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Physics Letters B 651 (2007) 352–356

PHYSICS LETTERS B

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Constraints on the DGP Universe using observational Hubble parameter

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Received 6 January 2007; received in revised form 28 March 2007; accepted 17 June 2007

Available online 28 June 2007

Editor: A. Ringwald

Abstract

In this work, we use observations of the Hubble parameter from the differential ages of passively evolving galaxies and the recent detection of the Baryon Acoustic Oscillations (BAO) at $z_1 = 0.35$ to constrain the Dvali–Gabadadze–Porrati (DGP) Universe. For the case with a curvature term, we set a prior $h = 0.73 \pm 0.03$ and the best-fit values suggest a spatially closed Universe. For a flat Universe, we set h free and we get consistent results with other recent analyses.

PACS: 98.80.Cq; 98.80.Es; 04.50.+h; 95.36.+x

1. Introduction

Observations of Wilkinson Microwave Anisotropy Probe (WMAP) [1], the type Ia Supernova (SN Ia) [2,3] and Sloan Digital Sky Survey (SDSS) [4,5] support an accelerated expanding Universe. Many cosmological models have been constructed to explain such a cosmology. Most of them concentrate on the dark energy term with a negative pressure, within the usual gravitation theory.

The observed accelerated expansion of the Universe is perhaps due to some unknown physical processes involving modifications of gravitation theory. Such modifications are usually related to the possible existence of extra dimensions, giving rise to the so-called braneworld cosmology. The braneworld cosmology is an example which excludes the dark energy term by modifying the gravitation theory [6–9]. One interesting braneworld cosmological model is the one proposed by Dvali et al., which is usually called the Dvali–Gabadadze–Porrati (DGP) braneworld [10–12]. For scales below a crossover radius r_c, the gravitational force experienced by two punctual sources is the usual 4-dimensional $1/r^2$ force whereas for scales larger than r_c the gravitational force follows the 5-dimensional $1/r^3$ behavior.

Although the theoretical consistency and especially its self-accelerating solution are still waiting for confirming [13,14], the DGP models have been successfully tested from the observations. Deffayet et al. discussed observational constraints from the Cosmic Microwave Background (CMB) and SN Ia [15]. Jain et al. presented a constraint from the viewpoint of gravitational lenses [16]. Alcaniz et al. used the estimated ages of high-z objects to constrain the cosmological parameters [17]. The Chandara measurements of the X-ray gas mass fraction in galaxy clusters were used to do a combinational analysis with other cosmological probes [18]. Pires et al. tested the viability of DGP scenarios from the cosmological time measurements, i.e., recent estimates of the total age of the Universe and observations of the lookback time to galaxy clusters at intermediary and high redshifts [19]. Guo et al. constrained the DGP model from recent supernova observations and BAO [20]. Zhu and Alcaniz did the similar work using SN Ia [21]. See [22,23] for more corresponding comments on the DGP Universe.

In this work, we examine the DGP Universe using the observational $H(z)$ data (sometimes we call them OHD for simplicity) [24,25]. The observational $H(z)$ data are related to the differential ages of the oldest galaxies, the derivative of redshift z with respect to the cosmic time t (i.e., $\mathrm{d}z/\mathrm{d}t$) [25]. A determina-

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doi:10.1016/j.physletb.2007.06.053

tion of $\mathrm{d}z/\mathrm{d}t$ provides a measurement of the Hubble parameter, which can be used as an effective cosmological probe. In addition, we do the combinational analysis using data of the size of the Baryonic Acoustic Oscillations (BAO) peak detected in the large-scale correlation function of luminous red galaxies from the Sloan Digital Sky Survey (SDSS) [26]. For a Universe with a curvature term, a prior for the dimensionless Hubble constant $h = 0.73 \pm 0.03$ is taken from the combinational WMAP three-year estimate [1]. And we find that the best-fit values for both two cases suggest a closed Universe. For a flat DGP Universe, we set h free and get the results consistent with other independent analyses. The values of the current deceleration parameter, the transition redshift at which the Universe switches from deceleration to acceleration and the current value of the effective equation of state are discussed too.

This Letter is organized as follows: In Section 2, we briefly review the DGP Universe. In Section 3, we introduce the observational $H(z)$ data and the BAO data. In Section 4, we present the constraints on the DGP Universe. Discussions and conclusions are given in Section 5.

2. Overview of the DGP Universe

The DGP theory has an important parameter r_c which is the crossover radius where the theory changes between a region that is effectively 4-dimensional to what is fully 5-dimensional. It is defined as

$$r_c = \frac{M_{\mathrm{Pl}}}{2M_5^3}, \tag{1}$$

where M_{Pl} is the Planck mass and M_5 is the 5-dimensional reduced Planck mass. In the DGP Universe, the modified Friedmann equation due to the presence of an infinite-volume extra dimension reads [15,28]

$$H^2 = \left[\sqrt{\frac{\rho}{3M_{\mathrm{Pl}}^2} + \frac{1}{4r_c^2}} + \frac{1}{2r_c}\right]^2 - \frac{k}{a(t)^2}, \tag{2}$$

where H is the Hubble parameter, ρ is the energy density of the cosmic fluid and $k = 0, \pm 1$ is the spatial curvature parameter.

If we use the definition

$$\Omega_{r_c} = \frac{1}{4r_c^2 H_0^2}, \tag{3}$$

the Hubble parameter can be rewritten as

$$H(z)^2/H_0^2 = \Omega_k(1+z)^2 + \left[\sqrt{\Omega_{r_c}} + \sqrt{\Omega_{r_c} + \Omega_m(1+z)^3}\right]^2, \tag{4}$$

where z is the redshift, $H_0 = 100h\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ is the current value of the Hubble parameter, Ω_m and Ω_k are the matter and curvature density parameters, respectively.

And we can get this relation from the above equation by setting $z = 0$,

$$\Omega_k + \left[\sqrt{\Omega_{r_c}} + \sqrt{\Omega_{r_c} + \Omega_m}\right]^2 = 1. \tag{5}$$

The current value of the deceleration parameter $q = -\ddot{a}/aH^2$ takes the form [20]

$$q_0 = \left(\frac{1}{2}\Omega_m - \Omega_{r_c}\right)\left(\frac{\sqrt{\Omega_{r_c}}}{\sqrt{\Omega_m + \Omega_{r_c}}} + 1\right) - \sqrt{\Omega_{r_c}^2 + \Omega_m \Omega_{r_c}}. \tag{6}$$

For a flat Universe with $\Omega_k = 0$, Eq. (5) reduces to $\Omega_{r_c} = (1 - \Omega_m)^2/4$, from which we get $0 \leqslant \Omega_{r_c} \leqslant 0.25$ for $0 \leqslant \Omega_m \leqslant 1$. The current value of the deceleration parameter can be written as

$$q_0 = -1 + \frac{3}{2}\frac{\Omega_m}{\sqrt{\Omega_m + \Omega_{r_c}}}. \tag{7}$$

If we define $s = \Omega_{r_c}/\Omega_m$, the transition redshift z_{tr} at which the Universe switches from deceleration to acceleration can be expressed as [23]

$$z_{\mathrm{tr}} = -1 + 2s^{1/3}. \tag{8}$$

Also, we can derive the DGP Universe expressed in Eq. (4) using the time-dependent effective equation of state [23]

$$\omega_{\mathrm{eff}}(z) = -1 + \frac{1}{2}\frac{(1+z)^3}{s + (1+z)^3 + \sqrt{s}\sqrt{s + (1+z)^3}}. \tag{9}$$

It is clear that $\omega_{\mathrm{eff}} \to 0.5$ at $z \to \infty$. The current value of $\omega_{\mathrm{eff}0}$ depends on s, and is always larger than -1.

3. The observational $H(z)$ data set and BAO

3.1. The observational $H(z)$ data

The Hubble parameter $H(z)$ depends on the differential age of the Universe in this form

$$H(z) = -\frac{1}{1+z}\frac{\mathrm{d}z}{\mathrm{d}t}, \tag{10}$$

which provides a direct measurement for $H(z)$ through a determination of $\mathrm{d}z/\mathrm{d}t$. By using the differential ages of passively evolving galaxies determined from the Gemini Deep Deep Survey (GDDS) [29] and archival data [30–33], Simon et al. determined a set of observational $H(z)$ data in the range $0 \lesssim z \lesssim 1.8$ and used them to constrain the dark energy potential and its redshift dependence [34]. Using this data set, one can constrain parameters of various cosmological models. Yi and Zhang first used them to analyze the holographic dark energy models in which the parameter c plays a significant role [24]. The cases with $c = 0.6, 1.0, 1.4$ and setting c free are discussed in detail and the results are consistent with others. Samushia and Ratra used this data set to constrain the ΛCDM, XCDM and ϕCDM models [35] and Wei and Zhang analyzed a series of other cosmological models with interaction between dark matter and dark energy [36]. But as pointed out by Wei and Zhang, the data point near $z \sim 1.5$ derives from the main trend seriously and dip down sharply [36]. We will omit this point in later discussion.