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Attractors with vanishing central charge

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Abstract

We consider the attractor equations of particular $\mathcal{N}=2$, $d=4$ supergravity models whose vector multiplets' scalar manifold is endowed with homogeneous symmetric cubic special Kähler geometry, namely of the so-called st^2 and stu models. In this framework, we derive explicit expressions for the critical moduli corresponding to non-BPS attractors with vanishing $\mathcal{N}=2$ central charge. Such formulæ hold for a generic black hole charge configuration, and they are obtained without formulating any *ad hoc* simplifying assumption. We find that such attractors are related to the $\frac{1}{2}$-BPS ones by complex conjugation of some moduli. By uplifting to $\mathcal{N}=8$, $d=4$ supergravity, we give an interpretation of such a relation as an exchange of two of the four eigenvalues of the $\mathcal{N}=8$ central charge matrix Z_{AB}. We also consider non-BPS attractors with non-vanishing $\mathcal{Z}$; for peculiar charge configurations, we derive solutions violating the ansatz usually formulated in literature. Finally, by group-theoretical considerations we relate Cayley's hyperdeterminant (the invariant of the stu model) to the invariants of the st^2 and of the so-called t^3 model.

1. Introduction

The attractor mechanism in extremal black holes (BHs) [1–5] has recently been investigated in depth, and various advances along such a line of research has been performed [6–39].

The horizon geometry of extremal black holes in $d=4$ space–time dimensions is the direct product of two spaces with non-vanishing, constant (and opposite) curvature, namely it is the Bertotti–Robinson geometry [40–42]:

$$AdS_2 \times S^2 = \frac{SO(1,2)}{SO(1,1)} \times \frac{SO(3)}{SO(2)}. \tag{1.1}$$

In the framework of $\mathcal{N}=2$, $d=4$ supergravity, such an horizon geometry is associated to the maximal $\mathcal{N}=2$ supersymmetry algebra $\mathfrak{psu}(1,1|2)$, which is an interesting example of superalgebra containing not Poincaré nor semisimple groups, but direct products of simple groups as maximal bosonic subalgebra. Indeed, in this case the maximal bosonic subalgebra is $\mathfrak{so}(1,2) \oplus \mathfrak{su}(2)$ (with related maximal spin bosonic subalgebra $\mathfrak{su}(1,1) \oplus \mathfrak{su}(2)$), matching the corresponding bosonic isometry group of the Bertotti–Robinson metric (1.1).

In this context, the attractor configurations of the scalars at the event horizon of the BH have been recognized to fall into three distinct classes ([17,18,21]; see also [43] for a recent review):

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1. the $\frac{1}{2}$-BPS class, known since [1–5], preserving four supersymmetries out of the eight pertaining to the asymptotical Minkowski space–time background;
2. the non-BPS class with non-vanishing $\mathcal{N}=2$ central charge at the horizon, which does not preserve any supersymmetry at all;
3. the non-BPS class with vanishing $\mathcal{N}=2$ central charge at the horizon; thus, for this class the complete breakdown of supersymmetry at the BH event horizon is associated with the lack of central extension of $\mathfrak{psu}(1,1|2)$.

In this work we address the issue of the explicit determination of the non-BPS scalar configuration with vanishing $\mathcal{N}=2$ central charge, in the framework of peculiar $\mathcal{N}=2$, $d=4$ supergravities coupled to $n_V=2$ and 3 vector multiplets, namely for the so-called st^2 and stu models [21].

Such models belong to the broad class of the so-called d-geometries ([44]; see Section 2 for elucidation), whose explicit $\frac{1}{2}$-BPS attractors, for generic BH charges and generic n_V, are known after [45] (the *stu* model was previously investigated in [46]). Recently, in [10] the d-geometries with generic n_V were reconsidered, and the non-BPS attractors with non-vanishing central charge were explicitly determined for a peculiar choice of BH charges.

In our investigation, we find that, for a general charge configuration, the non-supersymmetric attractors with vanishing $\mathcal{N}=2$ central charge always violate the ansatz used in [10]. Furthermore, due to the high symmetry of the scalar geometries analyzed, they turn out to be intimately related to the $\frac{1}{2}$-BPS attractors.

The plan of this work is as follows.

In Section 2 we briefly recall the foundations of the special Kähler (SK) geometry endowing the vector multiplets' scalar manifold of $\mathcal{N}=2$, $d=4$ supergravity, focussing on the so-called SK d-geometries, and limiting ourselves to the sole quantities needed in the subsequent treatment. Section 3 is devoted to the st^2 model, the simplest symmetric model in which non-supersymmetric attractors with vanishing central charge appear; the violation of the ansatz of [10] is pointed out, and the explicit form of such attractors, along with the relation with the well-known $\frac{1}{2}$-BPS ones, is derived. As a byproduct of our approach, we also obtain a one-parameter family of non-BPS attractors with non-vanishing central charge which violate the ansatz of [10], showing that it actually implies some loss of generality in the context of SK d-geometries. In Section 4 we perform an analogous analysis in the *stu* model [23,46,47]. We derive the explicit expression of non-supersymmetric attractors with vanishing central charge, and elucidate how they related with the supersymmetric ones. Finally, in Section 5 we relate, by simple group-theoretical considerations, Cayley's hyperdeterminant to the quartic (and unique) invariants of the U-duality groups of the models t^3 [32] and st^2. Concluding remarks and an outlook can be found in the final Section 6.

2. Special Kähler geometry

In this section we briefly recall some notions of the special Kähler (SK) geometry underlying the vector multiplets' scalar manifold of $\mathcal{N}=2$, $d=4$ supergravity coupled to n_V vector multiplets. Our treatment is far from exhaustive, as we introduce only the quantities needed in the subsequent computations (for the notation, explanation and extensive treatment, see e.g. [48] and references therein).

Once a holomorphic prepotential function $F(X)$ of the sections X^Λ ($\Lambda=0,1,\dots,n_V$) is given, one can derive all the fundamental quantities in the framework of SK geometry. The Kähler potential and the corresponding moduli space metric are found to be

$$K=-\ln\left[i\left(\bar{X}^\Lambda\partial_\Lambda F-X^\Lambda\overline{\partial_\Lambda F}\right)\right],\qquad g_{i\bar{j}}=\partial_i\partial_{\bar{j}}K, \tag{2.1}$$

where the indices i and $\bar{j}$ refer to the moduli z^i and $\bar{z}^{\bar{j}}$ ($i,\bar{j}=1,\dots,n_V$ throughout), respectively. The covariantly holomorphic $\mathcal{N}=2$ central charge function

$$\mathcal{Z}(z,\bar{z},p,q)=e^{\frac{1}{2}K(z,\bar{z})}W(z,p,q) \tag{2.2}$$

is given in terms of the holomorphic superpotential

$$W(z,p,q)=q_\Lambda X^\Lambda-p^\Lambda\partial_\Lambda F, \tag{2.3}$$

and it can be used to calculate the so-called BH effective potential [5]

$$V_{\mathrm{BH}}=e^K\left[g^{i\bar{j}}(\mathcal{D}_iW)\bar{\mathcal{D}}_{\bar{j}}\bar{W}+W\bar{W}\right]=|\mathcal{Z}|^2+g^{i\bar{j}}(\mathcal{D}_i\mathcal{Z})\bar{\mathcal{D}}_{\bar{j}}\bar{\mathcal{Z}}, \tag{2.4}$$

where $\mathcal{D}_i=\partial_i+\frac{p}{2}\partial_iK$ denotes the Kähler-covariant derivative acting on an object with holomorphic Kähler weight p.

The attractor mechanism in extremal BHs [1–5] yields that at the BH event horizon the moduli are stabilized in terms of the electric q_0, q_i and magnetic charges p^0, p^i of the BH as they are critical points of V_{BH}, i.e. they are solutions of the Attractor Equations (AEs) given by the criticality conditions of V_{BH}. Through the fundamental relations characterizing the special Kähler