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Probing high-density behavior of symmetry energy from pion emission in heavy-ion collisions

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ABSTRACT

Within the framework of the improved isospin dependent quantum molecular dynamics (ImIQMD) model, the emission of pion in heavy-ion collisions in the region 1 A GeV as a probe of nuclear symmetry energy at supra-saturation densities is investigated systematically, in which the pion is considered to be mainly produced by the decay of resonances $\Delta(1232)$ and $N^*(1440)$. The total pion multiplicities and the π^-/π^+ yields are calculated for selected Skyrme parameters SkP, SLy6, Ska and SIII, and also for the cases of different stiffness of symmetry energy with the parameter SLy6. Preliminary results compared with the measured data by the FOPI Collaboration favor a hard symmetry energy of the potential term proportional to $(\rho/\rho_0)^{\gamma_s}$ with $\gamma_s = 2$.

Heavy-ion collisions induced by radioactive beam at intermediate energies play a significant role to extract the information of nuclear equation of state (EoS) of isospin asymmetric nuclear matter under extreme conditions. Besides nucleonic observables such as rapidity distribution and flow of free nucleons and light clusters (such as deuteron, triton and alpha, etc.), also mesons emitted from the reaction zone can be probes of the hot and dense nuclear matter. The energy per nucleon in the isospin asymmetric nuclear matter is usually expressed as $E(\rho,\delta) = E(\rho,\delta=0) + E_{\text{sym}}(\rho)\delta^2 + \mathrm{O}(\delta^2)$ in terms of baryon density $\rho = \rho_n + \rho_p$, relative neutron excess $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$, energy per nucleon in a symmetric nuclear matter $E(\rho,\delta=0)$ and bulk nuclear symmetry energy $E_{\text{sym}} = \frac{1}{2}\frac{\partial^2 E(\rho,\delta)}{\partial\delta^2}|_{\delta=0}$. In general, two different forms have been predicted by some microscopical or phenomenological many-body approaches. One is the symmetry energy increases monotonically with density, and the other is the symmetry energy increases initially up to a supra-saturation density and then decreases at higher densities. Based on recent analysis of experimental data associated with transport models, a symmetry energy of the form $E_{\text{sym}}(\rho) \approx 31.6(\rho/\rho_0)^{\gamma}$ MeV with $\gamma = 0.69$–1.05 was extracted for densities between $0.1\rho_0$ and $1.2\rho_0$ [1,2]. The symmetry energy at supra-saturation densities can be investigated by analyzing isospin sensitive observables in theoretically, such as the neutron/proton ratio of emitted nucleons, π^-/π^+, Σ^-/Σ^+ and K^0/K^+ [2]. Recently, a very soft symmetry energy at supra-saturation densities was pointed out by fitting the FOPI data [3] using IBUU04 model [4]. With the establishment of high-energy radioactive beam facilities in the world, such as the CSR (IMP in Lanzhou, China), FAIR (GSI in Darmstadt, Germany), RIKEN (Japan), SPIRAL2 (GANIL in Caen, France) and FRIB (MSU, USA) [2], the high-density behavior of the symmetry energy can be studied more detail experimentally in the near future. The emission of pion in heavy-ion collisions in the region 1 A GeV is especially sensitive as a probe of symmetry energy at supra-saturation densities. Further investigations of the pion emissions in the 1 A GeV region are still necessary by improving transport models or developing some new approaches. The ImIQMD model has been successfully applied to treat heavy-ion fusion reactions near Coulomb barrier [5–7]. Recently, Zhang et al. analyzed the neutron–proton spectral double ratios to extract the symmetry energy per nucleon at sub-saturation density with a similar model [8]. To investigate the pion emission, we further include the inelastic channels in nucleon–nucleon collisions.

In the ImIQMD model, the time evolutions of the baryons and pions in the system under the self-consistently generated mean-field are governed by Hamilton's equations of motion, which read as

$$\dot{\mathbf{p}}_i = -\frac{\partial H}{\partial \mathbf{r}_i}, \qquad \dot{\mathbf{r}}_i = \frac{\partial H}{\partial \mathbf{p}_i}. \tag{1}$$

Here we omit the shell correction part in the Hamiltonian H as described in Ref. [6]. The Hamiltonian of baryons consists of the relativistic energy, the effective interaction potential and the momentum dependent part as follows:

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Table 1
ImIQMD parameters and properties of symmetric nuclear matter for Skyrme effective interactions after the inclusion of the momentum dependent interaction with parameters $\delta = 1.57$ MeV and $\epsilon = 500$ c^2/GeV^2.

Parameters	SkM*	Ska	SIII	SVI	SkP	RATP	SLy6
α (MeV)	−325.1	−179.3	−128.1	−123.0	−357.7	−250.3	−296.7
β (MeV)	238.3	71.9	42.2	51.6	286.3	149.6	199.3
γ	1.14	1.35	2.14	2.14	1.15	1.19	1.14
g_{sur} (MeV fm^2)	21.8	26.5	18.3	14.1	19.5	25.6	22.9
g_{sur}^{iso} (MeV fm^2)	−5.5	−7.9	−4.9	−3.0	−11.3	0.0	−2.7
g_τ (MeV)	5.9	13.9	6.4	1.1	0.0	11.0	9.9
C_{sym} (MeV)	30.1	33.0	28.2	27.0	30.9	29.3	32.0
a_{sym} (MeV)	62.4	29.8	38.9	42.9	94.0	79.3	130.6
b_{sym} (MeV)	−38.3	−5.9	−18.4	−22.0	−63.5	−58.2	−123.7
c_{sym} (MeV)	−6.4	−3.0	−3.8	−5.5	−13.0	−4.1	12.8
ρ_∞ (fm^{-3})	0.16	0.155	0.145	0.144	0.162	0.16	0.16
m^*_∞/m	0.639	0.51	0.62	0.73	0.77	0.56	0.57
K_∞ (MeV)	215	262	353	366	200	239	230

$$H_B = \sum_i \sqrt{\mathbf{p}_i^2 + m_i^2} + U_{\mathrm{int}} + U_{\mathrm{mom}}. \tag{2}$$

Here the $\mathbf{p}_i$ and m_i represent the momentum and the mass of the baryons.

The effective interaction potential is composed of the Coulomb interaction and the local interaction

$$U_{\mathrm{int}} = U_{\mathrm{Coul}} + U_{\mathrm{loc}}. \tag{3}$$

The Coulomb interaction potential is written as

$$U_{\mathrm{Coul}} = \frac{1}{2} \sum_{i,j,j\neq i} \frac{e_i e_j}{r_{ij}} \operatorname{erf}(r_{ij}/\sqrt{4L}) \tag{4}$$

where the e_j is the charged number including protons and charged resonances. The $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$ is the relative distance of two charged particles.

The local interaction potential is derived directly from the Skyrme energy-density functional and expressed as

$$U_{\mathrm{loc}} = \int V_{\mathrm{loc}}\big(\rho(\mathbf{r})\big)\, d\mathbf{r}. \tag{5}$$

The local potential energy-density functional reads

$$\begin{aligned} V_{\mathrm{loc}}(\rho) = {} & \frac{\alpha}{2}\frac{\rho^2}{\rho_0} + \frac{\beta}{1+\gamma}\frac{\rho^{1+\gamma}}{\rho_0^\gamma} + \frac{g_{\mathrm{sur}}}{2\rho_0}(\nabla\rho)^2 \\ & + \frac{g_{\mathrm{sur}}^{\mathrm{iso}}}{2\rho_0}\big[\nabla(\rho_n - \rho_p)\big]^2 \\ & + \left(a_{\mathrm{sym}}\frac{\rho^2}{\rho_0} + b_{\mathrm{sym}}\frac{\rho^{1+\gamma}}{\rho_0^\gamma} + c_{\mathrm{sym}}\frac{\rho^{8/3}}{\rho_0^{5/3}}\right)\delta^2 \\ & + g_\tau \rho^{8/3}/\rho_0^{5/3}, \end{aligned} \tag{6}$$

where the ρ_n, ρ_p and $\rho = \rho_n + \rho_p$ are the neutron, proton and total densities, respectively, and the $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$ is the isospin asymmetry. The coefficients α, β, γ, g_{sur}, $g_{\mathrm{sur}}^{\mathrm{iso}}$, g_τ are related to the Skyrme parameters t_0, t_1, t_2, t_3 and x_0, x_1, x_2, x_3 [6]. The parameters of the potential part in the symmetry energy term are also derived directly from Skyrme energy-density parameters as

$$\begin{aligned} a_{\mathrm{sym}} &= -\frac{1}{8}(2x_0+1)t_0\rho_0, \qquad b_{\mathrm{sym}} = -\frac{1}{48}(2x_3+1)t_3\rho_0^\gamma, \\ c_{\mathrm{sym}} &= -\frac{1}{24}\left(\frac{3}{2}\pi^2\right)^{2/3}\rho_0^{5/3}\big[3t_1x_1 - t_2(5x_2+4)\big]. \end{aligned} \tag{7}$$

The momentum dependent term in the Hamiltonian is the same of the form in Ref. [9] and expressed as

$$U_{\mathrm{mom}} = \frac{\delta}{2} \sum_{i,j,j\neq i} \frac{\rho_{ij}}{\rho_0} \big[\ln\big(\epsilon(\mathbf{p}_i - \mathbf{p}_j)^2 + 1\big)\big]^2, \tag{8}$$

with

$$\rho_{ij} = \frac{1}{(4\pi L)^{3/2}} \exp\left[-\frac{(\mathbf{r}_i - \mathbf{r}_j)^2}{4L}\right], \tag{9}$$

which does not distinguish between protons and neutrons. Here the L denotes the square of the pocket-wave width, which is dependent on the mass number of the nucleus. The parameters δ and ϵ were determined by fitting the real part of the proton–nucleus optical potential as a function of incident energy.

In Table 1 we list the ImIQMD parameters related to several typical Skyrme forces after including the momentum dependent interaction. The parameters α, β and γ are redetermined in order to reproduce the binding energy ($E_B = -16$ MeV) of symmetric nuclear matter at saturation density ρ_0 and to satisfy the relation $\frac{\partial E/A}{\partial \rho}|_{\rho=\rho_0} = 0$ for a given incompressibility. Combined Eq. (7) with the kinetic energy part, the symmetry energy per nucleon in the ImIQMD model is given by

$$\begin{aligned} E_{\mathrm{sym}}(\rho) = {} & \frac{1}{3}\frac{\hbar^2}{2m}\left(\frac{3}{2}\pi^2\rho\right)^{2/3} + a_{\mathrm{sym}}\frac{\rho}{\rho_0} \\ & + b_{\mathrm{sym}}\left(\frac{\rho}{\rho_0}\right)^\gamma + c_{\mathrm{sym}}\left(\frac{\rho}{\rho_0}\right)^{5/3}. \end{aligned} \tag{10}$$

More clearly compared with other transport models, the symmetry energy can be expressed as

$$E_{\mathrm{sym}}(\rho) = \frac{1}{3}\frac{\hbar^2}{2m}\left(\frac{3}{2}\pi^2\rho\right)^{2/3} + \frac{1}{2}C_{\mathrm{sym}}\left(\frac{\rho}{\rho_0}\right)^{\gamma_s}. \tag{11}$$

The value $\gamma_s = 1$ is used in IQMD model [10,11]. In Fig. 1 we show a comparison of the energy per nucleon in symmetric nuclear matter with and without the momentum dependent potentials in the left panel and the nuclear symmetry energy in the right panel for different cases of Skyrme forces SkP, Sly6, Ska and SIII from Eq. (10), $\gamma_s = 0.5$ (soft) and 2 (hard) with $C_{\mathrm{sym}} = 32$ MeV in Eq. (11), and also compared with the form $E_{\mathrm{sym}} = 31.6(\rho/\rho_0)^\mu$ MeV ($\mu = 0.5$ and $\mu = 2$) [1].

Analogously to baryons, the Hamiltonian of pions is represented as

$$H_\pi = \sum_{i=1}^{N_\pi}\left(\sqrt{\mathbf{p}_i^2 + m_\pi^2} + V_i^{\mathrm{Coul}}\right), \tag{12}$$

where the $\mathbf{p}_i$ and m_π represent the momentum and the mass of the pions. The Coulomb interaction is given by