Available online at www.sciencedirect.com
ScienceDirect
Physics Letters B 655 (2007) 132–140
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Flavor $S_4 \otimes Z_2$ symmetry and neutrino mixing

He Zhang

Institute of High Energy Physics, Chinese Academy of Sciences, PO Box 918, Beijing 100049, China

Received 28 February 2007; received in revised form 22 August 2007; accepted 9 September 2007

Available online 12 September 2007

Editor: T. Yanagida

Abstract

We present a model of the lepton masses and flavor mixing based on the discrete group $S_4 \otimes Z_2$. In this model, all the charged leptons and neutrinos are assigned to the $\underline{\mathbf{3}}_\alpha$ representation of S_4 in the Yamanouchi bases. The charged lepton and neutrino masses are mainly determined by the vacuum expectation value structures of the Higgs fields. A nearly tri–bimaximal lepton flavor mixing pattern, which is in agreement with the current experimental results, can be accommodated in our model. The neutrino mass spectrum takes the nearly degenerate pattern, and thus can be well tested in the future precise experiments.

1. Introduction

To understand the origin of fermion masses and flavor mixing is crucial and essential in modern particle physics. From the analyses [1] of recent neutrino oscillation experiments [2–6], we have confirmed the large solar mixing angle $\theta_{12} \simeq 34°$ [7], maximal atmospheric mixing angle $\theta_{23} \simeq 45°$ and a very tiny θ_{13}, $\theta_{13} < 10°$. A very small solar mass squared difference $\Delta m^2_{21} \equiv m^2_2 - m^2_1 \simeq 8.0 \times 10^{-5}\ \mathrm{eV}^2$ and an atmospheric mass squared difference $\Delta m^2_{32} \equiv m^2_3 - m^2_2 \simeq \pm 2.5 \times 10^{-3}\ \mathrm{eV}^2$ are given by the experimental data [1]. Here the plus and minus signs in front of Δm^2_{32} correspond to the normal mass hierarchy ($m_1 < m_2 < m_3$) and inverted mass hierarchy ($m_3 < m_1 < m_2$) case. Since the Standard Model predicts massless neutrinos, it is clear that we should extend the Standard Model to accommodate non-vanishing neutrino masses. Among all possible mechanisms, the seesaw mechanism [8] is a very interesting and elegant one to explain the light neutrino masses. In the framework of the seesaw mechanism, the light left-handed neutrino masses can well be understood by introducing heavy right-handed Majorana neutrinos, and the light neutrino mass matrix is given by

$$M_\nu = M_D M_R^{-1} M_D^T, \qquad (1)$$

where M_D and M_R are the Dirac and Majorana mass matrices, respectively. The typical mass scales of M_R are $10^{14} \sim 10^{16}$ GeV.

Since the Yukawa structures are not well constrained up to now, to identify the structure of the neutrino mass matrix is one of the main objects in neutrino physics. An interesting and natural way to study the Yukawa coupling and find the underlying physics is the flavor symmetry. Among all possible flavor symmetries, discrete non-Abelian groups such as S_3 [9], A_4 [10] and so on have attracted a lot of attention. Note that most of these models rely on the basis of their group representation, and contain more theoretical parameters than the observables.

The permutation group S_4, which is formed by the 4! permutations, totally contains 24 group elements which belong to five conjugate classes. Therefore it has five irreducible representations (*reps*). Among these irreducible *reps*, there are two one-dimensional ($\underline{\mathbf{1}}_S$ and $\underline{\mathbf{1}}_A$), one two-dimensional ($\underline{\mathbf{2}}$) and two three-dimensional ($\underline{\mathbf{3}}_\alpha$ and $\underline{\mathbf{3}}_\beta$) *reps*. Here the '$S$' and '$A$' mean symmetric and

E-mail address: zhanghe@mail.ihep.ac.cn.

0370-2693/$ – see front matter
doi:10.1016/j.physletb.2007.09.003

Table 1
The character table of S_4. Here N stands for the number of elements in class C_i, and C denotes the cycle structure of each class

Class	N	C	$\underline{\mathbf{1}}_S$	$\underline{\mathbf{1}}_A$	$\underline{\mathbf{2}}$	$\underline{\mathbf{3}}_\alpha$	$\underline{\mathbf{3}}_\beta$
C_1	1	1^4	1	1	2	3	3
C_2	6	21^2	1	−1	0	1	−1
C_3	8	31	1	1	−1	0	0
C_4	6	4	1	−1	0	−1	1
C_5	3	2^2	1	1	2	−1	−1

antisymmetric *reps*, respectively. The character table of S_4 is given in Table 1. In respect that the particles in our model are all assigned to the $\underline{\mathbf{1}}_S$, $\underline{\mathbf{2}}$ and $\underline{\mathbf{3}}_\alpha$, we list the relevant representation matrices in Appendix A.

The S_4 models for fermion masses have been discussed by several authors in Refs. [11,12]. In this note, we adopt the $S_4 \otimes Z_2$ flavor symmetry and assign all the charged leptons, neutrinos and Higgs into the irreducible Yamanouchi bases [13]. Such a compact scheme contains only a few parameters, and therefore it can be examined quite well in the future experiments.

In the following, we will present the main contents of our model in Section 2. Some detailed analytical and numerical analyses will be given in Section 3. In Section 4, the S_4 invariant Higgs potential are discussed. Finally, a brief summary is given in Section 5.

2. Particle assignment

In our model, all the leptons are assigned to the $\underline{\mathbf{3}}_\alpha$ *rep* of S_4, but their Z_2 charges are different. Under the group

$$G = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes S_4 \otimes Z_2, \tag{2}$$

the lepton contents in our model are placed as

$$\ell_L \sim (1,2,-1)(\underline{\mathbf{3}}_\alpha,+), \qquad e_R \sim (1,1,-2)(\underline{\mathbf{3}}_\alpha,+), \qquad \nu_R \sim (1,1,0)(\underline{\mathbf{3}}_\alpha,-), \tag{3}$$

where the plus or minus sign denotes the reflection properties under Z_2, i.e., $\Psi \to \pm\Psi$. The Higgs scalars in our model are placed as follows

$$H_u \sim (1,2,-1)(\underline{\mathbf{1}}_S,+), \qquad H_d \sim (1,2,-1)(\underline{\mathbf{1}}_S,-), \qquad \chi \sim (1,2,-1)(\underline{\mathbf{3}}_\alpha,-), \qquad \phi \sim (1,2,-1)(\underline{\mathbf{2}},+). \tag{4}$$

Here the Z_2 symmetry guarantees that the lepton doublet couples to (e_R, ϕ_i) and (ν_R, χ_i), respectively. It will be shown in Section 4 that the masses of Higgs scalars are almost unconstrained in this model, thus we can choose their masses at some high energy scales in order to avoid the tree level flavor charging neutral currents. Note that we do not introduce any $SU(2)_L$ singlet or triplet Higgs [14]. Hence the heavy right-handed neutrino masses are exactly degenerate.

By using the group algebra given in Appendix A, we write down the $S_4 \otimes Z_2$ invariant Yukawa couplings:

$$-\mathcal{L}_Y = \alpha_e \overline{\ell_{iL}} e_{iR} \tilde{H}_u + \beta_e f_{ijk} \overline{\ell_{iL}} e_{jR} \tilde{\phi}_k + \alpha_\nu \overline{\ell_{iL}} \nu_{iR} H_d + \beta_\nu g_{ijk} \overline{\ell_{iL}} \nu_{jR} \chi_k + \frac{1}{2} M \overline{\nu_{iR}^c} \nu_{iR} + \text{h.c.}, \tag{5}$$

where $\tilde{H}_u = i\tau_2 H_u^*$ and $\tilde{\phi} = i\tau_2 \phi^*$. The last term in Eq. (5) is the bare Majorana mass term with M being the typical mass scale of the heavy right-handed neutrinos. The coefficients $\alpha_{e,\nu}$ and $\beta_{e,\nu}$ are in general complex parameters. However, since their phases are all global phases, there will be no Dirac CP violating phases in the MNS matrix and only the Majorana phases can be accommodated up to now.[1] The structures of the traceless matrices f and g can be obtained from the CG coefficients of S_4. Following the CG coefficient tables given in Ref. [13], we arrive at

$$f_{ij1} = \begin{pmatrix} 0 & \sqrt{2} & 0 \\ \sqrt{2} & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \qquad f_{ij2} = \begin{pmatrix} 0 & 0 & \sqrt{2} \\ 0 & 0 & -1 \\ \sqrt{2} & -1 & 0 \end{pmatrix}, \tag{6}$$

and

$$g_{ij1} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \qquad g_{ij2} = \begin{pmatrix} 0 & -1 & 0 \\ -1 & \sqrt{2} & 0 \\ 0 & 0 & -\sqrt{2} \end{pmatrix}, \qquad g_{ij3} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & -\sqrt{2} \\ -1 & -\sqrt{2} & 0 \end{pmatrix}, \tag{7}$$

where all the coefficient matrices are symmetric and traceless. Hence both the charged lepton and neutrino mass matrices are symmetric.

[1] Here we do not consider the spontaneous CP violation [15].