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Topological black holes in Gauss–Bonnet gravity with conformally invariant Maxwell source

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ABSTRACT

In this Letter, we present a class of rotating solutions in Gauss–Bonnet gravity in the presence of cosmological constant and conformally invariant Maxwell field and study the effects of the nonlinearity of the Maxwell source on the properties of the spacetimes. These solutions may be interpret as black brane solutions with inner and outer event horizons provide that the mass parameter m is greater than an extremal value m_{ext}, an extreme black brane if $m = m_{\mathrm{ext}}$ and a naked singularity otherwise. We investigate the conserved and thermodynamics quantities for asymptotically flat and asymptotically *AdS* with flat horizon. We also show that the conserved and thermodynamic quantities of these solutions satisfy the first law of thermodynamics.

1. Introduction

The presence of higher curvature terms can be seen in many theories such as renormalization of quantum field theory in curved spacetime [1], or in construction of low energy effective action of string theory [2] and so on. These examples motivate one to consider the more general class of gravitational action

$$I_G = \frac{1}{16\pi}\int d^{n+1}x\sqrt{-g}\mathcal{F}\big(R, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\big),$$

where R, $R_{\mu\nu}$ and $R_{\mu\nu\rho\sigma}$ are Ricci scalar, Ricci and Riemann tensor respectively. Among the higher curvature gravity theories, the so-called Lovelock gravity is quite special, whose Lagrangian consist of the dimensionally extended Euler densities. This Lagrangian is obtained by Lovelock as he tried to calculate the most general tensor that satisfies properties of Einstein's tensor in higher dimensions [3]. Since the Lovelock tensor does not contain derivatives of metrics of order higher than second, the quantization of linearized Lovelock theory is free of ghosts [4]. The gravitational action of Lovelock theory can be written as [3]

$$I_G = \int d^dx\sqrt{-g}\sum_{k=0}^{[d/2]}\alpha_k\mathcal{L}_k, \tag{1}$$

where $[z]$ denotes integer part of z, α_k is an arbitrary constant and $\mathcal{L}_k$ is the Euler density of a $2k$-dimensional manifold,

$$\mathcal{L}_k = \frac{1}{2^k}\delta^{\mu_1\nu_1\cdots\mu_k\nu_k}_{\rho_1\sigma_1\cdots\rho_k\sigma_k}R_{\mu_1\nu_1}{}^{\rho_1\sigma_1}\cdots R_{\mu_k\nu_k}{}^{\rho_k\sigma_k}. \tag{2}$$

In Eq. (2) $\delta^{\mu_1\nu_1\cdots\mu_k\nu_k}_{\rho_1\sigma_1\cdots\rho_k\sigma_k}$ is the generalized totally anti-symmetric Kronecker delta. It is worthwhile to mention that in d dimensions, all terms for which $k > [d/2]$ are identically equal to zero, and the term $k = d/2$ is a topological term. So, only terms for which $k < d/2$ are contributing to the field equations. In this Letter we want to restrict ourself to the first three terms of Lovelock gravity. The first term is the cosmological term and the second and third terms are the Einstein and second order Lovelock (Gauss–Bonnet) terms respectively.

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In the last decades a renewed interest appears in Lovelock gravity. In particular, exact static spherically symmetric black hole solutions of the Gauss–Bonnet gravity have been found in Ref. [4]. Exact solutions of static and rotating of the Maxwell–Gauss–Bonnet and Born–Infeld–Gauss–Bonnet models have been investigated in Refs. [5–10]. The thermodynamics of the uncharged static spherically black hole solutions has been considered in [11]. Properties of solutions with nontrivial topology have been studied in [12,13] and charged black hole solutions have been found in [5,14].

In the conventional, straightforward generalization of the Maxwell field to higher dimensions one essential property of the electromagnetic field is lost, namely, conformal invariance. The first black hole solution derived for which the matter source is conformally invariant is the Reissner–Nordström solution in four dimensions. Indeed, in this case the source is given by the Maxwell action which enjoys the conformal invariance in four dimensions. Massless spin-1/2 fields have vanishing classical stress tensor trace in any dimension, while scalars can be "improved" to achieve $T^{\alpha}_{\alpha}=0$, thereby guaranteeing invariance under the special conformal (or full Weyl) group, in accord with their scale-independence [15–17]. Maxwell theory can be studied in a gauge which is invariant under conformal rescalings of the metric, and firstly, has been proposed by Eastwood and Singer [18]. Also, Poplawski [19] have been showed the equivalence between the Ferraris–Kijowski and Maxwell Lagrangian results from the invariance of the latter under conformal transformations of the metric tensor. Quantized Maxwell theory in a conformally invariant gauge have been investigated by Esposito [20]. Also, there exists a conformally invariant extension of the Maxwell action in higher dimensions (Generalized Maxwell Field, GMF), if one uses the Lagrangian of the $U(1)$ gauge field in the form [21]

$$I_{\mathrm{GMF}}=\kappa\int d^dx\sqrt{-g}\big(F_{\mu\nu}F^{\mu\nu}\big)^{d/4},\tag{3}$$

where $F_{\mu\nu}=\partial_{\mu}A_{\nu}-\partial_{\nu}A_{\mu}$ is the Maxwell tensor and κ is an arbitrary constant. It is straightforward to show that the action (3) is invariant under conformal transformation ($g_{\mu\nu}\to\Omega^2 g_{\mu\nu}$ and $A_{\mu}\to A_{\mu}$) and for $d=4$, the action (3) reduces to the Maxwell action as it should be. The energy–momentum tensor associated to I_{GMF} is given by

$$T_{\mu\nu}=\kappa\big(dF_{\mu\rho}F_{\nu}{}^{\rho}F^{(d/4)-1}-g_{\mu\nu}F^{d/4}\big),\tag{4}$$

where $F=F_{\mu\nu}F^{\mu\nu}$ and it is easy to show that $T^{\mu}_{\mu}=0$. In what follows, we consider the action (3) as the matter source coupled to the Gauss–Bonnet gravity. The idea is to take advantage of the conformal symmetry to construct the analogues of the four-dimensional Reissner–Nordström black hole solutions in higher dimensions. The form of the energy–momentum tensor (4), automatically restricts the dimensions to be only multiples of four.

The main aim of this work is to present analytical solutions for typical class of rotating spacetime in Gauss–Bonnet theory coupled to conformally invariant Maxwell source with negative cosmological constant. As we show later, these solutions have some interesting properties, specially in the electromagnetic fields, which do not occur in Gauss–Bonnet gravity in the present of ordinary Maxwell field. In this Letter we discuss black hole solutions of Gauss–Bonnet-conformally invariant Maxwell source and investigate their properties and calculate the conserved and thermodynamics quantities.

The outline of our Letter is as follows. In next section, we present the basic field equations and general formalism of calculating the conserved quantities. In Section 3, we present the topological black hole of Gauss–Bonnet gravity in the presence of conformally invariant Maxwell source. Then, we calculate the thermodynamic quantities of asymptotically flat solutions and investigate the first law of thermodynamics. In Section 3.3 we introduce the rotating solutions with flat horizon and compute the thermodynamic and conserved quantities of them. We also perform a stability analysis of this solutions both in canonical and grand canonical ensemble. Finally, we finish our Letter with some concluding remarks.

2. Field equations in Gauss–Bonnet gravity with conformally invariant Maxwell source

The action of Gauss–Bonnet gravity in the presence of conformally invariant electromagnetic field is

$$\begin{aligned}I_G={}&-\frac{1}{16\pi}\int_{\mathcal{M}} d^{4p+4}x\sqrt{-g}\big\{R-2\Lambda+\alpha\big(R_{\mu\nu\gamma\delta}R^{\mu\nu\gamma\delta}-4R_{\mu\nu}R^{\mu\nu}+R^2\big)-\kappa\big(F_{\mu\nu}F^{\mu\nu}\big)^{p+1}\big\}\\&-\frac{1}{8\pi}\int_{\partial\mathcal{M}} d^{4p+3}x\sqrt{-\gamma}\big\{K+2\alpha\big(J-2\hat{G}_{ab}K^{ab}\big)\big\},\end{aligned}\tag{5}$$

where $\Lambda=-(2p+1)(4p+3)/l^2$ is the negative cosmological constant for asymptotically AdS solutions and α is the Gauss–Bonnet coefficient with dimension $(\mathrm{length})^2$. The second integral in Eq. (5) is the Gobbons–Hawking surface term and its counterpart for the Gauss–Bonnet gravity which is chosen such that the variational principle is well defined [22]. In this term, γ_{ab} is induced metric on the boundary $\partial\mathcal{M}$, K is trace of extrinsic curvature K^{ab} of the boundary, $\hat{G}_{ab}(\gamma)$ is Einstein tensor of the metric γ_{ab}, and J is trace of the tensor

$$J_{ab}=\frac{1}{3}\big(K_{cd}K^{cd}K_{ab}+2KK_{ac}K^{c}_{b}-2K_{ac}K^{cd}K_{db}-K^2K_{ab}\big).\tag{6}$$

Varying the action with respect to the metric tensor $g_{\mu\nu}$ and electromagnetic field A_{μ} the equations of gravitation and electromagnetic fields are obtained as

$$G_{\mu\nu}+\Lambda g_{\mu\nu}-\alpha H_{\mu\nu}=2\kappa\left[(p+1)F_{\mu\rho}F_{\nu}{}^{\rho}F^{p}-\frac{1}{4}g_{\mu\nu}F^{p+1}\right],\tag{7}$$

$$\partial_{\mu}\big(\sqrt{-g}F^{\mu\nu}F^{p}\big)=0,\tag{8}$$