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α-particle condensate states in ^{16}O

S. Ohkubo [a,b,*], Y. Hirabayashi [c]

[a] Department of Applied Science and Environment, Kochi Women's University, Kochi 780-8515, Japan
[b] Research Center for Nuclear Physics, Osaka University, Ibaraki, Osaka 567-0047, Japan
[c] Information Initiative Center, Hokkaido University, Sapporo 060-0811, Japan

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ABSTRACT

The existence of a rotational band with the $\alpha + {}^{12}\mathrm{C}(0_2^+)$ cluster structure, in which three α particles in $^{12}\mathrm{C}(0_2^+)$ are locally condensed, is demonstrated near the four-α threshold of ^{16}O in agreement with experiment. This is achieved by studying structure and scattering for the $\alpha + {}^{12}\mathrm{C}(0_2^+)$ system in a unified way. A drastic reduction (quenching) of the moment of the inertia of the 0^+ state at 15.1 MeV just above the four-α threshold in ^{16}O suggests that it could be a candidate for the superfluid state in α-particle condensation.

α-particle condensation has been paid much attention in light nuclei. Up to now the 0_2^+ (7.65 MeV) Hoyle state of ^{12}C has been considered to be a candidate for a three α-particle condensate with a dilute density distribution [1]. The elaborate microscopic α-cluster model wave functions of the Hoyle state by Uegaki et al. [2] and Kamimura et al. [3], which reproduce many experimental data involving the Hoyle state are almost completely equivalent to the condensate wave function [4]. Many theoretical studies [1,5–9] support the dilute property of the Hoyle state. However, the typical physical modes such as superfluidity and/or a quantum vortex have not been observed.

The fundamental question that may arise is that superfluidity due to α-particle condensation is difficult to observe in nuclear systems like ^{12}C and ^{16}O, while superfluidity has clearly been observed in bulk systems such as He II and ^{3}He liquids. We note, however, that recent studies of parahydrogen [10] and He clusters [11] show that superfluidity can be observed in small systems with 10 or less particles. This encourages us to study the superfluidity of a small number of α particles in strong-interaction systems composed of protons and neutrons. One of the most convincing ways to demonstrate the existence of α-particle condensation is to show superfluidity of the system. It has been shown [12] theoretically and experimentally that a reduction (quenching) of the moment of inertia from the rigid-body value is characteristic to the superfluid behavior of a dilute Bose gas due to the occurrence of Bose–Einstein condensation. This reduction is also observed for liquid helium (the Hess–Fairbank effect) [13]. Path Integral Monte Carlo (PIMC) is a simulation which makes use of this reduction of the moment of inertia from the classical moment of inertia in calculating the superfluid density, for example, of quantum fluids in confined geometries [14].

In studies of α-particle condensation in ^{16}O, so far mostly the 0^+ state has been discussed [1,15,16]. Tohsaki et al. [1] thought that the 0^+ state at $E_x = 14.0$ MeV located below the four-α threshold energy is an α-particle condensate. Wakasa et al. and Funaki et al. [15] suggested that a newly observed 0^+ state at $E_x = 13.6$ MeV is an α-particle condensate. On the other hand, very recently Funaki et al. [16] performed a semi-microscopic four-α cluster model calculation in the OCM (Orthogonality Condition Model) and concluded that the 0^+ state at $E_x = 15.1$ MeV can probably be an α-particle condensate. These states were shown to have dilute density distributions in the frame of the *bound state approximation*. A dilute density distribution is not equivalent to α-particle condensation and no clear experimental evidence for α-particle condensation such as superfluidity and/or vortex excitation has been presented. α-particle condensation in ^{16}O has been controversial. We note that in experiment in the high excitation energy region above the four-α threshold well-developed α-cluster states (2^+, 4^+ and 6^+) have been observed in the $^8\mathrm{Be} + {}^8\mathrm{Be}$ and $\alpha + {}^{12}\mathrm{C}(0_2^+)$ decay channels of ^{16}O [17–20]. These states have been considered as linear chain states of four α particles for many years

* Corresponding author at: Department of Applied Science and Environment, Kochi Women's University, Kochi 780-8515, Japan.
E-mail address: shigeo@cc.kochi-wu.ac.jp (S. Ohkubo).

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[17]. We think that it is important to understand not only the resonant 0^+ state but also the other resonant higher spin states built on it in a unified way in the context of α-particle condensation. It is also important to study the α decaying resonant states above the α threshold by solving the *scattering equation* correctly.

In this Letter, from a unified description of structure and scattering for the $\alpha + {}^{12}$C system, we show that a rotational band with the $\alpha + {}^{12}\mathrm{C}(0_2^+)$ cluster structure is predicted near the four-α threshold in ^{16}O. The above α-cluster states observed in the four-α decay channel [17–20] can be understood consistently as fragmented states of the band. It is shown that the observed 0^+ state at $E_x = 15.1$ MeV just above the four-α threshold, which we interpret to be fragmented from the broad band head 0^+ state, has a reduced moment of inertia compared to the well-developed $\alpha + {}^{12}\mathrm{C}(0_2^+)$ cluster structure. It is suggested that this 0^+ state could be a candidate for the superfluid state in ^{16}O in four α-particle condensation.

In α-cluster studies a unified description of structure and scattering has been powerful because the interaction potential can be uniquely determined from rainbow scattering [21]. In fact, a unified study of structure and scattering of the $\alpha + {}^{40}$Ca system could disentangle a long-standing controversy about the α-cluster structure in ^{44}Ti [22,23]. This unification may be extended to the case where a target nucleus is excited because the nuclear rainbow and prerainbow appear also in inelastic scattering and the mechanism can be understood in a similar way to elastic scattering [24,21].

We study the elastic and inelastic $\alpha + {}^{12}$C scattering, and states with the $\alpha + {}^{12}$C cluster structure in a unified way using a double folding (DF) model in the coupled channel method by taking into account the excited states of ^{12}C, which has been shown to be successful in describing α and ^{3}He scattering at the high and low energy regions [8,9]. The diagonal and coupling potentials of the DF model for the $\alpha - {}^{12}$C system are calculated as follows:

$$V_{ij}(\mathbf{R}) = \int \rho_{00}^{(\alpha)}(\mathbf{r}_1)\rho_{ij}^{(\mathrm{C})}(\mathbf{r}_2)v_{\mathrm{NN}}(E, \rho, \mathbf{r}_1 + \mathbf{R} - \mathbf{r}_2)\,d\mathbf{r}_1\,d\mathbf{r}_2, \qquad (1)$$

where $\rho_{00}^{(\alpha)}(\mathbf{r})$ is the ground state density of the α particle, while v_{NN} denotes the density dependent M3Y effective interaction (DDM3Y) [25] usually used in the DF model. $\rho_{ij}^{(\mathrm{C})}(\mathbf{r})$ represents the diagonal ($i = j$) or transition ($i \neq j$) nucleon density of ^{12}C which is calculated using the resonating group method by Kamimura et al. [3]. In the calculation of densities of ^{12}C, the shell-like structure of the ground state 0_1^+ and the well-developed α-cluster structure of the Hoyle 0_2^+ state are simultaneously well reproduced. These wave functions have been checked against many experimental data including charge form factors, electric transition probabilities and reactions involving excitation to the 0_2^+ state [3]. The wave function for the Hoyle 0_2^+ state is very close to the α-condensate wave function. In the calculations the normalization factor $N_R + iN_I$ is introduced for the $\alpha - {}^{12}$C DF potential. The imaginary part takes into account absorption phenomenologically.

We have shown in Ref. [8] that elastic and inelastic $\alpha + {}^{12}$C rainbow scattering from the Hoyle state in the high energy region ($E_\alpha = 139$, 166 and 172.5 MeV) can be well described in the DF model with $N_R = 1.23$–1.26. We extend this analysis to the lowest energy $E_\alpha = 18.5$ MeV where experimental data of both inelastic scattering from the Hoyle state and elastic scattering are available [26]. We analyze the angular distributions in the coupled channel calculations including most of the channels of ^{12}C open at this energy, that is, g.s., 2_1^+(4.44 MeV), 3^-(9.65 MeV), 0_2^+(7.65 MeV) and 2_2^+(10.3 MeV). The absorption due to the coupling to all the other open channels, i.e., $p + {}^{15}$N, $n + {}^{15}$O and $d + {}^{14}$N channels, is introduced as a small imaginary potential with $N_I = 0.045$. The calculated angular distributions with $N_R = 1.398$ are compared with the experimental data in Fig. 1. The volume integral per nucleon pair for the real potential is $J_V = 427.3$ MeV fm^3 for the g.s. channel. The characteristic oscillations at the backward hemisphere of the experimental data are well reproduced by the calculation. The backward rise in elastic scattering, ALAS (Anomalous Large Angle Scattering), is caused by the internal waves [27], which penetrate deep into the internal region of the potential. The ALAS seen for inelastic scattering from the Hoyle state is also understood similarly in terms of internal waves [24]. These results suggest that the diagonal and coupling potentials in Eq. (1) work in the α-cluster structure study in the low energy region near the $\alpha + {}^{12}\mathrm{C}(0_2^+)$ threshold.

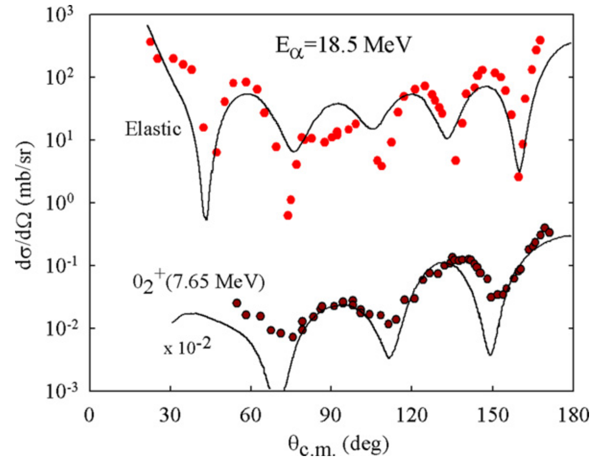

Fig. 1. Calculated angular distributions (solid lines) in elastic and inelastic $\alpha + {}^{12}$C scattering at $E_\alpha = 18.5$ MeV are compared with the experimental data (points) [26].

We study the resonant α-cluster structure of ^{16}O by solving the coupled channel scattering equations with use of the real part of the double folding potential. We take $N_R = 1.34$, which is chosen so that the calculated energy of the band head 1^- state of the $K = 0_1^-$, which has a well-developed $\alpha + {}^{12}$C(g.s.) structure, corresponds well with experimental energy. The volume integral J_V and rms radius of the potential are 409.6 MeV fm^3 and 3.44 fm for the g.s. channel, and 510.3 MeV fm^3 and 4.29 fm for the $\alpha - {}^{12}\mathrm{C}(0_2^+)$ channel, respectively. The energy-dependence of N_R, that is, $N_R = 1.23$ at $E_\alpha = 139$ MeV [8], $N_R = 1.389$ at $E_\alpha = 18.5$ MeV and $N_R = 1.34$ at $E_\alpha \simeq 0$, which increases toward the lower energy as the incident energy decreases and again increases toward zero energy, seems to be consistent with the threshold anomaly.

In Fig. 2 the calculated states are shown in comparison with the experimental data. The calculation reproduces the parity-doublet $K = 0_1^+$ band and $K = 0_1^-$ band with the $\alpha + {}^{12}$C(g.s.) structure well. A resonance energy E_r is defined as the energy where the elastic channel phase shift δ_J passes $\pi/2$. (The 0^+ state of the $K = 0_1^+$ band is well below the barrier and calculated in the single channel bound state approximation.) In Table 1 the calculated excitation energy of resonant states of the parity-doublet $K = 0_1^+$ and $K = 0_1^-$ bands and its widths derived from $\Gamma_\alpha = 2/\frac{d\delta_J}{dE}|_{E=E_r}$ are displayed in comparison with the experimental data [28]. Because the calculated widths are strongly dependent on excitation energy, the dimensionless reduced widths θ_α^2, which are more physically related to the degree of α-clustering, are also shown at the three channel radii $a = 5.2$, 5.6 and 6.0 fm. θ_α^2 is defined as $\Gamma_\alpha = 2P(a)\gamma^2(a)$ with $P(a)$ being the penetration factor, $\gamma^2(a) = \theta_\alpha^2(a)\gamma_W^2(a)$ and the Wigner limit $\gamma_W^2(a) = 3\hbar^2/2\mu a$ with μ denoting the reduced mass. The agreement of the calculated θ_α^2 with experiment is good especially for the $K = 0_1^-$ band states. The