Available online at www.sciencedirect.com
ScienceDirect
Physics Letters B 655 (2007) 97–103
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Holographic hessence models

Wen Zhao

Department of Applied Physics, Zhejiang University of Technology, Hangzhou, Zhejiang, People's Republic of China

Received 11 April 2007; received in revised form 15 June 2007; accepted 31 August 2007

Available online 14 September 2007

Editor: A. Ringwald

Abstract

We discuss the evolution of holographic hessence model, which satisfies the holographic principle and can naturally realize the equation of state crossing -1. By discussing the evolution of the models in the w–w' plane, we find that, if $c \geqslant 1$, $w_{\rm he} \geqslant -1$ and $\dot{V} < 0$ keep for all time, which are quintessence-like. However, if $c < -1$, which mildly favors the current observations, $w_{\rm he}$ evolves from $w_{\rm he} > -1$ to $w_{\rm he} < -1$, and the potential is a nonmonotonic function. In the earlier time, the potential must be rolled down, and then be climbed up. Considered the current constraint on the parameter c, we reconstruct the potential of the holographic hessence model.

PACS: 98.80.-k; 98.80.Es; 04.30.-w; 04.62.+v

1. Introduction

Numerous and complementary cosmological observations indicate that the expansion of the universe is undergoing cosmic acceleration at the present time [1]. This cosmic acceleration is viewed as due to a mysterious dominant component, dark energy, with negative pressure. The combined analysis of cosmological observations suggests that the universe is spatially flat, and consists of about 70% dark energy, 30% dust matter (cold dark matter plus baryons), and negligible radiation. Although we can affirm that the ultimate fate of the universe is determined by the feature of dark energy, the nature of dark energy as well as its cosmological origin remain enigmatic at present. Explanations have been sought within a wide range of physical phenomena, including a cosmological constant, exotic fields [2–6], a new form of the gravitational equation [7], etc. Recently, a new model stimulated by the holographic principle has been put forward to explain the dark energy [8,9]. According to the holographic principle, the number of degrees of freedom of a physical system scales with the area of its boundary. In the context, Cohen et al. [10] suggested that in quantum field theory a short distant cutoff is related to a long distant cufoff due to the limit set by formation of a black hole, which results in an upper bound on zero-point energy density. In line with this suggest, Hsu and Li [8,9] argued that this energy density could be view as the holographic dark energy satisfying

$$\rho_{\rm de} = 3c^2 M_P^2 L^{-2}, \tag{1}$$

where c is a numerical constant, and $M_P \equiv 1/\sqrt{8\pi G}$ is the reduced Planck mass. If we take L as the size of the current universe, for instance the Hubble scale H^{-1}, then the dark energy density will be close to the observed data. However, Hsu [8] pointed out that this yields a wrong equation of state for dark energy. Li [9] subsequently proposed that the IR cut-off L should be taken as the size of the future event horizon

$$L = R_{\rm eh}(a) = a\int_t^\infty \frac{d\tilde{t}}{a(\tilde{t})} = a\int_a^\infty \frac{d\tilde{a}}{H\tilde{a}^2}. \tag{2}$$

Then the problem can be solved nicely and the holographic dark energy model can thus be constructed successfully. The holographic dark energy scenario may provide simultaneously natural solutions to both dark energy problems as demonstrated in Ref. [9]. The only undetermined parameter c should be fixed by the observations. If $c \leqslant 1$, which satisfies the original bound $L^3\rho_{\rm de} \leqslant LM_p^2$, the equation of state (EOS) of dark energy evolves from the state of $w > -1$ to $w < -1$, and the critical

E-mail address: wzhao7@mail.ustc.edu.cn.

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2007.08.083

state of $w=-1$ must be crossed. If $c>1$, the EOS of dark energy keeps $w>-1$ [9], which naturally avoided the cosmic big rip. However, the original bound $L^3\rho_{\rm de}\leqslant LM_p^2$ will be violated. Since the model we discuss here is only a phenomenological framework and it is unclear whether it is appropriate to tightly constrain the value of c by means of the analogue to the black hole. As a matter of fact, the possibility of $c>1$ has been seriously dealt with and a modest value of c larger than one could be favored in the literature [11]. In this Letter, we consider the general case with c as a free parameter.

For a kind of realized dark energy model, the feature of EOS crossing -1 cannot be realized by the simple quintessence, phantom, or k-essence [12]. The quintom is one of the simplest models with EOS crossing -1, which is the combination of a quintessence ϕ_1 and a phantom ϕ_2. The hessence is a kind of simple quintom [13,14], which has the Lagrangian density

$$\mathcal{L}_{\rm he}=\frac{1}{2}(\partial_\mu\phi_1)^2-\frac{1}{2}(\partial_\mu\phi_2)^2-V\big(\phi_1^2-\phi_2^2\big), \tag{3}$$

where the potential function $V(\phi_1^2-\phi_2^2)$ is free for the models. Different choice of V follows a different evolution of the universe. In Ref. [15], the authors found that this kind of models may be the local effective approximation of the D3-brane Universe. In Ref. [14], we have proved that the evolution of potential function can be exactly determined by the EOS of hessence $w_{\rm he}(z)$ and its evolution $w'_{\rm he}(z)$. If considered the holographic constraint in Eq. (1), the EOS of the hessence can be exactly determined for a fixed c. So the potential function for the holographic hessence only depends on the parameter c. In this Letter, we first discuss the evolution of the EOS and potential of the holographic hessence models for the different c. Then considered the constraint on c from the current observations, we reconstruct the potential function of holographic hessence models.

2. Holographic hessence models

We consider the action

$$S=\int d^4x\sqrt{-g}\left(-\frac{\mathcal{R}}{16\pi G}+\mathcal{L}_{\rm he}+\mathcal{L}_m\right), \tag{4}$$

where g is the determinant of the metric $g_{\mu\nu}$, $\mathcal{R}$ is the Ricci scalar, $\mathcal{L}_{\rm he}$ and $\mathcal{L}_m$ are the Lagrangian densities of the hessence dark energy and matter, respectively. The Lagrangian density of hessence is in Eq. (3). One can easily find that this Lagrangian is invariant under the transformation

$$\phi_1\to\phi_1\cosh(i\alpha)-\phi_2\sinh(i\alpha), \tag{5}$$

$$\phi_2\to-\phi_1\sinh(i\alpha)+\phi_2\cosh(i\alpha), \tag{6}$$

where α is constant. This property makes one can rewrite the Lagrangian density (3) in another form

$$\mathcal{L}_{\rm he}=\frac{1}{2}\big[(\partial_\mu\phi)^2-\phi^2(\partial_\mu\theta)^2\big]-V(\phi), \tag{7}$$

where we have introduced two new variables (ϕ,θ), i.e.,

$$\phi_1=\phi\cosh\theta,\qquad \phi_2=\phi\sinh\theta. \tag{8}$$

Consider a spatially flat FRW (Friedmann–Robertson–Walker) universe with metric

$$ds^2=dt^2-a^2(t)\gamma_{ij}\,dx^i\,dx^j, \tag{9}$$

where $a(t)$ is the scale factor, and $\gamma_{ij}=\delta^i_j$ denotes the flat background space. Assuming ϕ and θ are homogeneous, from the action in (4), we obtain the equations of motion for ϕ and θ

$$\ddot\phi+3H\dot\phi+\phi\dot\theta^2+dV/d\phi=0, \tag{10}$$

$$\phi^2\ddot\theta+\big(2\phi\dot\phi+3H\phi^2\big)\dot\theta=0, \tag{11}$$

where $H\equiv\dot a/a$ is the Hubble parameter, an overdot denotes the derivatives with respect to cosmic time. Eq. (11) implies

$$Q=a^3\phi^2\dot\theta={\rm const}, \tag{12}$$

which is associated with the total conserved charge within the physical volume due to the internal symmetry [13]. This relation turns out

$$\dot\theta=\frac{Q}{a^3\phi^2}. \tag{13}$$

Substituting this into Eq. (10), we can rewrite the kinetic equation as

$$\ddot\phi+3H\dot\phi+\frac{Q^2}{a^6\phi^3}+\frac{dV}{d\phi}=0, \tag{14}$$

which is equivalent to the energy conservation equation of the hessence $\dot\rho_{\rm he}+3H(\rho_{\rm he}+p_{\rm he})=0$. The pressure, energy density and the EOS of the hessence are

$$p_{\rm he}=\frac{1}{2}\dot\phi^2-\frac{Q^2}{2a^6\phi^2}-V(\phi),$$

$$\rho_{\rm he}=\frac{1}{2}\dot\phi^2-\frac{Q^2}{2a^6\phi^2}+V(\phi), \tag{15}$$

$$w_{\rm he}=\left[\frac{1}{2}\dot\phi^2-\frac{Q^2}{2a^6\phi^2}-V(\phi)\right]\Big/\left[\frac{1}{2}\dot\phi^2-\frac{Q^2}{2a^6\phi^2}+V(\phi)\right], \tag{16}$$

respectively. It is easily seen that $w_{\rm he}\geqslant-1$ when $\dot\phi^2\geqslant Q^2/(a^6\phi^2)$, while $\omega_{\rm he}\leqslant-1$ when $\dot\phi^2\leqslant Q^2/(a^6\phi^2)$. The transition occurs when $\dot\phi^2=Q^2/(a^6\phi^2)$. In the case of $Q\equiv0$, the hessence becomes the quintessence model. From the expression of EOS of hessence, we can find it is only dependant of the potential function $V(\phi)$. If $V(\phi)$ is determined, w is also determined. On the contrary, if $w(z)$ is fixed, the potential function $V(\phi)$ also can be solved. Here we consider the holographic hessence models, which satisfies the holographic constraint in Eq. (1). Consider now a spatially flat FRW universe with matter component ρ_m (including both baryon matter and cold dark matter) and holographic hessence component $\rho_{\rm he}$. The Friedmann equation reads

$$3H^2M_p^2=\rho_m+\rho_{\rm he}, \tag{17}$$

or equivalently,

$$\frac{H^2}{H_0^2}=\Omega_{m0}a^{-3}+\Omega_{\rm he}\frac{H^2}{H_0^2}. \tag{18}$$