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The growth of linear perturbations in the DGP model

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ABSTRACT

We study the linear growth of matter perturbations in the DGP model with the growth index γ as a function of redshift. At the linear approximation: $\gamma(z) \approx \gamma_0 + \gamma_0' z$, we find that, for $0.2 \leqslant \Omega_{m,0} \leqslant 0.35$, γ_0 takes the value from 0.658 to 0.671, and γ_0' ranges from 0.035 to 0.042. With three low redshift observational data of the growth factor, we obtain the observational constraints on γ_0 and γ_0' for the ΛCDM and DGP models and find that the observations favor the ΛCDM model but at the 1σ confidence level both the ΛCDM and DGP models are consistent with the observations.

1. Introduction

Various observations show that our universe is undergoing an accelerating expansion [1–3] and many models have been proposed to explain this mysterious phenomenon. There are basically two main classes of models. One is dark energy which yields sufficient negative pressure to induce a late-time accelerated expansion; the other is the modified gravity, such as the scalar-tensor theory [4], the $f(R)$ theory [5] and the Dvali–Gabadadze–Porrati (DGP) braneworld scenarios [6,7], et al. However, these models may predict the same late time accelerated cosmological expansion, although they are quite different physically. So an important task is to discriminate one from another. Recently, some attempts have been made [8–14] in this regard. An interesting approach is to differentiate the dark energy and the modified gravity with the growth function $\delta(z) \equiv \delta\rho_m/\rho_m$ of the linear matter density contrast as a function of redshift z. While different models give the same late time expansion, they may produce different growth of matter perturbations [15].

To the linear order of perturbation, the matter density perturbation $\delta = \delta\rho_m/\rho_m$ satisfies the following equation [16] at the large scales

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\rm eff}\rho_m\delta = 0, \tag{1}$$

where $G_{\rm eff}$ is the effective Newton's constant and the dot denotes the derivative with respect to time t. In general relativity, $G_{\rm eff} = G_N$ where G_N is the Newton's constant. Defining the growth factor $f \equiv d\ln\delta/d\ln a$, one can obtain

$$\frac{df}{d\ln a} + f^2 + \left(\frac{\dot{H}}{H^2} + 2\right) f = \frac{3}{2}\frac{G_{\rm eff}}{G_N}\Omega_m, \tag{2}$$

where Ω_m is the fractional energy density of matter. In general, analytical solutions to Eq. (2) are hard to find, and we need to resort to numerical methods. It has been known for many years that there is a good approximation to the growth factor f, which is given by [17]

$$f \equiv \frac{d\ln\delta}{d\ln a} \simeq \Omega_m(z)^\gamma, \tag{3}$$

where γ is the growth index and is taken as a constant. This parameterized approach has been studied in some works recently, see e.g. [18–28]. For example, substituting the above equation into Eq. (2) and then expanding around $\Omega_m = 1$ (a good approximation at the high redshift), one can obtain $\gamma_\infty \simeq 0.5454$ [18,20] for the ΛCDM model and $\gamma_\infty \simeq 11/16 \approx 0.6875$ [18,19] for the flat DGP model. Therefore, in principle, one can distinguish the dark energy model from the modified gravity model with observational data on the growth factor. However, taking the index γ as a constant is only an approximation although it is a very good one in certain circumstances. More generically, one should rewrite Eq. (3) as

$$f \equiv \frac{d\ln\delta}{d\ln a} = \Omega_m(z)^{\gamma(z)}. \tag{4}$$

Defining a new quantity $\gamma' \equiv \frac{d\gamma(z)}{dz}$, we can expand γ at the low redshift, as follows

$$\gamma(z) \approx \gamma_0 + \gamma_0' z, \quad 0 \leqslant z \leqslant 0.5. \tag{5}$$

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This approximation has been studied in Refs. [29–31], and it was found that γ'_0 is a quasi-constant and $\gamma'_0 \simeq -0.02$ for dark energy models with a constant equation of state. However, for modified gravity models, such as some scalar-tensor models, γ'_0 is negative and can take absolute values larger than those in models inside General Relativity [30], while for the $f(R)$ model γ'_0 is also negative but its value is largely outside the range found for dark energy models in General Relativity [31]. Therefore, an accurate γ'_0 at the low redshift could provide another characteristic discriminative signature for these models.

In this Letter, we will mainly focus on the observational constraints on γ_0 and γ'_0 from data on the growth factor. Firstly, we will study the linear growth index with the form $\gamma \approx \gamma_0 + \gamma'_0 z$ for the DGP model. Then, with the best fit value $\Omega_{m,0}$ from the observational data we will discuss the theoretical values of γ_0 and γ'_0 and the observational constraints on them.

2. Growth index of DGP model

For the DGP model, in general, $G_{\rm eff}$ can be written as

$$G_{\rm eff} = G_N\left(1+\frac{1}{3\beta}\right), \tag{6}$$

where $\beta = 1 - 2r_c H(1+\frac{\dot{H}}{3H^2})$ [23,32–34] and the constant r_c is a scale which sets a length beyond which gravity starts to leak out into the bulk. According to Ref. [33], $\frac{G_{\rm eff}}{G_N}$ can be rewritten as

$$1+\frac{1}{3\beta} = \frac{4\Omega_m^2 - 4(1-\Omega_k)^2 + \alpha}{3\Omega_m^2 - 3(1-\Omega_k)^2 + \alpha}, \tag{7}$$

where $\alpha \equiv 2\sqrt{1-\Omega_k}(3 - 4\Omega_k + 2\Omega_m\Omega_k + \Omega_k^2)$, $\Omega_k \equiv -k/(a^2H^2)$, and $\Omega_m \equiv 8\pi G\rho_m/(3H^2)$. Here the spatial curvature $k = 0$, $k > 0$ and $k < 0$ correspond to a flat, closed and open universe respectively.

For the DGP model, the modified Friedmann equation takes the form [7,19]

$$H^2 + \frac{k}{a^2} - \frac{1}{r_c}\sqrt{H^2+\frac{k}{a^2}} = \frac{8\pi G}{3}\rho_m. \tag{8}$$

Defining $\Omega_{r_c} = \frac{1}{4r_c^2H_0^2}$, we have

$$E^2(z) \equiv \left(\frac{H}{H_0}\right)^2 = \left[\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{r_c}} + \sqrt{\Omega_{r_c}}\right]^2 + \Omega_{k0}(1+z)^2. \tag{9}$$

Setting $z = 0$ in the above gives rise to a constraint equation

$$1 = \left[\sqrt{\Omega_{m,0} + \Omega_{r_c}} + \sqrt{\Omega_{r_c}}\right]^2 + \Omega_{k0}. \tag{10}$$

Therefore, there are only two model independent parameters out of $\Omega_{m,0}$, Ω_{r_c} and Ω_{k0}.

The matter density perturbation in the DGP model satisfies the equation [16,25]:

$$\frac{d^2\ln\delta}{d(\ln a)^2} + \left(\frac{d\ln\delta}{d\ln a}\right)^2 + \left(2+\frac{d\ln H}{d\ln a}\right)\left(\frac{d\ln\delta}{d\ln a}\right) = \frac{3}{2}\left(1+\frac{1}{3\beta}\right)\Omega_m. \tag{11}$$

Using

$$\frac{d\ln H}{d\ln a} = \frac{\dot{H}}{H^2} = -\frac{3}{2} + \frac{\Omega_k}{2} - \frac{3}{2}\frac{-1+\Omega_k}{1+\Omega_m-\Omega_k}(1-\Omega_k-\Omega_m), \tag{12}$$

we obtain

$$\frac{d^2\ln\delta}{d(\ln a)^2} + \left(\frac{d\ln\delta}{d\ln a}\right)^2 + \frac{d\ln\delta}{d\ln a}\left(\frac{1}{2}(1+\Omega_k) - \frac{3}{2}\frac{-1+\Omega_k}{1+\Omega_m-\Omega_k}(1-\Omega_k-\Omega_m)\right) = \frac{3}{2}\left(1+\frac{1}{3\beta}\right)\Omega_m. \tag{13}$$

Thus, according to the definition of f, we have the following differential equation

$$\Omega_m\left[\frac{3(-1+\Omega_k)}{1+\Omega_m-\Omega_k}(1-\Omega_k-\Omega_m) - \Omega_k\right]\frac{df}{d\Omega_m} + f^2 + f\left[\frac{1}{2}(1+\Omega_k) - \frac{3}{2}\frac{-1+\Omega_k}{1+\Omega_m-\Omega_k}(1-\Omega_k-\Omega_m)\right] = \frac{3}{2}\left(1+\frac{1}{3\beta}\right)\Omega_m. \tag{14}$$

Substituting the generic expression for f, Eq. (4), into Eq. (14) we arrive at an equation on $\gamma(z)$

$$\frac{1}{2}\left[(1+\Omega_k-2\gamma\Omega_k) + \frac{3(-1+\Omega_k)}{1+\Omega_m-\Omega_k}(2\gamma-1)(1-\Omega_k-\Omega_m)\right] - (1+z)\gamma'\ln\Omega_m + \Omega_m^{\gamma} = \frac{3}{2}\left(1+\frac{1}{3\beta}\right)\Omega_m^{1-\gamma}. \tag{15}$$

If we only consider the linear expansion at the low redshift as given in Eq. (5), it is easy to derive

$$\gamma'_0 = \left(\ln\Omega_{m,0}^{-1}\right)^{-1}\left[-\Omega_{m,0}^{\gamma_0} + \frac{3}{2}\left(1+\frac{1}{3\beta}\right)\Omega_{m,0}^{1-\gamma_0} - \frac{1}{2}(1+\Omega_{k,0}-2\gamma_0\Omega_{k,0}) - 3\frac{-1+\Omega_{k,0}}{1+\Omega_{m,0}-\Omega_{k,0}}(1-\Omega_{k,0}-\Omega_{m,0})\left(\gamma_0-\frac{1}{2}\right)\right]. \tag{16}$$

This gives a constraint equation

$$g(\gamma_0, \gamma'_0, \Omega_{m,0}, \Omega_{k,0}) = 0. \tag{17}$$

So, for any given background parameters $\Omega_{m,0}$ and $\Omega_{k,0}$, the value of γ'_0 can be determined by that of γ_0. For the sake of simplicity, we will only consider the case of a spatially flat universe in this Letter ($\Omega_k = 0$). Thus from Eq. (16), we get

$$\gamma'_0 = \left(\ln\Omega_{m,0}^{-1}\right)^{-1}\left[-\Omega_{m,0}^{\gamma_0} + \frac{3}{2}\frac{4\Omega_{m,0}^2+2}{3\Omega_{m,0}^2+3}\Omega_{m,0}^{1-\gamma_0} - \frac{1}{2} + \frac{3}{1+\Omega_{m,0}}(1-\Omega_{m,0})\left(\gamma_0-\frac{1}{2}\right)\right]. \tag{18}$$

According to equation $f(z=0) = \Omega_{m,0}(0)^{\gamma_0}$, the value of γ_0 can be obtained by solving Eq. (14) numerically for an given value of $\Omega_{m,0}$. Then plugging this obtained γ_0 into Eq. (18), we can get the value of γ'_0. The results are shown in Fig. 1. We find, from the right panel, that the value of γ_0 increases from 0.658 to 0.671 for $0.2 \leqslant \Omega_{m,0} \leqslant 0.35$. This suggests that γ cannot really be regarded as a constant as Ω_m varies. Notice that our result is different from that obtained for the ΛCDM model where the value of γ_0 is found to decrease from 0.558 to 0.554 for $0.2 \leqslant \Omega_{m,0} \leqslant 0.35$ [29]. This feature of γ_0 also provides a distinctive signature for the DGP model from the ΛCDM model. From the right panel, we can see that the γ'_0 is positive and ranges approximately from 0.035 to 0.042, which is also different from the dark energy models, the scalar–tensor model and $f(R)$ model. For example for the wCDM model with $\Omega_{m,0} = 0.3$, γ'_0 is negative and quasi-constant $\gamma'_0 \simeq -0.02$. So, in principle, we can discriminate the DGP model from the dark energy model merely through the sign of γ'_0 if we can have an accurate value of γ'_0 from the observation data. Now we will discuss the observational constraints on γ_0 and γ'_0.