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Baxter equation beyond wrapping

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ABSTRACT

The Baxter-like functional equation encoding the spectrum of anomalous dimensions of Wilson operators in maximally supersymmetric Yang–Mills theory available to date ceases to work just before the onset of wrapping corrections. In this Letter, we work out an improved finite-difference equation by incorporating nonpolynomial effects in the transfer matrix entering as its ingredient. This yields a self-consistent asymptotic finite-difference equation valid at any order of perturbation theory. Its exact solutions for fixed spins and twists at and beyond wrapping order give results coinciding with the ones obtained from the asymptotic Bethe Ansatz. Correcting the asymptotic energy eigenvalues by the Lüscher term, we compute anomalous dimensions for a number of short operators beyond wrapping order.

1. Asymptotic Baxter equation and wrapping

To date [1] there is a significant body of data which suggests that the spectrum of all anomalous dimensions in planar maximally supersymmetric gauge theory can be computed overcoming complicated calculations Feynman diagrams. This finding [2–5] generalizes previous observations that the spectrum of one-loop maximal-helicity gluon $X = F_{+\perp}$ operators in pure gauge theory,

$$\mathcal{O} = \mathrm{tr}\big\{D_+^{N_1} X(0) D_+^{N_2} X(0) \cdots D_+^{N_L} X(0)\big\}, \tag{1}$$

can be calculated by identifying the dilation operator with the Hamiltonian of a noncompact Heisenberg magnet [6,7]. The correspondence works by placing the elementary fields $X(0)$ of the Wilson operator on the spin-chain sites and identifying spin generators with the ones of the collinear $SL(2)$ subgroup of the (super)conformal group. The noncompactness of the spin chain is a consequence of the fact that there are infinite towers of covariant derivatives D_+ acting on those fields. The $sl(2)$ subsector of the maximally supersymmetric gauge theory, which we study in this Letter, is spanned by the Wilson operators (1) with the elementary complex scalar field $X = \phi^1 + i\phi^2$ [4].

The one-loop integrable structure was generalized to all orders in 't Hooft coupling $g^2 = g^2_{\mathrm{YM}} N_c/(4\pi^2)$ and, though one is currently lacking a putative spin-chain picture for the dilatation operator, a set of Bethe Ansatz equations was put forward which survives a number of nontrivial spectral checks [8]. However, these equations allow one to calculate anomalous dimensions for Wilson operators as long as the order of the perturbative expansion does not exceed the length of the operator. Namely, when the interaction of the spins in the chain start to wrap around it, the aforementioned equations start to fail [9,10]. Thus the true anomalous dimension for the Wilson operators are given a sum of two terms

$$\gamma(g) = \gamma^{(\mathrm{asy})}(g) + \gamma^{(\mathrm{wrap})}(g). \tag{2}$$

The first contribution is determined by the solution to the asymptotic Bethe Ansatz equations and can be written as [11]

$$\gamma^{(\mathrm{asy})}(g) = ig^2 \int_{-1}^{1} \frac{dt}{\pi} \sqrt{1-t^2} \left(\ln \frac{Q(+\frac{i}{2} - gt)}{Q(-\frac{i}{2} - gt)} \right)', \tag{3}$$

in terms of a polynomial with zeroes determined by the Bethe roots u_k

$$Q(u) = \prod_{k=1}^{N} \big(u - u_k(g)\big). \tag{4}$$

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It has to be supplemented by the condition of the vanishing quasimomentum

$$i\vartheta = \frac{1}{\pi}\int_{-1}^{1}\frac{dt}{\sqrt{1-t^2}}\ln\frac{Q(+\frac{i}{2}-gt)}{Q(-\frac{i}{2}-gt)} = 0, \tag{5}$$

in order to pick out only cyclic physical states. For vanishing 't Hooft coupling, $u_k(0) = u_{k,0}$ coincide with the Bethe roots of the short-range $sl(2)$ XXX spin chain. However, at higher order of perturbation theory they acquire a nontrivial dependence on the 't Hooft coupling, and so does the Baxter function $Q(u) = Q_0(u) + g^2 Q_1(u) + \cdots$. The Baxter polynomial $Q(u)$ is a function of the spectral parameter and it is determined as a solution to a finite-difference equation known as the asymptotic Baxter equation

$$(x^+)^L e^{\sigma_+(u^+)-\Theta(u^+)} Q(u+i) + (x^-)^L e^{\sigma_-(u^-)-\Theta(u^-)} Q(u-i) = t(u)Q(u). \tag{6}$$

The dressing factors accompanying the polynomial depend on the renormalized spectral parameter $x = x[u] = \frac{1}{2}(u+\sqrt{u^2-g^2})$ with the used notation $x^\pm = x[u^\pm]$. The exponents σ and Θ encode nontrivial dynamics of the long-range field-theoretical "spin chain", with the first one

$$\sigma_\pm(u) = \int_{-1}^{1}\frac{dt}{\pi}\frac{\ln Q(\pm\frac{i}{2}-gt)}{\sqrt{1-t^2}}\left(1-\frac{\sqrt{u^2-g^2}}{u+gt}\right), \tag{7}$$

partially responsible for the renormalization of the conformal spin at higher orders of perturbation theory, and the second

$$\Theta(u) = -8i\sum_{r=2}^{\infty}\sum_{s=r+1}^{\infty}\left(\frac{g}{2}\right)^{r+s-2} C_{rs}(g)\int_{-1}^{1}\frac{dt}{\pi}\sqrt{1-t^2}\left(\ln\frac{Q(+\frac{i}{2}-gt)}{Q(-\frac{i}{2}-gt)}\right)'\left\{\left(-\frac{2}{g}\right)^{s-2}\frac{U_{s-2}(t)}{x^{r-1}} - \left(-\frac{2}{g}\right)^{r-2}\frac{U_{r-2}(t)}{x^{s-1}}\right\}, \tag{8}$$

providing smooth interpolation between the weak and strong-coupling expansions [12]. Here we presented the Θ-phase in a form of an infinite expansion with the transcendental coefficients

$$C_{rs}(g) = \sin\left(\frac{\pi}{2}(s-r)\right)\int_0^\infty dv\,\frac{J_{r-1}(gv)J_{s-1}(gv)}{v(e^v-1)}, \tag{9}$$

which is the most suitable form for perturbative analyses we perform in this Letter.

The onset of wrapping corrections starts from the order $g^{2(L+2)}$ for the Wilson operators (1) [9,10]. Thus the first few orders of the corresponding anomalous dimensions are free from these complications $\gamma^{(\mathrm{asy})} = g^2\gamma_0 + g^4\gamma_1 + \cdots$ and can be determined efficiently from the Baxter equation (6), $\gamma_\ell = \gamma_\ell^{(\mathrm{asy})}$ for $\ell \leqslant L+1$. The first wrapping correction to the anomalous dimensions is given by a multiparticle Lüscher formula, which was recently conjectured and tested for the Konishi operators and four-loop twist-two ($L=2$) operators in Ref. [13],[1]

$$\gamma^{(\mathrm{wrap})}(g) = -g^4\gamma_0^2 i\left|Q_0\left(\frac{i}{2}\right)\right|^2\sum_{n=1}^{\infty}\operatorname*{res}_{z=in}\left(\frac{g^2}{z^2+n^2}\right)^L\frac{T^2(z,n)}{R(z,n)} + \mathcal{O}\big(g^{2(L+3)}\big). \tag{10}$$

It is written in terms of

$$R(z,n) = Q_0\left(\frac{1}{2}z-\frac{i}{2}(n-1)\right)Q_0\left(\frac{1}{2}z+\frac{i}{2}(n-1)\right)Q_0\left(\frac{1}{2}z+\frac{i}{2}(n+1)\right)Q_0\left(\frac{1}{2}z-\frac{i}{2}(n+1)\right), \tag{11}$$

and the function

$$T(z,n) = \sum_{m=0}^{n-1}\frac{Q_0(\frac{1}{2}z-\frac{i}{2}(n-1)+im)}{[(m-\frac{1}{2}n)-\frac{i}{2}z][(m+1-\frac{1}{2}n)-\frac{i}{2}z]}, \tag{12}$$

which also contains a kinematical pole at $z = in$ in addition to the ones displayed explicitly in Eq. (10).

2. Transfer matrix revisited

The original proposal [11] for the transfer matrix $t(u)$, entering the right-hand side of the Baxter equation, as a polynomial of order L in the bare spectral parameter u yields an inconsistent equation one order before the wrapping corrections set in. Since the Baxter equation has a number of advantages over the Bethe Ansatz equations—with relative simplicity in its diagonalization, straightforward asymptotic solution for large values of quantum numbers of the Wilson operators, etc., being a few—one has to seek for modifications of Eq. (6) which yield a self-consistent equation at any order of perturbation theory. A systematic inspection demonstrates that the aforementioned limitations can be easily overcome by merely modifying analytical properties of the transfer matrix and assuming the following ansatz for it

$$t(u) = \Re\mathrm{e}(x^+)^L\left(2+\sum_{k\geqslant 1}\mathfrak{Q}_k(g)\Re\mathrm{e}(x^+)^{-k}\right) - \sum_{k\geqslant 1}\mathfrak{R}_k(g)\Im\mathrm{m}(x^+)^{-k}. \tag{13}$$

[1] Here we restored the multiplicative factor $|Q_0(\frac{i}{2})|^2$ required to make the wrapping correction invariant under rescaling of the Baxter function by an arbitrary constant, $Q \to \lambda Q$, allowed by the homogeneous Baxter equation. Its apparent absence in Ref. [13] is due to a particular normalization chosen in that paper. We would like to thank Romuald Janik for correspondence on this point.