Physics Letters B 676 (2009) 94–98

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

# Singularity of spherically-symmetric spacetime in quintessence/phantom dark energy universe

Shin'ichi Nojiri [a,*], Sergei D. Odintsov [b,1]

[a] Department of Physics, Nagoya University, Nagoya 464-8602, Japan
[b] Institució Catalana de Recerca i Estudis Avançats (ICREA) and Institut de Ciencies de l'Espai (IEEC-CSIC), Campus UAB, Facultat de Ciencies, Torre C5-Par-2a pl, E-08193 Bellaterra (Barcelona), Spain

ARTICLE INFO

*Article history:*
Received 30 March 2009
Received in revised form 25 April 2009
Accepted 28 April 2009
Available online 3 May 2009
Editor: T. Yanagida

*PACS:*
95.36.+x
98.80.Cq

ABSTRACT

We consider ideal fluid and equivalent scalar field dark energy universes where all four known types of finite time, future singularities occur at some parameter values. It is demonstrated that pressure/energy density of such quintessence/phantom dark energy diverges in spherically-symmetric spacetime at finite radius or at the center. This may cause the instability of the relativistic star or black hole in such universe. The resolution of the problem via the extra modification of the equation of state is briefly discussed.



## 1. Introduction

The discovery of the late-time universe acceleration brought to the playground the number of dark energy (DE) models with the effective equation of state (EoS) parameter $w$ being very close to $-1$ in accordance with observational data. It is known that phantom/quintessence models lead to the violation of all/some of the energy conditions. Such models unlike to $\Lambda$CDM with $w=-1$ lead to the number of quite surprising consequences in the remote future. For instance, phantom DEs are characterized by the future Big Rip singularity [1]. Some of quintessence DEs bring the universe to softer finite-time singularity in the future. Such finite-time future singularities may represent so-called sudden singularities [2,3] or some other singularity types which are classified in Ref. [4]. It is evident that the presence of finite-time future singularity in the course of the universe evolution may show up at the current epoch. One example has been given in Ref. [5] where it was conjectured that sudden singularity [3] of specific modified gravity DE may make the relativistic star formation process being unstable. The resolution of the problem is to introduce the higher-order curvature terms [3] relevant only at the early universe in such a way that future singularity disappears.

In the present Letter we consider the specific dark fluid which contains all four known finite-time singularity types [4]. The reformulation of it as scalar DE model with the same FRW asymptotic solutions for the corresponding scalar potentials is also made. The energy density/pressure of such singular DE may become divergent in the spherically-symmetric spacetime at finite radius or at the center. In a sense, that is the way the finite-time singularity manifests itself as singularity of spherically-symmetric space. This may lead to the instability of relativistic stars (in the same way as for modified gravity DE model in Ref. [5]) or instability of black holes located in such dark energy universe. It indicates that number of current DEs with such properties may be problematic for realistic description of current accelerating universe. Some extra EoS modification by the terms relevant at the very early universe maybe necessary in order to resolve this problem. Such modification is discussed briefly in the last section. The reconstruction method to find the specific dark energy responsible for any singularity of spherically-symmetric space is also presented.

## 2. Singularities of spherically-symmetric spacetime filled with dark energy

In Appendix A, the appearance of the finite-time singularities is shown for the sufficiently realistic dark fluid and scalar field theory. In this section we show that finite-time singularities of dark energy models in the appendix manifest themselves as radius singularities of spherically-symmetric spacetime filled with such dark energies. In [5], it has been pointed that curvature singularity is realized inside the relativistic star for a viable class of $f(R)$-gravities (for review of viable, realistic models of that sort, see [6]). Since such spherically-symmetric solution with a naked singularity is inconsistent, this result indicates that large star (or even planet)

* Corresponding author.
*E-mail address:* nojiri@phys.nagoya-u.ac.jp (S. Nojiri).
[1] Also at Center of Theoretical Physics, TSPU, Tomsk, Russia.

0370-2693/$ – see front matter 
doi:10.1016/j.physletb.2009.04.079

could not be formed, or such a relativistic star could be unstable in such a theory.

It is known that viable modified gravity also shows all four above types of finite-time singularity [3]. Hence, it is natural to expect that qualitatively similar situation should be typical for any dark energy which brings the universe to finite-time singularity. Motivated with these observations, we investigate the EoS dark fluid, which generates curvature singularity in spherically-symmetric spacetime. Especially in this section, we investigate the singularity, which is generated for a finite (non-vanishing) value of the radius. The singularity at the origin (vanishing radius) is investigated in the next section.

Let us first consider what kind of (perfect) fluid could generate a singularity. We concentrate on the singularity which occurs for a finite radius for spherically symmetric solution. Assume the metric has the following form:

$$ds^2 = -e^{\nu(r)}\,dt^2 + e^{2\lambda(r)}\,dr^2 + r^2\,d\Omega_2^2. \tag{1}$$

Here $d\Omega_2^2$ expresses the metric of two-dimensional sphere. Eq. (1) expresses the arbitrary spherically symmetric and static space-time also in the presence of some matter, that corresponds to the inside of the star or planet. Then the Einstein equations have the following form:

$$\frac{1}{r}\frac{d\lambda}{dr} + \frac{e^{2\lambda}-1}{r^2} = \kappa^2\rho e^{2\lambda}, \tag{2}$$

$$\frac{1}{r}\frac{d\nu}{dr} - \frac{e^{2\lambda}-1}{r^2} = \kappa^2 p_r e^{2\lambda}, \tag{3}$$

$$\frac{d^2\nu}{dr^2} + \left(\frac{d\nu}{dr} - \frac{d\lambda}{dr}\right)\left(\frac{d\nu}{dr} + \frac{1}{r}\right) = \kappa^2 p_a e^{2\lambda}. \tag{4}$$

Here $\rho$ is the energy density and $p_r$ and $p_a$ are the radial and angular components of the pressure. In the following, as for the usual perfect fluid, we assume $p_r = p_a$. If one does not impose this assumption, we can consider general types of singularity. By combining (3) and (4) and deleting $p = p_r = p_a$, it follows

$$0 = -\frac{d^2\nu}{dr^2} - \left(\frac{d\nu}{dr}\right)^2 + \frac{d\nu}{dr}\left(\frac{1}{r} + \frac{d\lambda}{dr}\right) + \frac{1}{r}\frac{d\lambda}{dr} - \frac{e^{2\lambda}-1}{r^2}. \tag{5}$$

The following kind of singularities at $r = r_0$ may be now considered:

$$\lambda(r) = \lambda_0 + \lambda_1 \ln\frac{r-r_0}{r_0} + \sum_{n=2}^{\infty}\lambda_n (r-r_0)^{n-1},$$
$$\nu(r) = \nu_0 + \nu_1 \ln\frac{r-r_0}{r_0} + \sum_{n=2}^{\infty}\nu_n (r-r_0)^{n-1}. \tag{6}$$

By substituting (6) into (5), one obtains

$$\frac{\nu_1 - \nu_1^2 + \nu_1\lambda_1}{(r-r_0)^2} + \frac{\frac{\nu_1+\lambda_1}{r} - 2\nu_1\nu_2 + \nu_1\lambda_2 + \nu_2\lambda_1}{r-r_0} - \nu_2^2 - \lambda_2\nu_2 + \frac{\nu_2+\lambda_2}{r} + \frac{1}{r^2} + \mathcal{O}\big((r-r_0)\big)$$
$$= -\frac{e^{2\lambda_0 + \sum_{n=2}^{\infty}\lambda_n (r-r_0)^{n-1}}}{r^2}\left(\frac{r-r_0}{r_0}\right)^{2\lambda_1}. \tag{7}$$

Since the l.h.s. in (7) contains only the power $(r-r_0)^m$, where $m$ is an integer greater than or equal to $-2$, $2\lambda_1$ should be also an integer greater than or equal to $-2$. Furthermore, if we assume $2\lambda_1 = -2$, from the coefficients of $(r-r_0)^{-2}$, we find $0 = \nu_1^2 + e^{2\lambda_0}$, which is inconsistent since the r.h.s. is positive definite. Therefore, $2\lambda_1$ must be an integer greater than or equal to $-1$: $2\lambda_1 \geqslant -1$. Then from the coefficients of $(r-r_0)^{-2}$, again, we find

$$0 = \nu_1(1 - \nu_1 + \lambda_1), \quad \text{that is,} \quad \nu_1 = 0 \text{ or } \nu_1 = \lambda_1 + 1. \tag{8}$$

If $\nu_1 = \lambda_1 = 0$, there is no singularity and we do not consider this case. In case $\nu_1 = 0$ and $2\lambda_1 = -1$, from the coefficients of $(r-r_0)^{-2}$ in (7), we find $-\frac{1}{2r_0} - \frac{\nu_2}{2} - \frac{e^{2\lambda_0}}{r_0} = 0$. In case $\nu_1 = 0$ and $2\lambda_1 \geqslant 1$, one finds

$$\nu_2 = -\frac{1}{r_0}. \tag{9}$$

The case $\nu_1 \neq 0$ and $\nu_1 = \lambda_1 + 1 = 1/2$ corresponds to the black hole, where $r_0$ corresponds to the horizon radius and it follows $-\frac{3}{2}\nu_2 + \frac{\lambda_2}{2} - \frac{e^{2\lambda_0}}{r_0} = 0$. In case $\nu_1 = 1$ and $\lambda_1 = 0$, we find $\frac{1}{r_0} - 2\nu_2 + \lambda_2 = 0$. In case $\nu_1 = \lambda_1 + 1 \geqslant 3/2$, one gets $\frac{\nu_1+\lambda_1}{r_0} - 2\nu_1\nu_2 + \nu_1\lambda_2 + \nu_2\lambda_1 = 0$.

We now investigate how $\rho$ and $p$ behave. In case $\nu_1 = 0$ and $2\lambda_1 = -1$, from Eqs. (2) and (3), $\rho$ and $p$ are not singular and behave as $\rho \sim -p \sim \frac{1}{\kappa^2 r_0^2}$. Therefore there could not be the singularity at $r = r_0$. In case $\nu_1 = 0$ and $2\lambda_1 = n \geqslant 1$, Eqs. (2) and (3) give $\rho \sim \frac{2n e^{-2\lambda_0} r_0^{n-1}}{\kappa^2 (r-r_0)^{n+1}}$, $p \sim -\frac{e^{-2\lambda_0} r_0^{n-2}}{\kappa^2 r_0 (r-r_0)^n}$. Here relation (9) is used. Then $\rho$ and $p$ satisfy the following asymptotic equation of state (EoS):

$$p \sim -C\rho^{\frac{n}{n+1}}. \tag{10}$$

Here $C$ is a positive constant. Since $\rho$ and $p$ diverge at $r = r_0$, there is a curvature singularity at $r = r_0$. As we will see soon, the EoS (10) corresponds to that in (A.5), which generates Type III singularity.

In case $\lambda_1 = 0$ and $\nu_1 = 1$ case, we find

$$\rho \sim \frac{1}{\kappa^2}\left(\frac{2e^{-2\lambda_0}\lambda_2}{r_0} + \frac{1}{r_0^2} - \frac{e^{-2\lambda_0}}{r_0^2}\right), \qquad p \sim \frac{2e^{-2\lambda_0}}{\kappa^2 r_0 (r-r_0)}.$$

Here $\rho$ is finite although $p$ diverges at $r = r_0$. The divergence of $p$ generates the curvature singularity. The EoS has the following form: $p(\rho - \rho_0) \sim \text{const}$.

In case $2\lambda_1 = n \geqslant 0$ and $\nu_1 = \lambda_1 + 1$, we find

$$\rho \sim \frac{n e^{-2\lambda_0} r_0^{n-1}}{\kappa^2 (r-r_0)^{n+1}}, \qquad p \sim \frac{(n+2) e^{-2\lambda_0} r_0^{n-1}}{\kappa^2 (r-r_0)^{n+1}}, \tag{11}$$

and therefore the following asymptotic EoS:

$$p = \left(1 + \frac{2}{n}\right)\rho. \tag{12}$$

The divergence of $\rho$ and $p$ at $r = r_0$ means the curvature singularity. Since the EoS parameter $w \equiv p/\rho = 1 + 2/n$ is positive, the EoS does not generate any finite-time singularity and corresponds to $\alpha = 1$ and $-1 + A = 1 + 2/n$.

Hence, rather general case with the singularity occurrence for finite $r$ is investigated. We found here essentially two types of singularity expressed by the EoS fluid (10) or (12). These EoS fluids could be compared with the asymptotic EoS dark fluid generating the finite time singularity (A.3), (A.4), and (A.5). First, one notices that EoS fluid (12) for finite radius singularity does not exist for dark fluid generating the finite time singularity as in (A.3), (A.4), and (A.5). (However, it could be that other dark fluid generating future singularity shows up radius singularity in this example.) Eq. (A.3) has a similar structure but in (A.3), $w \equiv p/\rho \leqslant -1$ although $w > 1$ in (12). The EoS (10) has a similar structure with those in (A.3) and (A.3) if we identify

$$\alpha = \frac{n}{n+1}. \tag{13}$$

In case of (A.3), $\alpha$ is negative but in case of (A.5), $\alpha$ is positive. Then the fluid with EoS (A.5), which generates Type III singularity, may generate finite radius singularity. Note, however, we do not have the explicit proof that the fluid generating finite-time singularity always generate finite radius singularity, and vice versa. That