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Shell evolution and nuclear forces

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ABSTRACT

We present a quantitative study of the role played by different components characterizing the nucleon–nucleon interaction in the evolution of the nuclear shell structure. It is based on the spin–tensor decomposition of an effective two-body shell-model interaction and the subsequent study of effective single-particle energy variations in a series of isotopes or isotones. The technique allows to separate unambiguously contributions of the central, vector and tensor components of the realistic effective interaction. We show that while the global variation of the single-particle energies is due to the central component of the effective interaction, the characteristic behavior of spin–orbit partners, noticed recently, is mainly due to its tensor part. Based on the analysis of a well-fitted realistic interaction in the *sdpf* shell-model space, we analyze in detail the role played by the different terms in the formation and/or disappearance of $N = 16$, $N = 20$ and $N = 28$ shell gaps in neutron-rich nuclei.

The shell structure is a common feature of finite quantum systems. Amongst them, atomic nuclei represent unique objects characterized by the appearance of a specific shell structure. In particular, the magic numbers which correspond to the shell closures, will change depending on the N/Z ratio, i.e. when we move from nuclei in the vicinity of the β-stability line towards the particle driplines. This has attracted a lot of attention nowadays because an increasing number of nuclei far from stability have become accessible experimentally (e.g., [1] and references therein). The hope to reach even more exotic nuclei demands for an improved modelization, i.e. in the context of nuclear astrophysics. Since the underlying shell structure determines nuclear properties in a major way, changes of nuclear shell closures and the mechanisms responsible for that should be much better understood.

Recently, the role of different components of the nucleon–nucleon (NN) interaction in the evolution of the shell structure has been actively discussed. Based on the analysis of the origin of a shell closure at $N = 16$, Otsuka et al. [2] have suggested that a central spin–isospin-exchange term, $f(r)(\vec{\sigma}\cdot\vec{\sigma})(\vec{\tau}\cdot\vec{\tau})$ of the NN interaction plays a decisive role in the shell formation.

However, from a systematic analysis of heavier nuclei, another conjecture has been put forward, namely, the dominant role played by the tensor force [3]. The evidence is based on the comparison of the position of experimental one-particle or one-hole states in nuclei adjacent to semi-magic configurations with the so-called effective single-particle energies (ESPE's). Within the shell-model framework, the latter ESPE's are defined [4] as one-nucleon separation energies for an occupied orbital (or extra binding gained by the addition of a nucleon to an unoccupied orbital) evaluated from a Hamiltonian containing nucleon single-particle energies (the bare single-particle energies with respect to a closed-shell core) plus the monopole part of the two-body residual interaction [5,6], i.e.

$$\hat{H}_{mon} = \sum_{j,\rho} \epsilon_j^{\rho} \hat{n}_j^{\rho} + \sum_{j,j',\rho,\rho'} V_{jj'}^{\rho\rho'} \frac{\hat{n}_j^{\rho}(\hat{n}_{j'}^{\rho'} - \delta_{jj'}\delta_{\rho\rho'})}{(1+\delta_{jj'}\delta_{\rho\rho'})}, \quad (1)$$

where j denotes a set of single-particle quantum numbers (nlj) and ρ refers to a proton (π) or to a neutron (ν), $\hat{n}_j^{\rho}$ are particle-number operators. $V_{jj'}^{\rho\rho'}$ are centroids of the two-body interaction defined as [5–7]

$$V_{jj'}^{\rho\rho'} = \frac{\sum_J \langle j_\rho j'_{\rho'}|V|j_\rho j'_{\rho'}\rangle_{JM}(2J+1)(1+(-1)^J\delta_{jj'}\delta_{\rho\rho'})}{(2j_\rho+1)(2j'_{\rho'}+1-\delta_{jj'}\delta_{\rho\rho'})}, \quad (2)$$

where the total angular momentum of a two-body state J runs over all possible values.

The monopole Hamiltonian represents a spherical mean field extracted from the interacting shell model. Its spherical single-particle states, or ESPE's, provide an important ingredient for the

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formation of shells and interplay between spherical configurations and deformation in nuclei. Large shell gaps obtained from a monopole Hamiltonian are a prerequisite to obtain certain magic numbers. A reduction of the spherical shell gaps may lead to formation of a deformed ground state, if the correlation energy of a given excited configuration and a decrease in the monopole part are large enough to make such an intruder excitation energetically favorable.

For example, the ESPE of the $\nu 0f_{7/2}$ orbital at $Z = 8$, $N = 20$ is the difference between total energy obtained, using Eq. (1), for ^{28}O in its ground state and ^{29}O with an extra neutron in the $0f_{7/2}$ state assuming normal filling of the orbitals (normal filling is used throughout this work). Considering a series of isotopes or isotones, it is clear that ESPE's will experience a shift provided by the monopole part of the proton–neutron matrix elements, mainly. The bigger the overlap of the proton and neutron radial wave functions and the higher the j-values of the orbitals considered will lead, in general, to more drastic changes. In the present study we take into account the mass dependence of the two-body matrix elements of the effective interaction according to the rule: $V(A) = (A_{core}/A)^{1/3} V(A_{core})$.

From the analysis of the experimental data and the ESPE's it has been noticed [3,8] that systematically

$$\left|V^{\pi\nu}_{j_> j'_<}\right| > \left|V^{\pi\nu}_{j_> j'_>}\right|, \qquad \left|V^{\pi\nu}_{j_< j'_>}\right| > \left|V^{\pi\nu}_{j_< j'_<}\right|, \tag{3}$$

where $j_> = l + 1/2$ and $j_< = l - 1/2$ are proton orbitals and $j'_> = l' + 1/2$ and $j'_< = l' - 1/2$ are neutron orbitals. Thus, an extra attraction is manifested between generalized spin–orbit partners (proton $j = l + 1/2$ and neutron $j' = l' - 1/2$ with $l \neq l'$ or vice versa).

This remarkable property is in line with the analytic relation valid for a pure tensor force [3], i.e. using the above notation, $(2j_> + 1)V^{\pi\nu}_{j_> j'} + (2j_< + 1)V^{\pi\nu}_{j_< j'} = 0$. To strengthen this idea, Otsuka et al. [3] have compared changes of the ESPE's in Ca, Ni and Sb isotopes, as due to the tensor force only and estimating its strength as resulting from a $(\pi + \rho)$-exchange potential with a cut-off at 0.7 fm, with available experimental data.

This work has stimulated a large number of investigations using mean-field approaches [9–23]. It is worth noting that phenomenological interactions, such as Skyrme and Gogny force, most frequently used in mean-field calculations, did not include a tensor term [24]. Provided its importance, a tensor term should be introduced and the parameters re-adjusted, what up to now, is not satisfactorily reached yet (see, e.g. Refs. [23,25]).

However, importance of the tensor force within the shell model [3,8] is mainly demonstrated in an empirical way. It is evident, that the choice of the particular cut-off that was used to fix the strength of the tensor force component plays a crucial role in obtaining quantitative result for shifts in the ESPE's as presented in Fig. 4 of Ref. [3]. It is also well known that the NN interaction is subjected to a strong renormalization before it can be handled as an effective interaction in many-body calculations within a restricted model space [26]. It is not straightforward to trace how the tensor component will become renormalized amongst the other terms contributing to the NN interaction. Moreover, many shell-model interactions having high descriptive and predictive power were obtained by a χ^2-fit of two-body matrix elements to reproduce known experimental levels for a wide range of nuclei studied within a given model space (e.g. [27,28]). Even the effective interactions, maximally preserving their microscopic origin (based on a G-matrix), need further phenomenological correction (see e.g., [6,29]). There is strong indication that inclusion of three-nucleon forces can heal the microscopically derived effective interaction, in particular, improve its monopole part (see Ref. [30] and references therein for ab-initio studies). However, there are still no systematic calculations available up to date for many-nucleon systems either within the shell model, or within the density-functional approach. This is why the present study of the two-nucleon case is of interest.

In spite of the indirect evidence at a two-body level [3], up to now, the role played by the tensor force is not well determined. For example, recent shell-model studies based on large-scale calculations using a realistic effective interaction in the heavy Sn nuclei region [31] conclude on the absence of a characteristic effect expected to result from a tensor force component.

In this Letter we present a quantitative study of the role played by different components of the *effective interaction*. It is based on the spin–tensor decomposition of the two-body interaction, which involves tensors of rank 0, 1 and 2 in spin and configuration space. The procedure allows to separate the central, vector and tensor parts of the effective interaction. The monopole properties of each component can be studied separately, elucidating unambiguously its role in the shell evolution. The method has already been applied in a similar context [32,33], however, the authors used different effective interactions in smaller model spaces, concluding on a second-order tensor effect only. Contrary to these results, we put into evidence an important first-order tensor effect in the present study.

A spin–tensor decomposition of the two-particle interaction has been known for many years [34–40]. In a given model space, a complete set of two-body matrix elements determines the properties of nuclei ranging within this space. For spin 1/2 fermions (nucleons), one can construct from their spin operators a complete set of linear operators in a two-particle spin space:

$$S^{(0)} = 1, \qquad S_2^{(0)} = [\sigma_1 \times \sigma_2]^{(0)}, \qquad S_3^{(1)} = \sigma_1 + \sigma_2,$$
$$S_4^{(2)} = [\sigma_1 \times \sigma_2]^{(2)}, \qquad S_5^{(1)} = [\sigma_1 \times \sigma_2]^{(1)}, \qquad S_6^{(1)} = \sigma_1 - \sigma_2.$$

By coupling the spin tensor operators with the corresponding rank tensors in the configuration space one can construct scalar interaction terms. The most general two-body interaction can then be written as

$$V(1,2) \equiv V = \sum_{k=0,1,2} \left(S^{(k)} \cdot Q^{(k)}\right) = \sum_{k=0,1,2} V^{(k)}. \tag{4}$$

Here, $V^{(0)}$ and $V^{(2)}$ represent the central and tensor parts of the effective NN interaction. The $V^{(k=1)}$ term contains the so-called symmetric ($S^{(1)}_{i=3}$) and antisymmetric ($S^{(1)}_{i=5,6}$) spin–orbit operators [37], which we will denote as LS and ALS, respectively. To obtain the matrix elements for the different multipole components in jj coupling, first, one transforms two-body matrix elements between normalized and antisymmetrized states from jj coupling to LS coupling in the standard way. The LS-coupled matrix elements of $V^{(k)}$ can be calculated from the LS coupled matrix elements of V as

$$\begin{aligned}
&\left\langle (ab) : LS, JMTM_T \middle| V^{(k)} \middle| (cd) : L'S', JMTM_T \right\rangle \\
&\quad = (2k+1)(-1)^J \begin{Bmatrix} L & S & J \\ S' & L' & k \end{Bmatrix} \\
&\qquad \times \sum_{J'} (-1)^{J'} (2J'+1) \begin{Bmatrix} L & S & J' \\ S' & L' & k \end{Bmatrix} \\
&\qquad \times \left\langle (ab) : LS, J'MTM_T \middle| V \middle| (cd) : L'S', J'MTM_T \right\rangle,
\end{aligned} \tag{5}$$

where $a \equiv (n_a, l_a)$. Finally, starting from the LS coupled matrix elements of $V^{(k)}$, for each k, we arrive at a set of jj coupled matrix elements to be used for further investigation. It is important to