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Evolution of mixed Dirac particles interacting with an external magnetic field

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Abstract

We study in the framework of relativistic quantum mechanics the evolution of a system of two Dirac neutrinos that mix with each other and have non-vanishing magnetic moments. The dynamics of this system in an external magnetic field is determined by solving the Pauli–Dirac equation with a given initial condition. We consider first neutrino spin-flavor oscillations in a constant magnetic field and derive an analytical expression for the transition probability of spin-flavor conversion in the limit of small magnetic interactions. We then investigate ultrarelativistic neutrinos in a transversal magnetic field and derive their wave functions and transition probabilities with no limitation for the size of transition magnetic moments. Although we consider neutrinos, our formalism is straightforwardly applicable to any spin-$1/2$ particles.

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1. Introduction

In the neutrino oscillation phenomena observed so far oscillations occur between different neutrino flavors ν_λ ($\lambda = e, \mu, \tau$) so that an active left-handed neutrino goes to another active left-handed neutrino (e.g. $\nu_e^{\mathrm{L}} \leftrightarrow \nu_\mu^{\mathrm{L}}$). In some situations another type of oscillations is possible, namely spin-oscillation, where oscillation happens between an active left-handed neutrino and its inert right-handed counterpart (e.g. $\nu_e^{\mathrm{L}} \leftrightarrow \nu_e^{\mathrm{R}}$). Also a combination of these two oscillation types, so-called spin-flavor oscillations, can happen. There oscillations take place between an active left-handed neutrino and an inert right-handed neutrino of different flavor (e.g. $\nu_e^{\mathrm{L}} \leftrightarrow \nu_\mu^{\mathrm{R}}$) [1,2].

In this Letter we shall consider spin-flavor oscillations in an external magnetic field. We assume that neutrinos are Dirac particles with non-vanishing magnetic moments. Let us remind that Dirac particles can have both ordinary (diagonal) magnetic moments and transition (non-diagonal) magnetic moments, whereas for Majorana particles only transition magnetic moments are possible [3]. Note that despite the recent claims of the experimental discovery of the Majorana nature of neutrinos [4], it is still an open question whether neutrinos are Dirac or Majorana particles [5]. We shall use the formalism of relativistic quantum mechanics, that is, we use the Dirac equation as a starting point, which is a proper approach for spin-$1/2$ particles. One of us (M.D.) has previously used this formalism for studying neutrino flavor oscillations in vacuum [6] and in an external axial-vector field [7] (neutrino interaction

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with matter). Neutrino spin-flavor oscillations in electromagnetic fields of various configurations have been studied in Ref. [8]. The present Letter is a continuation of these previous works.

Let us consider a system of two Dirac neutrinos with non-vanishing masses and magnetic moments. In general, both the mass matrix and the matrix of magnetic moments are non-diagonal in the flavor basis of neutrino wave functions. When the mass part of the Hamiltonian is diagonalized by a unitary transformation and the original flavor basis is replaced by a new set of wave functions in the mass eigenstate basis, the matrix of magnetic moments is generally not diagonal in the new basis. This means that there will be transition magnetic moments between the mass eigenstates. We will consider magnetic moment matrices of neutrinos in various bases in Section 2. In Section 3 we will discuss the situation where the resulting magnetic moment matrix is close to diagonal, i.e. the transition magnetic moments are small compared with the diagonal magnetic moments in the mass eigenstate basis, in which case one can apply the formalism developed in Refs. [6,7]. In Section 4 we will consider the effects of transition magnetic moments for ultrarelativistic neutrinos in a transversal magnetic field with no limitations on the size of any magnetic moments. In particular, we apply our result for studying the situation, where the transition magnetic moment is large compared with the diagonal ones. We will derive the transition probability for the process like $\nu_\beta^{\mathrm{L}} \to \nu_\alpha^{\mathrm{R}}$, where α, β denote two different flavors, in transversal magnetic field. In Section 5 we summarize our results.

2. Electrodynamics of mixed particle states possessing magnetic moments

The magnetic moments of Dirac neutrinos are usually non-vanishing in both flavor and mass eigenstates bases [9]. This is why the flavor oscillations of neutrinos under the influence of an external magnetic field are associated generally with spin-flavor conversions. One has to analyze the behavior of a two Dirac neutrino system in a four-dimensional basis $\Psi^{\mathrm{T}} = (\nu_\beta^{\mathrm{L}}, \nu_\alpha^{\mathrm{L}}, \nu_\beta^{\mathrm{R}}, \nu_\alpha^{\mathrm{R}})$, which includes both chiral components of neutrinos [10]. The analytic solution of the Schrödinger evolution equation for this system appears to be quite complicated generally.

Let us denote the magnetic moments of the two Dirac neutrinos ν_α, ν_β as $M_{\alpha\alpha}$, $M_{\beta\beta}$ and $M_{\alpha\beta}$, where the last one is called as a transition magnetic moment. The Lagrangian of the neutrinos in the presence of an external electromagnetic field $F_{\mu\nu} = (\mathbf{E}, \mathbf{B})$ is then given by

$$\mathcal{L}(\nu_\alpha, \nu_\beta) = \sum_{\lambda=\alpha,\beta} \mathcal{L}_0(\nu_\lambda) - (m_{\beta\alpha}\bar{\nu}_\beta \nu_\alpha + \text{h.c.}) - \frac{1}{2} \sum_{\lambda\lambda'=\alpha,\beta} M_{\lambda\lambda'} \bar{\nu}_\lambda \sigma_{\mu\nu} \nu_{\lambda'} F^{\mu\nu}. \tag{2.1}$$

Here, $\mathcal{L}_0(\nu_\lambda) = \bar{\nu}_\lambda(\mathrm{i}\gamma^\mu \partial_\mu - m_{\lambda\lambda})\nu_\lambda$ and $\sigma_{\mu\nu} = (\mathrm{i}/2)(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$, and the parameters $m_{\lambda\lambda}$ and $m_{\beta\alpha}$ (a vacuum mixing parameter) have dimension of mass.

To analyze the dynamics of the system one has to set the initial condition by giving the initial wave functions of neutrinos ν_λ $(\lambda = \alpha, \beta)$ and then analytically determine the field distributions at later moments of time. The analogous problem was studied in Ref. [6] for mixed neutrino states in vacuum and in Ref. [7] for mixed neutrino states in an external axial-vector field.

We assume an initial condition of the form

$$\nu_\alpha(\mathbf{r}, 0) = 0, \qquad \nu_\beta(\mathbf{r}, 0) = \xi(\mathbf{r}), \tag{2.2}$$

where $\xi(\mathbf{r})$ is a function to be specified. If we identify ν_α as ν_e or ν_τ and ν_β as ν_μ, for example, this initial condition might correspond a situation where the source of neutrinos consists of pions and kaons which decay into muon neutrinos.

In order to eliminate the vacuum mixing term in Eq. (2.1) we introduce a new basis of the wave functions, the mass eigenstate basis ψ_a, $a = 1, 2$, related to the original flavor basis ν_λ through a unitary transformation

$$\nu_\lambda = \sum_{a=1,2} U_{\lambda a} \psi_a, \tag{2.3}$$

where the matrix $U = (U_{\lambda a})$ is parameterized in terms of a mixing angle θ in the usual manner:

$$U = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}. \tag{2.4}$$

The Lagrangian (2.1) rewritten in terms of the fields ψ_a takes the form

$$\mathcal{L}(\psi_1, \psi_2) = \sum_{a=1,2} \mathcal{L}_0(\psi_a) - \frac{1}{2} \sum_{ab=1,2} \mu_{ab} \bar{\psi}_a \sigma_{\mu\nu} \psi_b F^{\mu\nu}, \tag{2.5}$$

where $\mathcal{L}_0(\psi_a) = \bar{\psi}_a(\mathrm{i}\gamma^\mu \partial_\mu - m_a)\psi_a$ is the Lagrangian for the free neutrino ψ_a with the mass m_a and

$$\mu_{ab} = \sum_{\lambda\lambda'=\alpha,\beta} U^{-1}_{a\lambda} M_{\lambda\lambda'} U_{\lambda' b} \tag{2.6}$$