for vector and tensor unparticles should be modified appropriately to preserve the conformal symmetry [24]. To obtain the unparticle potential in non-relativistic limit, we use the propagator of the scalar unparticle operator given by [18,19]

$$\int e^{iqx} \langle 0|T\mathcal{O}_{\mathcal{U}}(x)\mathcal{O}_{\mathcal{U}}(0)|0\rangle = i\frac{A_{d_{\mathcal{U}}}}{2\sin d_{\mathcal{U}}\pi}\frac{1}{(-q^2)^{2-d_{\mathcal{U}}}}, \quad \text{where}$$

$$A_{d_{\mathcal{U}}} = \frac{16\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}}}\frac{\Gamma(d_{\mathcal{U}}+1/2)}{\Gamma(d_{\mathcal{U}}-1)\Gamma(2d_{\mathcal{U}})}. \tag{2}$$

Combining Eqs. (1) and (2), the four-fermion effective interacting term in momentum space could be expressed by

$$\bar{\chi}^{(c)}\chi\left[\left(\frac{\lambda}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}}\right)^2\frac{A_{d_{\mathcal{U}}}}{2\sin d_{\mathcal{U}}\pi}\frac{1}{(-q^2)^{2-d_{\mathcal{U}}}}\right]\bar{\chi}^{(c)}\chi. \tag{3}$$

By Fourier transformation and with $q^0 = 0$, the static unparticle potential in WIMP interaction resulted by Eq. (3) is found by

$$V(r) = -\frac{\alpha}{r^t} \tag{4}$$

with $t = 2d_{\mathcal{U}} - 1$,

$$\alpha = \frac{\xi_\Gamma}{2\pi^{2d_{\mathcal{U}}}}\left(\frac{\lambda}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}}\right)^2,$$

$$\xi_\Gamma = \frac{\Gamma(d_{\mathcal{U}}+1/2)\Gamma(d_{\mathcal{U}}-1/2)}{\Gamma(2d_{\mathcal{U}})}. \tag{5}$$

Interestingly, we see that the power of unparticle potential is associated with the scaling dimension $d_{\mathcal{U}}$ and not an integer.

Since the unparticle potential is independent of the polar and azimuth angles in spherical coordinate system, the relevant piece for the Sommerfeld factor is the radial Schrödinger equation, read by

$$\left[\frac{d^2}{dr^2} + \left(-\frac{\ell(\ell+1)}{r^2} + \frac{2\mu\alpha}{r^t} + k^2\right)\right]u_{k\ell}(r) = 0, \tag{6}$$

where we have used the nature unit with $\hbar = c = 1$, $E = k^2/2\mu$ with μ being the reduced mass of system and $R_{k\ell}(r) = u_{k\ell}(r)/r$. Once we solve the differential equation with the proper boundary condition, the Sommerfeld effect associated with each angular momentum ℓ is obtained by [15]

$$S_\ell = \left|\frac{(2\ell+1)!!}{|k|^\ell \ell!}\frac{\partial^\ell R_{k\ell}(r)}{\partial r^\ell}\bigg|_{r=0}\right|^2. \tag{7}$$

Due to the power t of unparticle potential in r being not an integer, there is no hope to find a general close form for the solution. Nevertheless, since the required Sommerfeld factor is estimated at $r = 0$, our strategy for finding the solution is to look for a good approximation to extract the behavior of radial wave function at $r \sim 0$.

Before discussing the solution to the differential equation, first we analyze the limit on $t = 2d_{\mathcal{U}} - 1$. To avoid the crossing of the branching cut at $1/\sin d_{\mathcal{U}}\pi$ in Eq. (2), we require $1 < d_{\mathcal{U}} < 2$ ($1 < t < 3$). In order to further understand whether the upper limit of t can be bounded by the boundary condition when solving differential equation, we examine the case with $t = 2$, in which the potential could be exactly solved at $r \to 0$. Hence, by taking $t = 2$ and $u_{k\ell} \sim r^\sigma$ and considering $r \to 0$, from Eq. (6) we get